Investigation of an analytical Aberration Description for Alvarez-Lohmann type Phase Plates

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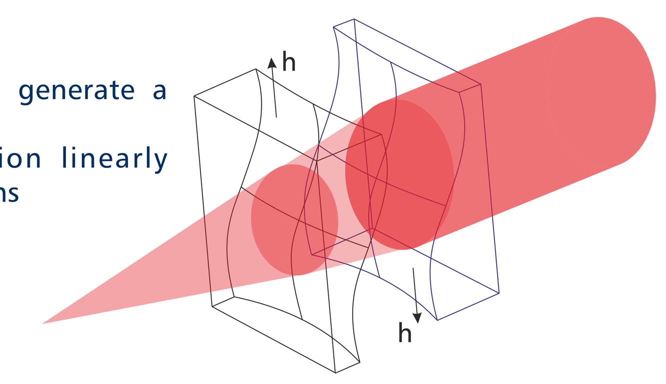
Motivation - Alvarez Lohmann Lenses

Alvarez-Lohmann Lenses [1,2]
- two cubic functions in combination generate a parabolic function
- curvature of the parabolic function linearly dependent on lateral shift of the functions



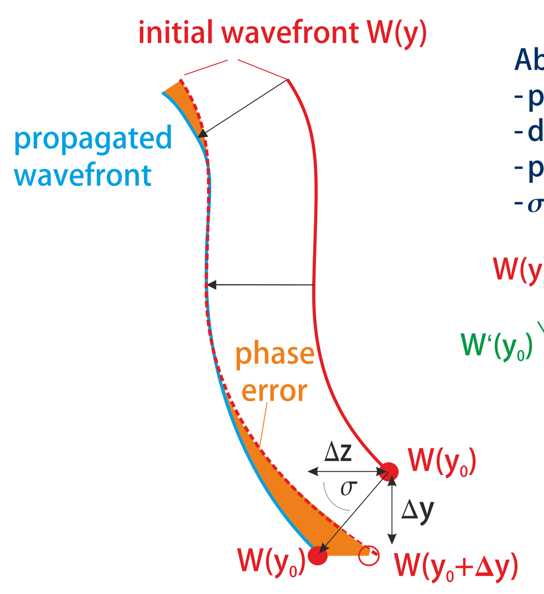
$$W(x,y) = A(x^2 y + \frac{y^3}{3})$$
$$W(x,y-h) - W(x,y+h)$$
$$= A(x^2(y-h) + \frac{(y-h)^3}{3}) - A(x^2(y+h) + \frac{(y+h)^3}{3})$$
$$= -2hA(x^2+y^2) - \frac{2Ah^3}{3}$$

Problem:
- model based on thin element approximation
- distance between elements and element thickness induce aberrations to the system [3]

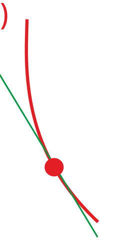
Goal:
- analytical approximation of induced aberrations
- comparison of the analytical model with other models:
  - -> commercial ray tracing software
  - -> Fresnel wave propagation (propagator: $\exp(1j\ 2\pi\Delta z\sqrt{\frac{1}{\lambda^2} - f_x^2 - f_y^2})$)

Approximations:
- thin phase elements (DOEs) with distances in between
- near paraxial region

Simplified Propagation Model



Aberration Approximation[4]:
- phase value $W(y_0)$ is moved to position $y_0+\Delta y$ at Δz
- difference $W(y_0+\Delta y)$-$W(y_0)$ is the phase error
- paraxial approximation of Δy $\Delta y = \Delta z * \sigma$
- σ is calculated by local grating periods



$$W'(y_0) = \frac{2\pi n}{mm}$$
$$\frac{W'(y_0)}{2\pi} = \frac{n \text{ line pairs}}{mm}$$
$$\sigma \approx \sin\sigma = \frac{\lambda}{2\pi} W'(y_0)$$
$$W(y_0+\Delta y) = W(y_0 + \Delta z \frac{\lambda}{2\pi} W'(y_0))$$

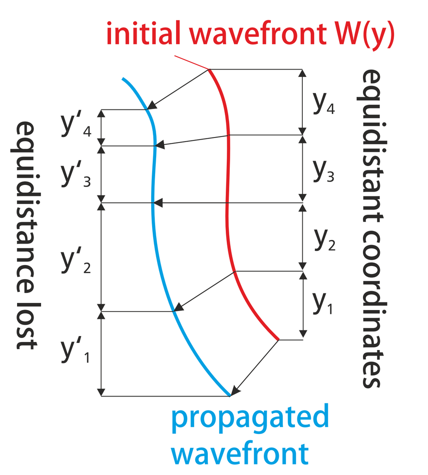
- different angles of incidence can be considered in the lateral offset Δy
- an expansion to 3D functions is acheived by partial derivatives

$$W(x_0+\Delta x, y_0+\Delta y) = W(x_0 + \Delta z(\frac{\lambda}{2\pi} W_x'(x_0,y_0)+\alpha),\ y_0 + \Delta z(\frac{\lambda}{2\pi} W_y'(x_0,y_0)+\beta))$$

Application to Alvarez-Lohmann lenses for $\alpha=\beta=0$

$$W(x_0+\Delta x, y_0+\Delta y) = A((x_0+\Delta x)^2(y_0+\Delta y) + \frac{(y_0+\Delta y)^3}{3})$$
$$= A((x_0 + \Delta z(\frac{\lambda}{2\pi}(2Ax_0y_0)))^2(y_0 + \Delta z(\frac{\lambda}{2\pi}A(x_0^2+y_0^2))) + \frac{(y_0+\Delta z(\frac{\lambda}{2\pi}A(x_0^2+y_0^2)))^3}{3})$$
$$= A((x_0^2 y_0) + \frac{y_0^3}{3}) + \Delta z\frac{\lambda}{2\pi}A^2(x_0^4 + 6x_0^2y_0^2 + y_0^4)$$
$$+ (\Delta z\frac{\lambda}{2\pi})^2 A^3(5x_0^4y_0 + 10x_0^2y_0^3 + y_0^5) + (\Delta z\frac{\lambda}{2\pi})^3 A^4(\frac{x_0^6}{3} + 5x_0^4y_0^2 + 5x_0^2y_0^4 + \frac{y_0^6}{3})$$

Coordinate system deformations - a source of residual errors



- equidistance of coordinate system is lost through propagation
- deformation is neglected by using:

$$W_{error}(x,y) = \frac{W(x',y') - W(x,y)}{2}$$

- residual error is composed of shift and deformation
- the residual error is estimated by comparing the analytical approximation with wave optical propagations and ray tracing

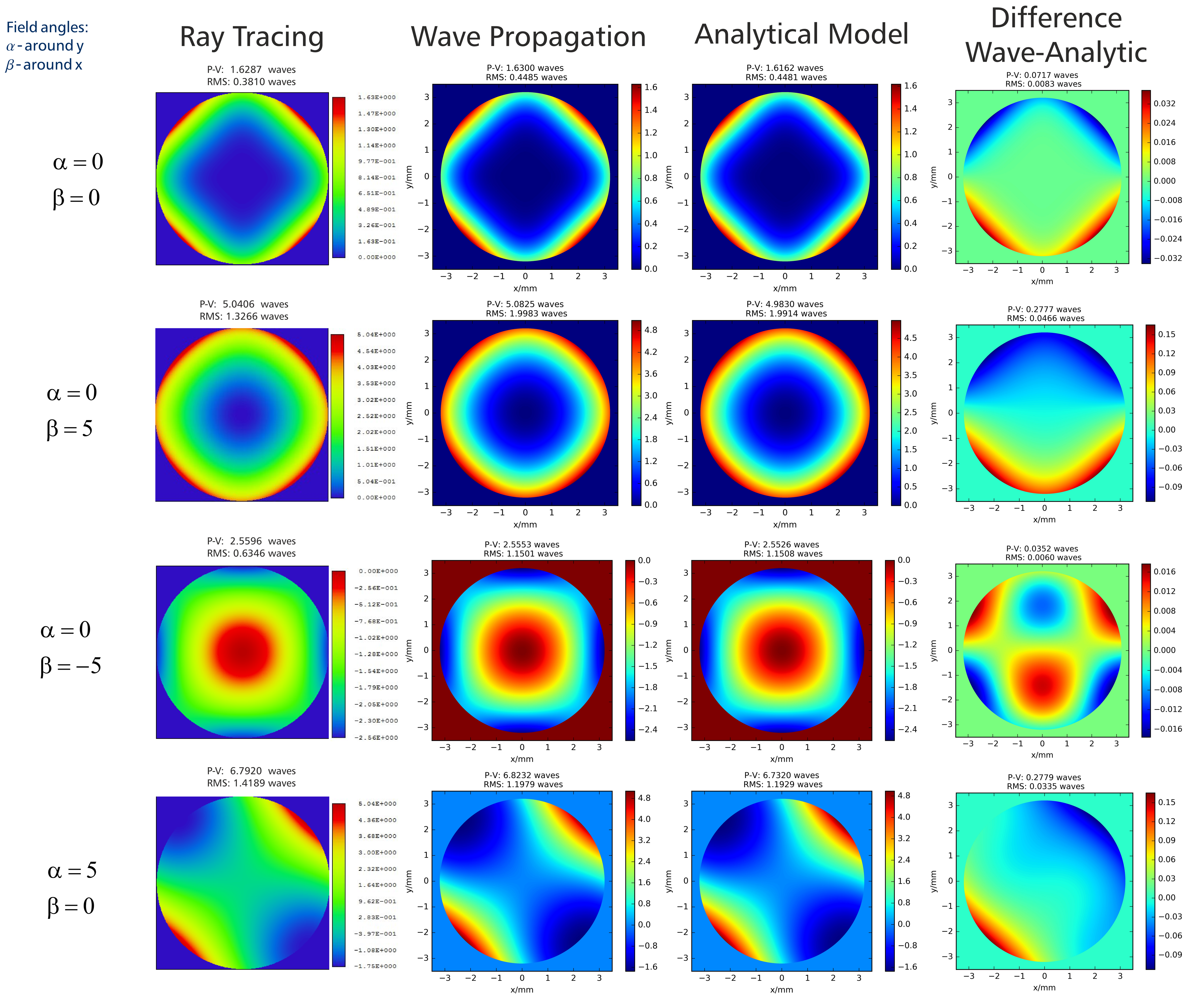
Conclusion & Outlook

- an analytical approximation for aberrations in Alvarez-Lohmann type phase plates was derived from [4] and compared to ray tracing and wave propagation methods
- due to the paraxial character of the model its accuracy decreases for large angles

- next steps are to investigate an analytical weighting of the error function for different field points and check the accuracy of the model for refractive elements
- analyzing the weighting factor „2" in the error description

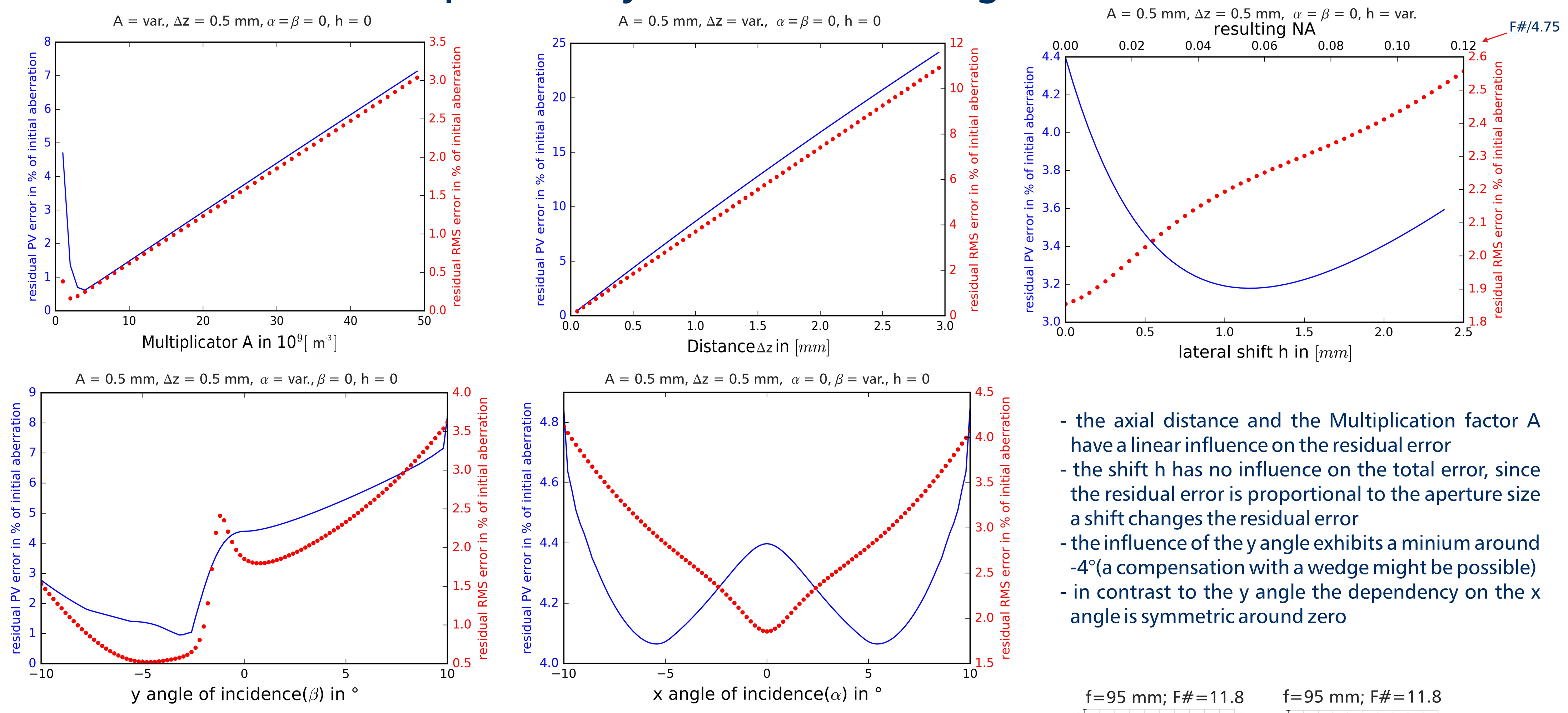
Aberrations: Ray Tracing, Wave Propagation and Analytical Model

Field angles:
α - around y
β - around x



System Data: axial distance $\Delta z = 0.5$ mm; object at infinity; $A = 30*10^9/m^3$ -> min. grating period 9 μm; h = 0

Residual Error - Dependency on distance, angle, shift and coefficient A

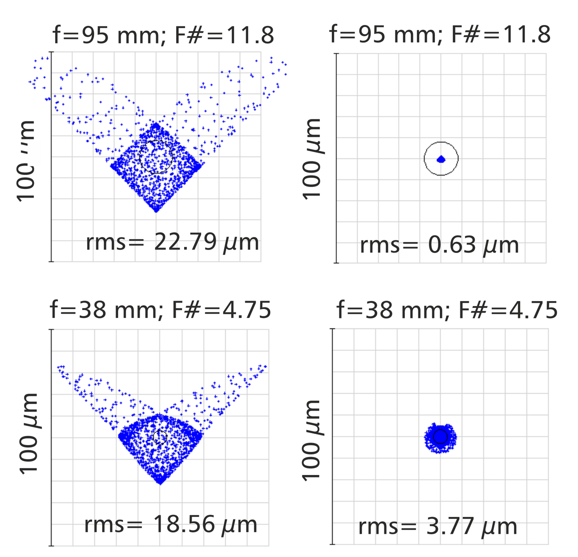


- the axial distance and the Multiplication factor A have a linear influence on the residual error
- the shift h has no influence on the total error, since the residual error is proportional to the aperture size a shift changes the residual error
- the influence of the y angle exhibits a minium around -4°(a compensation with a wedge might be possible)
- in contrast to the y angle the dependency on the x angle is symetric around zero

Error Compensation

- the analytical error function can be substracted from the description of the first phase element
- since each angle of incidence generates a different error function an analytical compensation over a range of view is not possible
- the compensation function is a compromise for all field positions

on axis spot diagrams for a system of Alvarez Lohmann DOEs
- element thickness 1mm, Δz = 0.5mm and ø = 8mm
- left: original element function
- right: error subtracted from first element function



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