Heterogeneous Multiple Bank Financing: Does it Reduce Inefficient Credit-Renegotiation Incidences?[*]

Christina E. Bannier[†]

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Abstract

Small and medium-sized firms often obtain capital via a mixture of relationship and arm's-length bank lending. We show that such heterogeneous multiple bank financing leads to a lower probability of inefficient credit foreclosure than both monopoly relationship lending and homogeneous multiple bank financing. Yet, in order to reduce hold-up and coordination-failure risk, the relationship bank's fraction of total firm debt must not become too large. For firms with intermediate expected profits, the probability of inefficient credit-renegotiation is shown to decrease along with the relationship bank's information precision. For firms with extremely high or extremely low expected returns, however, it increases.

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[†]Commerzbank Professor of Corporate Finance / SME Finance, Frankfurt School of Finance and Management, Department of Banking and Finance, Sonnemannstr. 9-11, 60314 Frankfurt, Germany, Phone: +49 69 154008 755, Fax: +49 69 154008 4755, E-mail: c.bannier@frankfurt-school.de

1 Introduction

This paper addresses the question of why many firms, particularly in European countries, borrow from several banks of which one often takes the position of a "relationship" lender, whereas the others act as "arm's-length" lenders. While early empirical studies on the number of bank relationships conclude that firms obtain a large fraction of their debt from mainly one institution (Petersen and Rajan, 1994; Harhoff and Körting, 1998), more recent work on both large and small- to medium-sized European companies finds that firms make use of a substantial number of bank relationships. Ongena and Smith (2000), for instance, show that in their sample of 1079 large firms from 20 European countries, more than 20% use eight or more banks, while less than 15% maintain single-bank relationships. For Germany, Brunner and Krahnen (2006) find an average number of 5.7 bank relationships per company, which is supported by Machauer and Weber (2001). Additionally, it has been shown for Portuguese data that even if firms borrow from only one bank when starting their business, a large fraction of companies substitutes single with multiple bank lending during the course of their lifes (Farinha and Santos, 2002).

The prevalence of heterogeneous multiple bank financing has received scant attention in the theoretical literature, however.[1] The early literature on financial intermediation mainly concentrated on either of the two types of bank financing: single relationship banking on the one hand and multiple bank lending via homogeneous lending institutions on the other. Regarding the characteristics of the two financing regimes, single banking has generally been seen as beneficial for opaque firms because it saves on monitoring costs (Diamond, 1984). Particularly young and small firms may find it difficult to credibly signal their quality in order to access the capital markets. A bank can solve this problem of information asymmetry by gathering information about the borrower prior to making the financing decision and by monitoring the firm over the course of the lending relationship. Increasing information precision along the duration of the relationship should allow a more efficient renegotiation of credit conditions, particularly for financially-distressed companies who, eventually, may find their credit constraints eased (Elsas and Krahnen, 1998). However, repeated borrowing gives the single bank an opportunity to threaten to cut future lending to the firm. Whenever information about the firm's credit quality cannot be credibly conveyed to other lending institutions, the firm will have to pay a lemon's premium if it tries to approach other banks. This hold-up problem gives the incumbent bank the chance to extract ex-post rents and makes single relationship banking particularly costly for highly-profitable firms (Sharpe, 1990; Rajan, 1992).[2]

Competition among banks limits a single bank's ability to extract rents ex-post. Additionally, it may protect the debtor against a sudden deterioration of the liquidity position of a single bank (Diamond, 1991).[3] Yet, multiple banks usually act as "arm's-

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[1] For an overview on general key characteristics of banking in Europe, see Schmidt (2001) or Brittain (2001).

[2] Ex-post rent extraction by the creditor may additionally distort entrepreneurial incentives ex-ante and therefore lead to an inefficient choice of investment projects (Von Thadden, 1992).

[3] Detragiache et al. (2000) consider the adverse selection problem that arises if a firm needs to borrow from non-relationship banks at the refinancing stage, because the relationship bank is unable to prolong

length lenders": as competition does not allow individual rent-extraction from the firm, they will not invest into costly borrower-monitoring and hence rely on less precise information than a relationship bank.[4] While this saves on banks' monitoring costs, it raises the risk of inefficient credit-renegotation decisions particularly for firms of lower credit quality. If firms obtain funds from several small banks, a coordinated foreclosure of credit by a significant fraction of lenders may force illiquid but otherwise solvent companies into default. Coordination "failures" of this type have been characterized as one of the main risks associated with a multiple lending regime (Morris and Shin, 2004).

Combining the two strands of the literature, Hubert and Schäfer (2002) compare the isolated effects of single relationship lending on the one hand with those of a multitude of small, homogeneous lending institutions on the other hand. They consider the case where short-term debt holders obtain noisy information about the borrower's quality and have to decide whether or not to prolong credit. Due to indivisibilities and the sunk-cost character of a large fraction of a firm's investment, a withdrawal of funds may be troublesome even for a relatively healthy firm, as refunding from new sources is costly to find on short notice. This gives rise to strategic complementarities in lenders' decisions: since credit withdrawal reduces the likelihood of the firm's success, lenders fear to be left alone, thereby resorting to the very preemptive action which undermines the funded business project. Yet, the authors can show that the cost of this coordination failure may still be lower than the cost arising from the renegotiating potential of a single relationship lender. This holds in particular for the case of low information asymmetry between the firm and its lenders. Generally, they find that the costs from a large monopolistic lender are highest for firms with bright business prospects since this allows the lender to appropriate substantial quasi-rents in the renegotiation process ex-post, while the risk of coordination failure is highest for firms with low expected returns. These results are comparable to Bolton and Scharfstein (1996) who discuss the optimal number of creditors from the perspective of ex-post bargaining problems and conclude that firms with high credit quality are better off with two creditors rather than one.

Among the first papers taking into account the coexistence of relationship and arm's-length bank lending in a theoretical model is the work by Elsas et al. (2004). The authors derive the optimal debt structure, i.e. the optimal degree of relationship versus arm's-length bank financing, from the trade-off between the bargaining power of the relationship bank and the risk of coordination failure from arm's-length banks. They find that firms with high expected cash-flows prefer a homogeneous multiple bank financing regime without a relationship bank, while firms with low expected profits or high asset specificity tend to be financed within a heterogeneous multiple banking system. However, the authors assume that there is no information asymmetry between the relationship bank and arm's-length banks. Yet, the higher precision of informa-

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credit due to own liquidity problems.

[4]In this sense, arm's-length banks offer a similar type of financing as the capital markets do. Since small and medium-sized European firms, in particular, are still shy to use the capital markets and prefer to receive funds from banks, we refer to arm's-length bank lenders in the model rather than bond holders. Only recently did German Mittelstand firms, for instance, open up to other forms of lending via the capital markets, as has been commented on by The Economist (November 2003): "Though starved of bank loans, German companies are reluctant to tap alternative sources of capital."

tion about the firm's credit quality has continuously been mentioned as the distinctive feature of relationship banking, so that the paper by Elsas et al. (2004) falls short of this aspect. The distinguishing characteristics between the two lender types in their study are simply the size of the banks' funds and the ability, respectively disability, to renegotiate credit.

This paper, in contrast, explicitly considers information asymmetries between relationship and arm's-length banks in a heterogeneous financing system.[5] We assume that the relationship bank is a large creditor by holding a substantial fraction of the firm's total debt and that she disposes of more precise information about the firm. Our study hence accounts for the particular importance that the degree of uncertainty in information about borrower quality has for credit decisions. Additionally, we presume that the relationship bank may renegotiate financing conditions, while arm's-length banks cannot. We then analyze the banks' interaction with regard to the efficiency of credit-refinancing decisions.[6] Furthermore, we compare the heterogeneous multiple bank financing regime to both the case of single relationship lending and homogeneous multiple lending. In this respect, we complement the earlier work by Hubert and Schäfer (2002) by a richer structure of lender types.

In our model, two different effects, pertaining to the different characteristics of the relationship bank, interplay and influence the results. First, due to her ability to renegotiate repayment conditions, the relationship bank may ease or tighten the firm's financial constraints by asking for a lower or higher repayment rate. While the former leads to a beneficial smoothing of the firm's budget constraints, the latter may be seen as a manifestation of the hold-up effect. Second, since the result of the renegotiation process between firm and relationship bank is observable to arm's-length banks, the relationship bank's actions allow for a signalling function that may help to coordinate the decisions of arm's-length banks. The coordination-effect gains momentum in the relationship bank's information precision, such that arm's-length banks will simply follow the relationship bank's action if she is known to possess completely precise information. Still, coordination may lead to both a more or less efficient outcome. This is due to the fact that a reduction in the charged repayment rate by the relationship bank is taken as a signal for a low firm quality, while an increase in the requested repayment rate signals high expected firm returns.

Our results are multi-faceted and differentiate between a system where the relationship bank disposes of very but not infinitely precise information and the limit where she possesses completely precise information about her borrower. If we consider that the bank's precision of information about the firm's credit quality rises with the duration of the relationship, we may assume that the former case is found in early stages, the latter in more mature stages of the lending relationship.

In the early stages, incidences of inefficient credit-renegotiation, i.e. of premature

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[5]In contrast to Carletti (2004), who analyzes how the number of bank relationships influences banks' monitoring incentives, we assume that only one of the banks truly acts as a relationship lender and, hence, may obtain more precise information about the firm than all other bank lenders.

[6]In a more general global games setting, the influence of large players on financial crises has been analyzed by Corsetti et al. (2001) and Bannier (2005). In contrast to these models, however, in our case the large player may additionally renegotiate the financial contract.

credit termination, decrease along with the relationship bank's precision of information for firms with intermediate expected profits. The net-effect of the relationship bank's renegotiation- and coordination-influence hence turns out to be beneficial and to strengthen in the bank's information precision. By guaranteeing access to finance even for firms of less than very high quality,[7] the relationship bank thus injects a degree of strategic solidity. On the downside, however, firms with extremely high or extremely low expected returns face a higher likelihood of inefficient credit termination the more precise the relationship bank's information becomes. For firms with extremely high expected returns, the relationship bank renegotiates an extremely high repayment rate, such that the adverse hold-up effect overcompensates any beneficial coordination effect. For firms with extremely low expected cash-flows, in contrast, the intertemporal-smoothing effect via a reduction of the renegotiated repayment by the relationship bank is not sufficiently strong to outweigh a coordination failure by arm's-length banks.

Comparing heterogeneous multiple bank financing to monopolistic relationship banking, we find that inefficient credit termination is always less likely under the first regime. This holds irrespective of the firm's expected quality. As an explanation, consider that a monopolistic relationship bank will always request the highest feasible repayment rate such that the hold-up problem is maximized. In a heterogeneous bank financing system, in contrast, the renegotiation-effect is limited as it affects only the relationship bank's (comparably smaller) fraction of firm debt. At the same time, an increase in the repayment requested by the relationship bank is perceived as a signal of high firm quality by arm's-length banks. The subsequent beneficial coordination-effect reduces the initial negative hold-up effect. However, a reduction in the repayment to the relationship bank will be interpreted as a signal of low firm quality, so that the risk of coordination-failure increases. Compared to homogeneous multiple bank financing, therefore, a heterogeneous regime only fares better if the renegotiation-effect is guaranteed to be relatively small, i.e. if the relationship bank's fraction of total firm debt and the charged repayment rate may not become too large.

In the mature stages of the relationship, when the relationship bank disposes of completely precise private information about the firm, our model shows that she always charges the maximum feasible repayment. Thus, the renegotiation-influence always gives rise to a hold-up problem, but its impact is limited due to the existence of arm's-length loans. Additionally, arm's-length banks will always follow the relationship bank's strategy as she is known to be perfectly informed about the firm. Heterogeneous bank financing therefore reduces the likelihood of inefficient credit termination compared to monopoly relationship banking. Yet, due to the maximum repayment request, the relationship bank is no longer able to signal the firm's quality via variations in the repayment rate. Therefore, when compared to a homogeneous multiple banking context, a heterogeneous system leads to more efficient credit-renegotiation only if the relationship bank's fraction of debt is relatively small, such that the hold-up effect is limited sufficiently strongly.

From the banks' point of view, a heterogeneous multiple bank financing system is advantageous as well, given the limitations with regard to the relationship bank's influence as mentioned above. Generally, banks benefit from the reduction of the borrower's de-

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[7]In the following, we refer to the expected firm profits and the firm's "quality" interchangeably.

fault probability. Additionally, we can show that the relationship bank earns higher expected profits from the credit relationship if the firm obtains loans from multiple arm's-length banks as well. As a reason, consider that in a heterogeneous bank financing system, apart from renegotiating the repayment rate, she may also try to coordinate arm's-length banks' behavior in her favor, thereby trying to maximize the amount of repayment while minimizing the probability of default at the same time.

Our results match a number of stylized facts established from empirical studies on relationship lending in a heterogeneous banking system. Elsas and Krahnen (1998) find that relationship banks, "housebanks" in their terminology, develop a truly distinct lending behavior despite competition from other banking institutions. Based on data from the German credit market, they show that relationship banks increase financing to borrowers of deteriorating, but still sustainable, quality. Similarly, Foglia et al. (1998), based on Italian data, conclude that the existence of relationship lending from one specific banking institution increases firms' general access to credit. Even some of our more detailed results are matched by empirical findings. In particular, D'Auria et al. (1999) concur that with regard to the pricing of loans, a privileged relationship to one particular lender may be preferable, i.e. leading to lower cost of capital for the firm, as long as the relationship bank's share of total borrowing does not get so large as to represent a virtual monopoly.

The model may also be extended into other directions. For instance, our results may be important for the optimal structure of lending syndicates, in which often one lead-bank coordinates the decisions of other syndicate members.[8] One further parallel may be drawn with respect to the role of banks in out-of-court restructuring of debt and the resolution of financial distress. James (1995, 1996) finds that bank participation in debt restructuring transactions facilitates public debt exchange offers.[9] However, bank concessions are usually contingent upon the successful completion of the public debt exchange offer.[10] This has been taken up in a theoretical examination by Pagratis (2005), who finds that the bank, by drawing on collective wisdom of the other lenders' restructuring decisions, injects a degree of strategic stability. For German firms facing distress, Brunner and Krahnen (2006) have found that bank pools may overcome coordination problems and increase the probability of workout success, provided that the number of pool members remains small. This may well point into the same direction as our results because a smaller number of pool members may be expected to lead to higher heterogeneity between banks. As our model shows, a sufficiently heterogeneous group of banks may reduce the probability of inefficient debt restructuring, since it enables efficient coordination at lowest (hold-up) costs.

We will proceed as follows. Section 2 describes the general model of heterogeneous multiple bank financing in which a firm obtains credit from a relationship bank and a multitude of arm's-length banks. Section 3 examines the characteristics of this financing system provided that the relationship bank disposes of finitely precise private information. The results are compared to a market where the relationship bank acts

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[8]I am very thankful to an anonymous referee for pointing out the relation to this topic.

[9]Likewise, Gilson, Kose and Lang (1990) point out that the probability of out-of-court restructuring is positively related to a firm's reliance on bank debt.

[10]This is in contrast to our model where the less-informed participants make their decisions contingent on the informed bank's action rather than the other way round.

as a monopolist and to a homogeneous multiple banking regime. Section 4 studies the limiting case where the relationship bank obtains private information of infinitely high precision. Again, a comparative statics analysis is followed by a comparison of different banking regimes. Section 5 concludes.

2   The Model

The model considers a simple economy with three types of risk-neutral agents:[11] a firm, a relationship bank and a continuum of (small) arm's-length banks.[12] The firm has no financial resources and intends to implement a risky project that requires an investment of 1. Both types of bank lenders are approached by the firm in order to raise short-term funds. Provided that initial financing decisions are successful, the project starts in $t = 1$ and matures in $t = 3$. At an intermediate stage, lenders have to decide whether or not to prolong credit, so that the project is threatened by early liquidation.[13] Early liquidation is inefficient if the project's quality is sufficiently high.

The bank financing system is heterogeneous in three respects. First, it is assumed that the relationship bank has been maintaining long-term relations with the firm and therefore disposes of more precise information about the firm than arm's-length banks. In this respect, we assume that while the project's quality θ is unobservable to any bank, the relationship bank observes a noisy private signal $x_R = \theta + \varepsilon$ with $\varepsilon \sim N(0, 1/\alpha)$, while small banks observe individual private signals $x_{AL_i} = \theta + \varepsilon_i$ with $\varepsilon_i \sim N(0, 1/\beta)$. It is assumed that ε and ε_i are independent of each other and of θ and that $\alpha \geq \beta$. Second, the relationship bank grants a loan of non-negligible size. In particular, her loan provides a fraction λ of total firm debt. Arm's-length banks, in contrast, grant loans that are negligibly small individually but amount to a combined proportion of $1 - \lambda$ of total firm debt.[14] Third, the relationship bank may renegotiate financing conditions, while arm's-length banks cannot. Arm's-length banks are therefore distinguished from the relationship bank by a smaller financial "size", a lower precision of information about the borrower's credit quality and the disability to renegotiate.

The model has the following time structure:

- In $t = 0$, the firm approaches the banks in order to raise capital for a risky

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[11]The assumption of risk-neutrality greatly helps to simplify equilibrium derivation and to focus on the pure interactions between banks without additional moral hazard concerns due to risk-aversion. Risk-neutrality should certainly be a reasonable characterization for well-diversified banks. With regard to the firm, it is arguably more difficult to defend the assumption of risk-neutrality. Still, several experimental studies have been able to show that agents often tend to display features of risk-neutrality rather than of risk-aversion (Neugebauer and Selten, 2006; Neugebauer and Perote, 2006).

[12]Empirical studies found that for medium-sized European firms, the number of banking relationships varies between 1 and 70 (Ongena and Smith, 2000). In the model, the assumption of a continuum of arm's length lenders is made for simplicity and does not qualitatively influence the results.

[13]In essence, the model considers short-term, revolving loans. Alternatively, we may think of longer-term loans that include covenants. These may allow the banks to foreclose their loans in certain circumstances. In order to keep the model as simple as possible, we assume all loans to have identical maturity.

[14]Due to their negligible financial mass, we refer to arm's-length banks also as "small" banks.

business project. The firm offers a repayment of $r(> 1)$ per unit of capital in $t = 3$. Provided that financing decisions have been successful, the firm invests in the project.[15]

- In $t = 1$, project quality θ is realized from a normal distribution $N(\mu_\theta, \sigma_\theta)$ and banks receive noisy private signals about the realized value of θ. Based on her private information, the relationship bank decides whether to renegotiate the repayment and extend credit or to foreclose the loan. The arm's-length banks observe her actions and decide whether to extend or withdraw their loans as well.[16] Early foreclosures deliver a liquidation value of $K < r$ per unit of capital for both types of lenders.

- In $t = 2$, project quality θ becomes uncovered. The firm makes her choice whether to invest additional effort V into the project and refinance the foreclosed fraction of loans or to terminate the project prematurely. In the latter case, the final liquidation value of the project is normalized to zero.

- In $t = 3$, if the project has not been terminated prematurely, the project's cash-flow is realized and equals θ. Credit is repaid with the firm holding the residual claim.[17]

All bargaining power is supposed to rest with the relationship bank. The repayment rate renegotiated by the relationship bank is referred to as r_R. Early liquidation value K may also be interpreted as the collateral that may be seized by the banks in the intermediate period. In order to keep the model as simple as possible, K is assumed to be the same for all banks, while in reality it would certainly be an additional negotiation parameter of the relationship bank. However, we consider that it is more costly for the firm to refinance a loan withdrawn by the relationship bank, W_R, than to refinance arm's-length banks' loans, i.e. $W_R \geq W_{AL} \geq r$. Note that W_R is the maximum feasible amount of repayment that the relationship bank can demand and hence puts an upper boundary on the hold-up cost per unit of relationship-credit. In order to simplify the analysis, effort V is normalized to zero. Finally, we assume $\sigma_\theta \to \infty$, i.e. we examine the limiting case where the variance of θ becomes very large, so that banks dispose of hardly any prior information about the project quality. This assumption should be justified particularly for small and medium-sized firms that are informationally opaque. Furthermore, since the focus of our study is on the banks' conditional beliefs after observing their noisy private signals, imposing an improper prior distribution on θ considerably simplifies the analysis without altering the core results.[18]

The equilibrium of the game can be obtained by solving the sequential model backwards. At the last stage of the game, the firm has to decide whether to refinance the withdrawn fraction of loans or to terminate the project altogether. If the project is terminated, the firm receives a payoff of zero. If the withdrawn money is refinanced, the firm receives

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[15]Note that we do not analyze the initial financing decisions by the banks. Our model is focussed solely on the credit continuation decision in $t = 1$.

[16]The relationship bank may therefore use her actions as a signal to the arm's-length banks.

[17]Note that the non-identical maturity of loans and firm project is not critical given a discount factor of zero, which we assume here.

[18]For a more detailed discussion of the improper prior assumption see Morris and Shin (2003).

payoff $\pi_F(\text{refinance}) = \theta - \lambda r_R - (1-\lambda)(1-l)r - (1-\lambda)lW_{AL}$ whenever the relationship bank decides to renegotiate the repayment and extends credit. Here, l denotes the proportion of small banks that foreclose their loans. If, instead, the relationship bank chooses to withdraw her loan prematurely, the firm's payoff from refinancing is given by $\pi_F(\text{refinance}) = \theta - \lambda W_R - (1-\lambda)(1-l)r - (1-\lambda)lW_{AL}$. Note that the firm's payoff from continuing the project and refinancing withdrawn credits decreases in the number of banks withdrawing their loans. In this respect, the model gives rise to strategic complementarities among banks' actions: the larger the number of banks that withdraw, the higher is the incentive of the individual bank to foreclose the loan, since it becomes less likely that the firm will decide to refinance and continue the project.

Assuming that it is optimal for the firm to employ a trigger strategy,[19] we find the following. If the relationship bank extends credit, the firm will terminate the project rather than refinance the withdrawn funds whenever the realized project value θ is lower than

$$\theta_1^* = \lambda r_R + (1-\lambda)r + (1-\lambda)(W_{AL} - r)l.$$

If, instead, the relationship bank chooses to foreclose the loan, the firm will terminate the project for values of θ lower than

$$\theta_2^* = \lambda W_R + (1-\lambda)r + (1-\lambda)(W_{AL} - r)l.$$

In order to interpret the equilibrium values θ_1^* and θ_2^*, it is useful to introduce two boundary values for θ. Let $\underline{\theta_1}$ be the threshold value of the project cash-flow that renders the firm indifferent between terminating the project and refinancing the withdrawn part of the loan, if all financiers (including the relationship bank) extend their loan, i.e. $\underline{\theta_1} = \lambda r_R + (1-\lambda)r$. Similarly, let $\overline{\theta_2}$ be the threshold value that makes the firm indifferent between terminating and refinancing if all financiers withdraw their loan, i.e. $\overline{\theta_2} = \lambda W_R + (1-\lambda)W_{AL}$. Hence, for $l \to 0$, θ_1^* converges to $\underline{\theta_1}$ from above, for $l \to 1$, θ_2^* converges to $\overline{\theta_2}$ from below.

Fig. 1 shows the structure of project cash-flows and the corresponding equilibrium outcomes. Whenever the realized return of the firm's project is below $\underline{\theta_1}$, the project
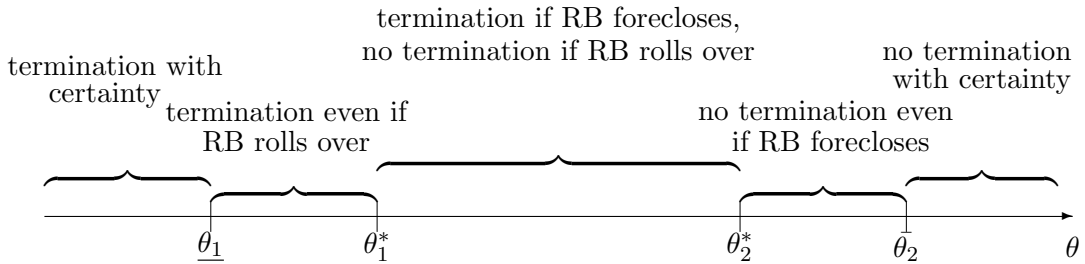

Figure 1: Range of project values

will be terminated with certainty, even if all small banks and the relationship bank (RB) extend their loans. For this range of very low project qualities, terminating the project is the efficient choice for the firm. If $\underline{\theta_1} < \theta < \theta_1^*$, the proportion of arm's-length banks optimally withdrawing their loans is still so high that the firm will decide not to continue the project even if the relationship bank renegotiates the repayment and rolls over the loan. However, terminating the project is not efficient any more, since it is not warranted by the project's quality: the firm would have proceeded with the project if only the proportion of withdrawn credit had been smaller. For $\theta_1^* < \theta < \theta_2^*$, the outcome depends entirely on the behavior of the relationship bank. If she decides to renegotiate and extend credit, the firm will choose to refinance the money withdrawn from the small banks. If, however, the relationship bank forecloses the loan, the firm will choose to terminate the project. Whenever the project quality is higher than θ_2^* but lower than $\overline{\theta_2}$, the firm will decide to proceed with the project even if the relationship bank chooses to foreclose the loan. For $\theta > \overline{\theta_2}$, the project cash-flow is so high that the project succeeds with certainty, no matter what the small banks and the relationship bank decide to do.[20]

In the following, we will derive the optimal strategies for the banks. Relying on trigger strategies, small banks are indifferent between foreclosing and extending their loans exactly after observing a private signal of x_{AL}^*. The proportion l of arm's-length banks withdrawing their money in $t = 1$ is then given by those who receive private information lower than x_{AL}^*, i.e. $l = \text{prob}(x_{AL} \leq x_{AL}^* | \theta)$.[21] The equilibrium values for the project cash-flow, θ_1^* and θ_2^*, are then resolved as

$$\theta_1^* = \lambda r_R + (1 - \lambda)r + (1 - \lambda)(W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_1^*)) \tag{1}$$

and

$$\theta_2^* = \lambda W_R + (1 - \lambda)r + (1 - \lambda)(W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_2^*)) \,, \tag{2}$$

where $\Phi(\cdot)$ denotes the cumulative normal distribution.[22] Note that threshold values θ_1^* and θ_2^* cannot be derived explicitly, but are given as implicit functions. Still, their features may be recognized from a graphical representation, see figure 2 as an example, or from simulations.

For x_{AL}^* to be a trigger for the arm's-length banks' decision of whether to withdraw or extend credit, both actions have to deliver the same expected payoff when a private signal of $x_{AL_i} = x_{AL}^*$ is observed, i.e.

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\begin{aligned}
\pi_{AL}(\text{foreclose}|x_{AL_i} = x_{AL}^*) &= \pi_{AL}(\text{extend}|x_{AL_i} = x_{AL}^*) \\
K &= r \cdot \text{prob}(\theta \geq \theta_2^* | x_{AL_i} = x_{AL}^*) \\
&\quad + r \cdot \text{prob}(\theta_1^* \leq \theta \leq \theta_2^* | x_{AL_i} = x_{AL}^*, x_R \geq x_R^*) \,.
\end{aligned}
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[20]The interval $[\underline{\theta_1}, \overline{\theta_2}]$ is the typical range where multiple equilibria occur in a homogeneous banking regime whenever the project value θ is common knowledge. Hence, in this interval both credit withdrawal and credit extension are self-sustaining equilibria that lead to (inefficient) project termination or (efficient) project continuation, respectively.

[21]Note that, due to the assumed independence of signal noise, the proportion of banks receiving private information lower than x_{AL}^* is equal to the probability with which an individual bank observes a signal lower than x_{AL}^*.

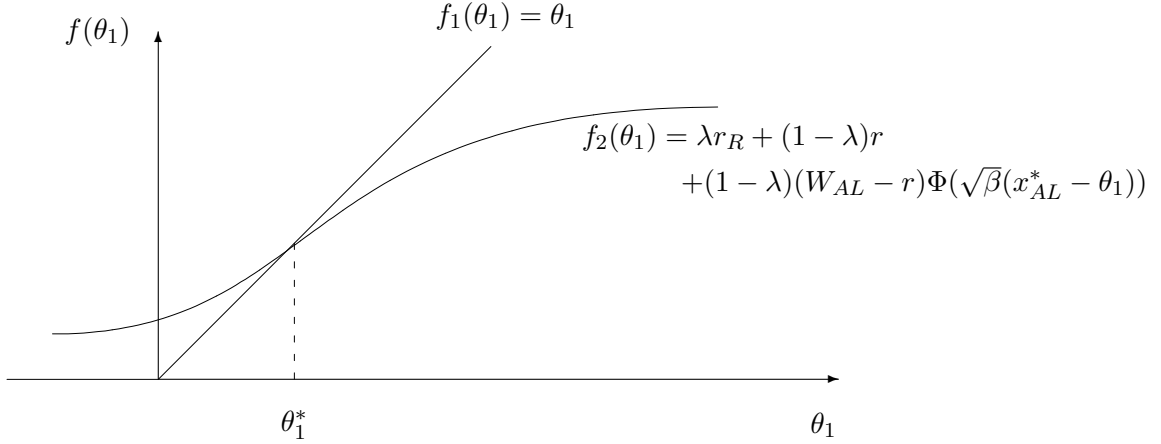[22]Likewise, $\varphi(\cdot)$ will denote the normal density.

Figure 2: Equilibrium value θ_1^*, determined by the intersection of the two functions f_1 and f_2

Whenever a project value higher than θ_2^* is realized, the firm will continue the project irrespective of the relationship bank's decision. For project values between θ_1^* and θ_2^*, however, continuation of the project critically depends on the relationship bank's action. Even though arm's-length banks can observe her decision, they still have to calculate the probability that the project value lies in the interval $[\theta_1^*, \theta_2^*]$, conditional on their observation and private information x_{AL}.

The trigger value x_R^* for the relationship bank is determined by a similar condition of indifference

$$\pi_R(\text{foreclose}|x_R = x_R^*) = \pi_R(\text{renegotiate and extend}|x_R = x_R^*)$$
$$K = r_R \cdot \text{prob}(\theta \geq \theta_1^*|x_R = x_R^*), \qquad (3)$$

where

$$r_R = \arg\max_{r \leq W_R} r \cdot \text{prob}(\theta \geq \theta_1^*|x_R),$$

which delivers[23]

$$r_R = \frac{1 - \Phi(\sqrt{\alpha}(\theta_1^* - x_R))}{\sqrt{\alpha}\frac{\partial \theta_1^*}{\partial r_R}\varphi(\sqrt{\alpha}(\theta_1^* - x_R))} \quad \text{for all} \quad x_R \geq x_R^*. \qquad (4)$$

The optimal repayment for the relationship bank is then the r_R that balances two different effects on her expected payoff. First, there is a positive effect of an increase in r_R due to higher repayment per unit of capital whenever credit is repaid. Second, however, a higher repayment to the relationship bank affects the threshold value θ_1^* that has to be exceeded by cash-flow θ for the project to be continued and hence for credit to be repaid. Interestingly, the effect of an increase in r_R on θ_1^* may be both positive

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[23]See also appendix A.

and negative. On the one hand, a higher repayment to the relationship bank leads to higher cost of capital to the firm. This, in turn, requires a higher project cash-flow θ for the project to be continued. Hence, it reduces the incidence of credit repayment via an increase in θ_1^*, which may be denoted as the "cost"-effect of r_R on θ_1^*. On the other hand, small banks, knowing about the relationship bank's informational advantage, may interpret an increase in r_R as a sign that the relationship bank observed more optimistic information about the project value. This, in turn, makes it more reasonable for small banks to extend their loans, so that θ_1^* decreases. This second impact may be referred to as the "information"-effect of r_R on θ_1^*. Both effects are strengthened by the relationship bank's fraction of total firm debt, λ, since it determines both the fraction of capital that succumbs to the renegotiation process but also the proportion of credit that will be extended.[24]

The relationship bank's indifference condition (3) leads to the following equation defining her trigger signal

$$ x_R^* = \theta_1^* + \frac{1}{\sqrt{\alpha}}\Phi^{-1}\left(\frac{K}{r_R}\right) . \tag{5} $$

Whenever the relationship bank observes private information lower than x_R^* she will foreclose the loan. For private information higher than x_R^*, however, she will renegotiate the repayment rate and roll over the loan.

Based on the anticipated strategy of the firm and the observed behavior of the relationship bank, arm's-length banks are indifferent between foreclosing and extending their loans, whenever they observe a private signal of[25]

$$ x_{AL}^* = x_R^* + \sqrt{\frac{\alpha+\beta}{\alpha\beta}}\Phi^{-1}\left[\frac{\int_{\theta_1^*}^{\theta_2^*}\Phi(\sqrt{\alpha}(\theta-x_R^*))\varphi(\sqrt{\beta}(\theta-x_{AL}^*))d\theta}{\frac{K}{r} - \frac{\theta_2^*-\lambda W_R-(1-\lambda)r}{(1-\lambda)(W_{AL}-r)}}\right] . \tag{6} $$

For signals lower than x_{AL}^* they will withdraw, for higher signals they will extend credit. The equilibrium in trigger strategies is hence described by equations (1), (2), (5) and (6).

Note that the relationship bank's effects on the firm's budget constraints are manifested in r_R. Whenever the relationship bank requests a higher repayment rate, i.e. $r_R > r$, this may be attributed to her ability to hold-up the firm, whereas a decrease in repayment, i.e. $r_R < r$, clearly eases the firm's budget constraints. The coordination-effect, in contrast, shows up in threshold value θ_1^* (and also in θ_2^*) and may be decomposed into the cost-effect and the information-effect as described above.

The following section analyzes the equilibrium in more detail, with emphasis on the relationship bank's action serving as a (coordinating) signal. We furthermore compare the model of heterogeneous multiple credit relations to a setting with a relationship bank acting as monopolist and also to a completely homogeneous credit market consisting solely of arm's-length banks.

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[24]Note that the effect of r_R on θ_1^* captures the net-effect of the initially introduced impact of the relationship bank on the firm's financial constraints and on the coordinated behavior by arm's-length banks.

[25]For the derivation of the equilibrium equation for arm's-length banks see appendix B.

3 Analysis of Financing Regimes - Finitely Precise Information

3.1 Heterogeneous Multiple Bank Financing

One of the key features of relationship lending is that the relationship bank may renegotiate the rate that has to be repaid on her fraction of debt.[26] Since the bargaining result is observable, the relationship bank may try to signal her information to the rest of the market. Before we investigate the impact of the bargaining process on the equilibrium, we will analyze in which way the relationship bank's information influences the renegotiated repayment rate r_R. Based on these results, we will then examine how her decision to foreclose or to renegotiate the repayment rate and extend the loan impacts the threshold values θ_1^* and θ_2^* of the project. Since for all $\theta \geq \underline{\theta_1}$ credit termination is inefficient, we will interpret a decrease in θ_1^* and θ_2^* as a reduction in the (ex-ante) probability of inefficient credit termination, respectively of project liquidation, as it reduces the state space of inefficient credit decisions.

For the effect of the relationship bank's private information x_R on the repayment rate r_R, we find the following:

Lemma 1 *For $\partial^2 \theta_1^*/(\partial r_R \partial x_R)$ sufficiently small (high), the relationship bank requests a higher (lower) repayment rate with improving information, i.e. with higher signal x_R.*

For proof, see appendix C.

A higher signal x_R increases the relationship bank's posterior expectation with respect to project cash-flow. Hence, she may try to charge a higher repayment in order to extract the higher expected net profit from the firm. However, she knows that threshold θ_1^* is influenced by her choice of r_R. As a consequence, $\partial r_R/\partial x_R$ is positive only if the cross derivative, $\partial^2 \theta_1^*/(\partial r_R \partial x_R)$, is sufficiently small or even negative. I.e., the relationship bank will increase the interest rate along with an improvement in signal value x_R only if she expects her information to either reduce the cost-effect ($\partial \theta_1^*/\partial r_R > 0$) or to increase the information effect ($\partial \theta_1^*/\partial r_R < 0$). Otherwise, she will demand a lower repayment rate along with an increase in x_R.

Likewise, the impact of the relationship bank's information precision α on the renegotiated repayment rate r_R depends on the repayment rate's effect on threshold θ_1^*:

Lemma 2 *For $\partial^2 \theta_1^*/(\partial r_R \partial \alpha)$ sufficiently small (high), more precise private information induces the relationship bank to bargain for a higher (lower) repayment rate. For intermediate private information x_R, the renegotiated repayment rate r_R will very likely decrease along with information precision α, while for extreme signals x_R the opposite holds.*

For proof, see appendix D.

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[26]Note that due to the assumption that all the bargaining power rests with the relationship bank, the renegotiated repayment, in a take-it-or-leave-it offer, extracts all the ex-ante expected surplus from the firm.

Again, the relationship bank increases the repayment rate along with increasing information precision only if an increase in α goes along with a reduction of the cost effect, respectively with a strengthening of the information effect. This will most likely be the case for extremely high or low signals x_R. For intermediate private information x_R, in contrast, the relationship bank will tend to reduce the repayment rate with increasing precision of information.

In the following, we will analyze the influence of the renegotiated repayment rate r_R on the equilibrium values θ_1^* and θ_2^* that the cash-flow has to exceed for the project to be continued.

Lemma 3 *Equilibrium values θ_1^* and θ_2^* increase (decrease) along with r_R if x_{AL}^* rises (falls) in r_R.*

Proof:

$$\frac{\partial \theta_1^*}{\partial r_R} = \frac{\lambda + (1-\lambda)(W_{AL} - r)\sqrt{\beta}\varphi(\sqrt{\beta}(x_{AL}^* - \theta_1^*))}{1 + (1-\lambda)(W_{AL} - r)\sqrt{\beta}\varphi(\sqrt{\beta}(x_{AL}^* - \theta_1^*))} \cdot \frac{\partial x_{AL}^*}{\partial r_R}$$

$$\frac{\partial \theta_2^*}{\partial r_R} = \frac{(1-\lambda)(W_{AL} - r)\sqrt{\beta}\varphi(\sqrt{\beta}(x_{AL}^* - \theta_2^*))}{1 + (1-\lambda)(W_{AL} - r)\sqrt{\beta}\varphi(\sqrt{\beta}(x_{AL}^* - \theta_2^*))} \cdot \frac{\partial x_{AL}^*}{\partial r_R} .$$

□

The influence of r_R on the strategy of arm's-length banks plays a pivotal role for the model. In particular, it determines whether the information-effect or the cost-effect of the renegotiation process dominates, so that the threshold values θ_1^* and θ_2^* either decrease or increase in r_R. However, the sign of the derivative $\partial x_{AL}^*/\partial r_R$ is not fully determined. Yet, a combination of the results derived so far allows interesting interpretations with regard to the efficiency of credit decisions and is given in proposition 1. It sums up the effects of the relationship bank's information precision on credit extension and project continuation. The results differentiate between extreme profit expectations and intermediate expectations as perceived by the relationship bank, i.e. extreme or intermediate values of x_R.

Proposition 1 *Increasingly (decreasingly) precise private information held by the relationship bank reduces (raises) the incidence of inefficient credit termination for firms perceived as of intermediate quality. The opposite holds for firms whose expected cash-flows are extremely high or low.*

Table 1 delineates the mechanisms for the case of an increase in the relationship bank's information precision.

Table 1: Effects of an increase in α

	intermediate values x_R	extreme values x_R (good or bad)
$\frac{\partial x_{AL}^*}{\partial r_R} > 0$	$r_R \downarrow \Rightarrow x_{AL}^* \downarrow, \theta_1^* \downarrow, \theta_2^* \downarrow$	$r_R \uparrow \Rightarrow x_{AL}^* \uparrow, \theta_1^* \uparrow, \theta_2^* \uparrow$
$\frac{\partial x_{AL}^*}{\partial r_R} < 0$	$r_R \uparrow \Rightarrow x_{AL}^* \downarrow, \theta_1^* \downarrow, \theta_2^* \downarrow$	$r_R \downarrow \Rightarrow x_{AL}^* \uparrow, \theta_1^* \uparrow, \theta_2^* \uparrow$

No matter whether cost-effect ($\partial x_{AL}^*/\partial r_R > 0$) or information-effect ($\partial x_{AL}^*/\partial r_R < 0$) dominate, whenever the relationship bank obtains more precise information, the risk of

inefficient credit termination is reduced for firms perceived as of intermediate quality by the relationship bank, but is increased for firms with extreme expected profits. The opposite holds for a decreasing precision of information held by the relationship bank.

How can this result be interpreted? Let us consider the case of firms perceived to be of intermediate value first. The relationship bank will always try to use both cost- and information-effect in order to maximize her proceeds from lending to the firm. Hence, whenever the cost-effect dominates, she will be cautious and tend to lower the repayment rate with increasing information precision. This induces the small banks to extend their loans for even lower signals x_{AL}, so that the incidence of inefficient liquidation is reduced. If the information-effect dominates, in contrast, the relationship bank will raise the repayment rate. This signals a sufficiently good firm quality to the arm's-length banks, so that again they will extend credit for a larger range of signal values x_{AL} and the probability of firm default will be reduced.

For firms with either extremely high or extremely low expected cash-flows, however, the relationship bank's strategy may be characterized as the trial to maximize her proceeds *despite* cost- and information-effect. E.g., if the relationship bank expects an extremely high firm cash-flow (observes a high signal x_R) and the cost-effect dominates, she will ask for a higher repayment along with α, even though she knows that this will have a detrimental effect on the threshold θ_1^* that decides on the firm's default. But as x_R is extremely high, she hopes that the firm's quality will be sufficiently high to surpass θ_1^* even though this threshold will be raised by an increase in r_R. If, in contrast, the relationship bank observes an extremely low value x_R, she will have to ask for a high repayment rate despite the cost-effect in order to maximize her repayment from a loan that is prolonged. If the information-effect dominates in this case, she will request a lower repayment rate in the hope of stimulating the firm's finances sufficiently strongly so that the threshold θ_1^* will be passed even though she knows that the arm's-length banks will take her action as a signal that the firm is of low quality and will therefore tend to cease credit prolongation.

In order to reduce inefficient credit termination incidences, firms with intermediate expected profits should therefore disclose information of maximum precision to the relationship bank. Firms with extremely low or extremely high expected cash-flows, in contrast, should disclose information of minimum precision to the relationship lender. While by doing so the probability of project liquidation can be minimized, the development of the requested repayment by the relationship bank depends on whether cost-effect (reduction of r_R) or information-effect (increase in r_R) dominate.

3.2 Comparing Financing Regimes

In the following, we will first compare the system of heterogeneous multiple bank financing to single relationship banking, i.e. to a monopoly of the relationship bank ($\lambda = 1$). Secondly, we will analyze the case where only arm's-length debt is available ($\lambda = 0$), so that the firm holds multiple credit relations with homogeneous small banks.

Monopoly Relationship Lending

From the firm's indifference condition it follows that with single relationship lending the

critical value of the project up to which the firm always chooses to terminate the project rather than refinance is $\hat{\theta} = W_R$. By renegotiating the repayment rate, the relationship bank tries to maximize her profit from extending the loan. Feasibility requires $\hat{r}_R \leq W_R$, so that

$$\hat{r}_R = \arg\max_{r \leq W_R} r \cdot \text{prob}(\theta \leq \hat{\theta} | x_R = \hat{x}_R) = W_R .$$

Hence, the relationship bank always demands the maximum repayment, so that the intertemporal-smoothing effect vanishes and the firm succumbs to maximal hold-up. The relationship bank is then indifferent between withdrawing and extending the loan if

$$\begin{aligned} \pi_R(\text{withdraw}) &= \pi_R(\text{extend}) \\ K &= W_R \cdot \text{prob}(\theta \geq \hat{\theta} | x_R = \hat{x}_R) \\ \hat{x}_R &= W_R + \frac{1}{\sqrt{\alpha}} \Phi^{-1}(\frac{K}{W_R}) . \end{aligned}$$

Based on equilibrium values $\hat{r}_R$, $\hat{\theta}$ and $\hat{x}_R$, the following lemma holds:

Lemma 4 *The incidence of inefficient project termination with single relationship banking is at least as high as with heterogeneous multiple bank financing.*

Proof:

$$\theta_1^* = \lambda r_R + (1 - \lambda)[(1 - l)r + l W_{AL}] \leq W_R = \hat{\theta},$$

since due to feasibility of the bargaining process $r_R \leq W_R$. $\quad\square$

Additionally, we find that the relationship bank herself may have a material interest in competition by small banks.

Lemma 5 *As long as the renegotiated repayment rate r_R does not fall too short of the monopoly rate $\hat{r}_R = W_R$, the relationship bank has an explicit interest in competition by arm's-length banks.*

Proof:

$$\begin{aligned} r_R \cdot \text{prob}(\theta \geq \theta_1^* | x_R) &> W_R \cdot \text{prob}(\theta \geq \hat{\theta} | x_R) \\ r_R \Phi(\sqrt{\alpha}(x_R - \theta_1^*)) &> W_R \Phi(\sqrt{\alpha}(x_R - \hat{\theta})) \\ r_R &> W_R \frac{\Phi(\sqrt{\alpha}(x_R - W_R))}{\Phi(\sqrt{\alpha}(x_R - \theta_1^*))} \end{aligned}$$

Since $\frac{\Phi(\sqrt{\alpha}(x_R - W_R))}{\Phi(\sqrt{\alpha}(x_R - \theta_1^*))}$ is lower than or at most equal to 1, the inequality holds as long as the renegotiated repayment rate in a heterogeneous multiple banking regime, r_R, does not fall too short of the monopoly rate $\hat{r}_R = W_R$. $\quad\square$

Even though the relationship bank charges the highest repayment rate in a monopoly situation (i.e. $r_R \leq \hat{r}_R = W_R$), so that hold-up costs are maximal, her expected payoff is not necessarily higher than in a competitive situation. This is due to the fact that the very lowering of the renegotiated repayment rate may coordinate small banks' behavior towards extending rather than foreclosing their loans. This, in turn, increases

the probability that the firm will continue the project and credit will be repaid. Single relationship lending lacks this coordination-effect that may contribute strongly to the relationship bank's profit.

Homogeneous Multiple Bank Financing

If the only source of financing is a continuum of small banks, the firm's indifference condition delivers a trigger value of $\tilde{\theta} = r + (W_{AL} - r)l$, up to which she terminates the project but proceeds for higher cash-flows. The small banks' indifference condition yields the following equilibrium value for their private signals

$$\tilde{x}_{AL} = \tilde{\theta} + \frac{1}{\sqrt{\beta}}\Phi^{-1}\Big(\frac{K}{r}\Big) \ .$$

Hence, since all banks with private information lower than $\tilde{x}_{AL}$ will foreclose their loans, equilibrium value $\tilde{\theta}$ can be expressed as $\tilde{\theta} = r + (W_{AL} - r)\Phi(\sqrt{\beta}(\tilde{x}_{AL} - \tilde{\theta}))$. Substituting for $\tilde{x}_{AL}$ then delivers

$$\tilde{\theta} = r - K + \frac{W_{AL}K}{r} \ .$$

When comparing homogeneous multiple bank financing to the heterogeneous system of section 3.1, we have to distinguish two different scenarios: either the relationship bank renegotiated the repayment rate and rolled over the loan or she foreclosed the loan. Let us first analyze the case of "efficient coordination" of lenders, in which the relationship bank extends the loan. It might be expected that in a heterogeneous system the incidence of inefficient credit termination is lower than in a homogeneous setting, i.e. $\theta_1^* < \tilde{\theta}$, since due to the relationship bank's non-negligible fraction of debt a lower amount of coordination among the individual small banks is necessary to yield the efficient outcome. We find that this is indeed the case, if

$$r_R < r + (1 - \frac{1}{\lambda})(W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_1^*)) - \frac{K}{\lambda} + \frac{W_{AL}K}{r\lambda} \ .$$

In contrast, if the relationship bank withdraws the loan, we find that the event of inefficient termination of projects is lower with heterogeneous than homogeneous multiple bank financing, i.e. $\theta_2^* < \tilde{\theta}$, if

$$\lambda < \frac{\frac{W_{AL}K}{r} - K - (W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_2^*))}{W_R - r - (W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_2^*))} \ .$$

Lemma 6 *Whenever the relationship bank causes relatively low hold-up costs for the firm, i.e. if either the repayment rate r_R or her fraction of total debt λ are sufficiently small, the incidence of inefficient project liquidation is lower in a system with heterogeneous rather than homogeneous multiple credit relations.*

4 Analysis of Financing Regimes - Infinitely Precise Information

4.1 Heterogeneous Multiple Bank Financing

Apart from being able to renegotiate the repayment and providing a non-negligible fraction of total firm debt, the relationship bank disposes of relatively more precise private information about the firm and its project. In the extreme, we may analyze the case in which the relationship bank's information is completely precise, i.e. $\alpha \to \infty$. It is easy to see that equilibrium values x_R^{**}, x_{AL}^{**}, θ_1^{**} and θ_2^{**} then converge in the following way:

$$
\begin{aligned}
x_R^{**} &\to \theta_1^{**} \\
x_{AL}^{**} &\to \theta_1^{**} \\
\theta_1^{**} &\to \lambda r_R + \frac{1}{2}(1-\lambda)(W_{AL} + r) \\
\theta_2^{**} &\to \lambda W_R + (1-\lambda)r + (1-\lambda)(W_{AL} - r)\Phi(\sqrt{\beta}(\theta_1^{**} - \theta_2^{**})) \\
&= \lambda W_R + \frac{1}{2}(1-\lambda)(W_{AL} + r) .
\end{aligned}
\tag{7}
$$

With completely precise information, the relationship bank demands the maximum feasible repayment, i.e. $r_R = W_R$, so that hold-up costs are maximized and intertemporal smoothing no longer exists. Since arm's-length banks know that the relationship bank disposes of fully precise information, their optimal strategy is to behave in the same way as she does. The formerly individual threshold values θ_1^{**}, θ_2^{**}, x_R^{**} and x_{AL}^{**} therefore converge to one identical value

$$
\lambda W_R + \frac{1}{2}(1-\lambda)(W_{AL} + r) = \theta_1^{**}
$$

This threshold value increases in the relationship bank's "size" λ, in refinancing costs W_{AL} and W_R, and in the repayment rates r and r_R. As the repayment requested by the relationship bank does no longer vary with the bank's expectations about firm quality, the information-effect vanishes. The renegotiation process is then completely dominated by the cost-effect.

4.2 Comparing Financing Regimes

A completely informed relationship bank acting as monopolist is characterized by the following equilibrium values:

$$
\hat{\hat{\theta}} = \hat{\hat{x}}_R = W_R .
$$

As before, we find that the event of inefficient project liquidation is never higher with heterogeneous multiple bank financing than when the relationship bank acts as monopolist, i.e. $\theta_1^{**} \leq \hat{\hat{\theta}}$.

When comparing threshold value θ_1^{**} from a heterogeneous system, given that the relationship bank disposes of completely precise private information, with the threshold

level for the project's value in a homogeneous system, we find that $\theta_1^{**} < \tilde{\tilde{\theta}}$ whenever

$$
\begin{aligned}
r_R = W_R \quad &< \quad r + (1 - \frac{1}{\lambda})(W_{AL} - r)\frac{1}{2} - \frac{K}{\lambda} + \frac{W_{AL}K}{r\lambda} \\
\Leftrightarrow \lambda \quad &< \quad \frac{W_{AL}\frac{K}{r} - K - (W_{AL} - r)\frac{1}{2}}{W_R - \frac{1}{2}(W_{AL} + r)} \ ,
\end{aligned}
$$

i.e. for sufficiently low cost of capital caused by the relationship bank. Note that this condition is more restrictive than the one under finitely precise information whenever $x_{AL}^* > \theta_2^*$.

Since with completely precise information the relationship bank will charge the maximum feasible repayment rate irrespective of the project value, heterogeneous multiple bank financing can only reduce the event of inefficient project liquidation compared to a homogeneous banking regime, if the relationship bank's fraction of total firm debt is sufficiently small, so that the severity of the hold-up problem for the firm's cost of capital does not become too strong.

Lemma 7 *With completely precise information held by the relationship bank, the incidence of inefficient project liquidation is always lower in a system of heterogeneous multiple bank financing than with single relationship banking. Compared to a homogeneous system, however, this holds only if the relationship bank's fraction of total firm debt is sufficiently low.*

5 Conclusion

Heterogeneous bank financing systems predominate in many countries. In particular small and medium-sized firms seem to prefer obtaining capital from both a well-informed relationship bank and a multitude of less well-informed arm's-length banks. Our study points to the asymmetric informational positions of the involved banking institutions as an explanatory factor for the prevalence of these financing systems.

Assuming that the relationship bank's information precision rises with the duration of the credit relation, we find that heterogeneous multiple bank financing is a self-sustaining regime for firms with moderate expected profits. For firms conducting businesses with extremely high or extremely low expected cash-flows, the opposite holds. With increasing length of the credit relation, they face a higher probability of inefficient liquidation. If we accept that small and medium-sized businesses typically run a moderate risk of financial distress, we might conjecture that particularly those smaller firms are interested in keeping up multiple credit relations with asymmetric banking institutions.[27] At the same time, our model provides an explanation why fluctuations in companies' return expectations (over time) may lead to frequent changes in the chosen financing structure.

As long as the relationship bank disposes of an informational advantage over arm's-length banks, heterogeneous multiple bank financing leads to a fewer incidences of inefficient credit termination and hence of firm liquidation as compared to homogeneous

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[27]For a detailed analysis of the characteristics of SME loan portfolios, see for instance Jobst (2003).

multiple bank financing or a monopoly lending regime, irrespective of borrower quality. Yet, the relationship bank's fraction of total firm debt has to remain sufficiently small as to not represent a virtual monopoly. This result is in contrast to Elsas et al. (2004), who found that with symmetric information, firms with high expected cash-flows tend to finance without a relationship bank. Obviously, a heterogeneous banking system offers particular advantages pertaining to information asymmetries between banks. Influencing the degree of information asymmetry may therefore be a policy device for firms of varying quality. For firms with intermediate cash-flow expectations, it may be profitable to disclose very precise information to the relationship bank.

Interestingly, the banks themselves may have material interest in a heterogeneous bank financing regime, and hence in competition by other banking institutions. While all banks benefit from a reduction in the firm's default probability, the relationship bank, in particular, profits from higher expected gains as compared to monopolistic relationship banking. These follow from the beneficial interplay of her renegotiation- and coordination-influence.

Given the analysis of different financial systems, it should be interesting to establish the optimal structure of bank debt from the firm's point of view. Obviously, the optimal financing structure will depend not only on the borrower quality, as one main result by Elsas et al. (2004), but also on the transparency of this parameter. A detailed examination of this question is, however, outside the scope of this paper and is left for future research.

Appendix A

For r_R to yield a maximum payoff to the relationship bank, the second-order condition demands that the second-order derivative $\frac{\partial^2 R_R}{\partial r_R^2} < 0$, where R_R denotes the relationship bank's expected payoff from extending the loan: $R_R = \pi_R(\text{renegotiate and extend}|x_R) = r_R \cdot \text{prob}(\theta \geq \theta_1^*|x_R) = r_R \Phi(\sqrt{\alpha}(x_R - \theta_1^*))$. The first derivative with respect to r_R is then given as:

$$\frac{\partial R_R}{\partial r_R} = \Phi(\sqrt{\alpha}(x_R - \theta_1^*)) + r_R \varphi(\sqrt{\alpha}(x_R - \theta_1^*))\sqrt{\alpha}(-\frac{\partial \theta_1^*}{\partial r_R}) \tag{8}$$

The second derivative is:

$$\begin{aligned}
\frac{\partial^2 R_R}{\partial r_R^2} &= \varphi(\sqrt{\alpha}(x_R - \theta_1^*))\sqrt{\alpha}(-\frac{\partial \theta_1^*}{\partial r_R}) + \varphi(\sqrt{\alpha}(x_R - \theta_1^*))\sqrt{\alpha}(-\frac{\partial \theta_1^*}{\partial r_R}) \\
&\quad + r_R(-\sqrt{\alpha}(x_R - \theta_1^*))\sqrt{\alpha}(-\frac{\partial \theta_1^*}{\partial r_R})\varphi(\sqrt{\alpha}(x_R - \theta_1^*))\sqrt{\alpha}(-\frac{\partial \theta_1^*}{\partial r_R}) \\
&\quad + r_R \varphi(\sqrt{\alpha}(x_R - \theta_1^*))\sqrt{\alpha}(-\frac{\partial^2 \theta_1^*}{\partial r_R^2}) \\
&= \sqrt{\alpha}\varphi(\sqrt{\alpha}(x_R - \theta_1^*))\Big[-2\frac{\partial \theta_1^*}{\partial r_R} - r_R\alpha(x_R - \theta_1^*)(\frac{\partial \theta_1^*}{\partial r_R})^2 - r_R\frac{\partial^2 \theta_1^*}{\partial r_R^2}\Big]. \tag{9}
\end{aligned}$$

Note that for $\frac{\partial^2 R_R}{\partial r_R^2}$ to be negative, it has to hold that:

$$0 < 2\frac{\partial \theta_1^*}{\partial r_R} + r_R\alpha(x_R - \theta_1^*)(\frac{\partial \theta_1^*}{\partial r_R})^2 + r_R\frac{\partial^2 \theta_1^*}{\partial r_R^2}$$

$$\Leftrightarrow \quad \frac{\partial \theta_1^*}{\partial r_R} > \frac{-1 + \sqrt{1 - r_R^2\alpha(x_R - \theta_1^*)\frac{\partial^2 \theta_1^*}{\partial r_R^2}}}{r_R\alpha(x_R - \theta_1^*)} \tag{10}$$

$$\text{or} \quad \frac{\partial \theta_1^*}{\partial r_R} < \frac{-1 - \sqrt{1 - r_R^2\alpha(x_R - \theta_1^*)\frac{\partial^2 \theta_1^*}{\partial r_R^2}}}{r_R\alpha(x_R - \theta_1^*)}. \tag{11}$$

I.e. feasibility requires $\frac{\partial \theta_1^*}{\partial r_R}$ to be either sufficiently large or sufficiently small. Note that the main results of our model in proposition 1 go through even if we restrict r_R to take on strictly positive values, so that the relationship bank's coordinating impact can only take the form of the cost-effect.

Appendix B

The information structure of the model is as follows. Given the realized project value θ, private signals are distributed as $x_R|\theta \sim N(\theta, \frac{1}{\alpha})$ and $x_{AL}|\theta \sim N(\theta, \frac{1}{\beta})$. For the banks, after observing their private information, the unknown project value θ is distributed according to $\theta|x_R \sim N(x_R, \frac{1}{\alpha})$ and $\theta|x_{AL} \sim N(x_{AL}, \frac{1}{\beta})$. Moreover, each arm's-length bank believes the relationship bank's information to be given by $x_R|x_{AL} \sim N(x_{AL}, \frac{\alpha+\beta}{\alpha\beta})$. Additionally, $\text{Cov}(x_R, \theta|x_{AL}) = \frac{1}{\beta}$, so that $\text{Corr}(x_R, \theta|x_{AL}) = \frac{\frac{1}{\beta}}{\sqrt{\frac{\alpha+\beta}{\alpha\beta}}\sqrt{\frac{1}{\beta}}} = \sqrt{\frac{\alpha}{\alpha+\beta}}$.

The arm's-length banks' indifference condition and threshold x^*_{AL} can be derived along the following steps:

$$\frac{K}{r} = 1 - \text{prob}(\theta \le \theta^*_2 | x_{AL} = x^*_{AL}) + \frac{\text{prob}(\theta^*_1 \le \theta \le \theta^*_2, x_R > x^*_R | x_{AL} = x^*_{AL})}{\text{prob}(x_R > x^*_R | x_{AL} = x^*_{AL})} \quad (12)$$

Denote $\frac{\text{prob}(\theta^*_1 \le \theta \le \theta^*_2, x_R > x^*_R | x_{AL} = x^*_{AL})}{\text{prob}(x_R > x^*_R | x_{AL} = x^*_{AL})}$ by m. We then have:

$$m = \frac{\int_{\theta^*_1}^{\theta^*_2} \int_{x^*_R}^{\infty} \frac{1}{2\pi \frac{1}{\sqrt{\beta}} \sqrt{\frac{\alpha+\beta}{\alpha\beta}} \sqrt{1 - \frac{\alpha}{\alpha+\beta}}} e^{-\frac{1}{2(1-\frac{\alpha}{\alpha+\beta})} \cdot \left[\left(\frac{\theta - x^*_{AL}}{\frac{1}{\sqrt{\beta}}}\right)^2 - 2\sqrt{\frac{\alpha}{\alpha+\beta}} \frac{\theta - x^*_{AL}}{\frac{1}{\sqrt{\beta}}} \frac{x_R - x^*_{AL}}{\sqrt{\frac{\alpha+\beta}{\alpha\beta}}} + \left(\frac{x_R - x^*_{AL}}{\sqrt{\frac{\alpha+\beta}{\alpha\beta}}}\right)^2\right]} dx_R d\theta}{1 - \Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x^*_R - x^*_{AL}))}$$

$$m = \frac{\int_{\theta^*_1}^{\theta^*_2} \int_{x^*_R}^{\infty} \frac{\sqrt{\alpha\beta}}{2\pi} e^{-\frac{1}{2}\alpha \left[(\theta - x_R)^2 + \frac{\beta}{\alpha}(\theta - x^*_{AL})^2\right]} dx_R d\theta}{1 - \Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x^*_R - x^*_{AL}))}$$

$$m = \frac{\int_{\theta^*_1}^{\theta^*_2} \left[1 - \int_{-\infty}^{x^*_R} \frac{1}{\sqrt{2\pi}\sqrt{\frac{1}{\alpha}}} e^{-\frac{1}{2}\left(\frac{x_R - \theta}{\sqrt{\frac{1}{\alpha}}}\right)^2} dx_R\right] \frac{1}{\sqrt{2\pi}\sqrt{\frac{1}{\beta}}} e^{-\frac{1}{2}\left(\frac{\theta - x^*_{AL}}{\sqrt{\frac{1}{\beta}}}\right)^2} d\theta}{1 - \Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x^*_R - x^*_{AL}))}$$

$$m = \frac{\int_{\theta^*_1}^{\theta^*_2} \left[1 - \Phi(\sqrt{\alpha}(x^*_R - \theta))\right] \varphi(\sqrt{\beta}(\theta - x^*_{AL})) d\theta}{1 - \Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x^*_R - x^*_{AL}))}$$

$$m = \frac{\int_{\theta^*_1}^{\theta^*_2} \Phi(\sqrt{\alpha}(\theta - x^*_R)) \varphi(\sqrt{\beta}(\theta - x^*_{AL})) d\theta}{\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x^*_{AL} - x^*_R))} .$$

Plugging this in (12) delivers[28]

$$\frac{K}{r} = 1 - \Phi(\sqrt{\beta}(\theta^*_2 - x^*_{AL})) + \frac{\int_{\theta^*_1}^{\theta^*_2} \Phi(\sqrt{\alpha}(\theta - x^*_R)) \varphi(\sqrt{\beta}(\theta - x^*_{AL})) d\theta}{\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x^*_{AL} - x^*_R))}$$

$$\frac{K}{r} - \Phi(\sqrt{\beta}(x^*_{AL} - \theta^*_2)) = \frac{\int_{\theta^*_1}^{\theta^*_2} \Phi(\sqrt{\alpha}(\theta - x^*_R)) \varphi(\sqrt{\beta}(\theta - x^*_{AL})) d\theta}{\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x^*_{AL} - x^*_R))}$$

$$\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x^*_{AL} - x^*_R)) = \frac{\int_{\theta^*_1}^{\theta^*_2} \Phi(\sqrt{\alpha}(\theta - x^*_R)) \varphi(\sqrt{\beta}(\theta - x^*_{AL})) d\theta}{\frac{K}{r} - \Phi(\sqrt{\beta}(x^*_{AL} - \theta^*_2))}$$

$$x^*_{AL} = x^*_R + \sqrt{\frac{\alpha+\beta}{\alpha\beta}} \Phi^{-1}\left(\frac{\int_{\theta^*_1}^{\theta^*_2} \Phi(\sqrt{\alpha}(\theta - x^*_R)) \varphi(\sqrt{\beta}(\theta - x^*_{AL})) d\theta}{\frac{K}{r} - \Phi(\sqrt{\beta}(x^*_{AL} - \theta^*_2))}\right) .$$

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[28]I am very thankful to an anonymous referee for pointing out an efficient way of simplifying this equation

Appendix C

Proof of Lemma 1:

The effect of the relationship bank's private information on the renegotiated repayment rate is given by:

$$\frac{\partial r_R}{\partial x_R} = -\frac{\sqrt{\alpha}\frac{\partial \theta_1^*}{\partial r_R}(\frac{\partial \theta_1^*}{\partial x_R} - 1)[\varphi(\cdot) - [1 - \Phi(\cdot)]\sqrt{\alpha}(\theta_1^* - x_R)] + [1 - \Phi(\cdot)]\frac{\partial^2 \theta_1^*}{\partial r_R \partial x_R}}{\sqrt{\alpha}\varphi(\cdot)[\frac{\partial \theta_1^*}{\partial r_R}]^2},$$

where $(\cdot) = (\sqrt{\alpha}(\theta_1^* - x_R))$. This derivative will be positive for sufficiently small $\partial^2 \theta_1^*/(\partial r_R \partial x_R)$, i.e. for

$$\frac{\partial^2 \theta_1^*}{\partial r_R \partial x_R} < \frac{\sqrt{\alpha}[(1 - \Phi(\cdot))\sqrt{\alpha}(\theta_1^* - x_R) - \phi(\cdot)]}{1 - \Phi(\cdot)}\frac{\partial \theta_1^*}{\partial r_R}(\frac{\partial \theta_1^*}{\partial x_R} - 1).$$

Otherwise, r_R decreases in x_R.

Appendix D

Proof of Lemma 2:

The partial derivative:

$$\begin{aligned}\frac{\partial r_R}{\partial \alpha} &= -\frac{\sqrt{\alpha}\frac{\partial \theta_1^*}{\partial r_R}[\frac{1}{2\sqrt{\alpha}}(\theta_1^* - x_R) + \sqrt{\alpha}\frac{\partial \theta_1^*}{\partial \alpha}][\phi(\cdot) - [1 - \Phi(\cdot)]\sqrt{\alpha}(\theta_1^* - x_R)]}{\alpha\phi(\cdot)(\frac{\partial \theta_1^*}{\partial r_R})^2} \\ &\quad -\frac{[1 - \Phi(\cdot)][\frac{1}{2\sqrt{\alpha}}\frac{\partial \theta_1^*}{\partial r_R} + \sqrt{\alpha}\frac{\partial^2 \theta_1^*}{\partial r_R \partial \alpha}]}{\alpha\phi(\cdot)(\frac{\partial \theta_1^*}{\partial r_R})^2}\end{aligned}$$

is negative provided that $\partial^2 \theta_1^*/(\partial r_R \partial \alpha)$ is sufficiently high, i.e. for:

$$\frac{\partial^2 \theta_1^*}{\partial r_R \partial \alpha} > \frac{\partial \theta_1^*}{\partial r_R}\Big[\frac{[\frac{\theta_1^* - x_R}{2\sqrt{\alpha}} + \sqrt{\alpha}\frac{\partial \theta_1^*}{\partial \alpha}][(1 - \Phi(\cdot))\sqrt{\alpha}(\theta_1^* - x_R) - \phi(\cdot)]}{1 - \Phi(\cdot)} - \frac{1}{2\alpha}\Big].$$

Otherwise, r_R increases in α.

Note that if the cost-effect dominates, i.e. $\partial \theta_1^*/\partial r_R > 0$, the r.h.s. of the above inequality will be negative for intermediate values of x_R:

$$\frac{[\frac{\theta_1^* - x_R}{2\sqrt{\alpha}} + \sqrt{\alpha}\frac{\partial \theta_1^*}{\partial \alpha}][(1 - \Phi(\cdot))\sqrt{\alpha}(\theta_1^* - x_R) - \phi(\cdot)]}{1 - \Phi(\cdot)} - \frac{1}{2\alpha} < 0$$

$$\frac{1 - \Phi(\cdot)}{2}(\theta_1^* - x_R)^2 + [\alpha(1 - \Phi(\cdot))\frac{\partial \theta_1^*}{\partial \alpha} - \frac{\phi(\cdot)}{2\sqrt{\alpha}}](\theta_1^* - x_R) - \alpha\phi(\cdot)\frac{\partial \theta_1^*}{\partial \alpha} - \frac{1 - \Phi(\cdot)}{2\alpha} < 0.$$

This inequality is satisfied for:

$$\theta_1^* + \frac{\alpha(1 - \Phi(\cdot))\frac{\partial \theta_1^*}{\partial \alpha} - \frac{\phi(\cdot)}{2\sqrt{\alpha}}}{1 - \Phi(\cdot)} - \sqrt{\Big(\frac{\alpha(1 - \Phi(\cdot))\frac{\partial \theta_1^*}{\partial \alpha} - \frac{\phi(\cdot)}{2\sqrt{\alpha}}}{1 - \Phi(\cdot)}\Big)^2 + \frac{2\alpha\phi(\cdot)\frac{\partial \theta_1^*}{\partial \alpha}}{1 - \Phi(\cdot)} - \frac{1}{\alpha}} < x_R$$

$$< \theta_1^* + \frac{\alpha(1 - \Phi(\cdot))\frac{\partial \theta_1^*}{\partial \alpha} - \frac{\phi(\cdot)}{2\sqrt{\alpha}}}{1 - \Phi(\cdot)} + \sqrt{\Big(\frac{\alpha(1 - \Phi(\cdot))\frac{\partial \theta_1^*}{\partial \alpha} - \frac{\phi(\cdot)}{2\sqrt{\alpha}}}{1 - \Phi(\cdot)}\Big)^2 + \frac{2\alpha\phi(\cdot)\frac{\partial \theta_1^*}{\partial \alpha}}{1 - \Phi(\cdot)} - \frac{1}{\alpha}},$$

i.e., for intermediate values of x_R. This will ease the condition on $\partial^2\theta_1^*/(\partial r_R \partial\alpha)$. Hence, for intermediate values of x_R it is very likely that r_R decreases in α.

For sufficiently extreme values of x_R it is likely that r_R increases in α as the condition on $\partial^2\theta_1^*/(\partial r_R \partial\alpha)$ is then tightened:

$$\frac{[\frac{\theta_1^* - x_R}{2\sqrt{\alpha}} + \sqrt{\alpha}\frac{\partial\theta_1^*}{\partial\alpha}][(1-\Phi(\cdot))\sqrt{\alpha}(\theta_1^* - x_R) - \phi(\cdot)]}{1 - \Phi(\cdot)} - \frac{1}{2\alpha} \; > \; 0$$

is given for:

$$x_R \; < \; \theta_1^* + \frac{\alpha(1-\Phi(\cdot))\frac{\partial\theta_1^*}{\partial\alpha} - \frac{\phi(\cdot)}{2\sqrt{\alpha}}}{1 - \Phi(\cdot)} - \sqrt{\left(\frac{\alpha(1-\Phi(\cdot))\frac{\partial\theta_1^*}{\partial\alpha} - \frac{\phi(\cdot)}{2\sqrt{\alpha}}}{1 - \Phi(\cdot)}\right)^2 + \frac{2\alpha\phi(\cdot)\frac{\partial\theta_1^*}{\partial\alpha}}{1 - \Phi(\cdot)} - \frac{1}{\alpha}}$$

and for:

$$x_R \; > \; \theta_1^* + \frac{\alpha(1-\Phi(\cdot))\frac{\partial\theta_1^*}{\partial\alpha} - \frac{\phi(\cdot)}{2\sqrt{\alpha}}}{1 - \Phi(\cdot)} + \sqrt{\left(\frac{\alpha(1-\Phi(\cdot))\frac{\partial\theta_1^*}{\partial\alpha} - \frac{\phi(\cdot)}{2\sqrt{\alpha}}}{1 - \Phi(\cdot)}\right)^2 + \frac{2\alpha\phi(\cdot)\frac{\partial\theta_1^*}{\partial\alpha}}{1 - \Phi(\cdot)} - \frac{1}{\alpha}}$$

i.e. for extreme values of x_R.

If the information-effect dominates, however, exactly the opposite holds.

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