DISCUSSION PAPER SERIES

IZA DP No. 4285

A Model of Overconfidence

Bruce A. Weinberg

July 2009

Forschungsinstitut
zur Zukunft der Arbeit
Institute for the Study
of Labor

A Model of Overconfidence

Bruce A. Weinberg
Ohio State University,
NBER and IZA

Discussion Paper No. 4285
July 2009

IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
E-mail: iza@iza.org

IZA Discussion Paper No. 4285
July 2009

ABSTRACT

A Model of Overconfidence*

People use information about their ability to choose tasks. If more challenging tasks provide more accurate information about ability, people who care about and who are risk averse over their perception of their own ability will choose tasks that are not sufficiently challenging. Overestimation of ability raises utility by deluding people into believing that they are more able than they are in fact. Moderate overestimation of ability and overestimation of the precision of initial information leads people to choose tasks that raise expected output, however extreme overconfidence leads people to undertake tasks that are excessively challenging. Consistent with our results, psychologists have found that moderate overconfidence is both pervasive and advantageous and that people maintain such beliefs by underweighting new information about their ability.

JEL Classification: D03, D08

Keywords: overconfidence, behavioral economics, information processing

Corresponding author:

Bruce A. Weinberg
Department of Economics
Ohio State University
410 Arps Hall
1945 North High Street
Columbus, Ohio 43210
USA
E-mail: weinberg.27@osu.edu

* The author is grateful for comments from Eric Fisher, Amy Glass, Masanori Hashimoto, David Hirshleifer, Peter Howitt, Carolyn Moehling, Howard Marvel, Jim Peck, Sherwin Rosen, David Schmeidler and seminar participants at Ohio State University. The author is responsible for all errors.

A Model of Overconfidence

"The over-weening conceit which the greater part of men have of their own abilities, is an ancient evil remarked by the philosophers and moralists of all ages (page 120)."

"Without regarding the danger, however, young volunteers never enlist so readily as at the beginning of a new war; and though they have scarce any chance of preferment, they figure to themselves, in their youthful fancies, a thousand occasions of acquiring honour and distinction which never occur (page 122)."

--Adam Smith, *The Wealth of Nations.*

I. Introduction

Observers have noted that people tend to approach their endeavors with excessive confidence[1]. The literature in psychology, which is discussed in the next section, points to two aspects of overconfidence. First, while the fate of Smith's military volunteers vividly illustrates the costs of extreme overconfidence, researchers have consistently found that moderate overestimation of ability is advantageous relative to a realistic assessment. Second, in updating their beliefs, people disregard much of the information that could be inferred from the outcomes of their actions.

This paper develops a simple, single-period, rational model in which it is optimal to be overconfident and to down weight new information about ability. We assume that people care about their beliefs about their ability and are risk averse in them. As discussed below, both assumptions are consistent with research in psychology. In our model people choose whether to undertake a challenging task, the realization of which depends on their ability, or to decline it in favor of a sure-thing, with a fixed payoff that

[1] DeBondt and Thaler (1995, p. 389) state, "Perhaps the most robust finding in the psychology of judgement is that people are overconfident."

does not depend on ability. This structure captures the general phenomenon that ability matters more in more challenging tasks, so that more challenging tasks provide more accurate information about ability (at least locally). The choice of tasks depends on the person's priors over his ability. Risk aversion over ability leads people to limit their information about their ability. Because choosing the sure-thing provides less information, people choose the sure-thing too often.

We study how expected utility changes as the mean of a person's priors over his ability or his perception of the precision of his priors increases. We show that increasing the mean of a person's priors over his ability raises his expected utility when the mean is below, equal to, or even slightly above his actual ability. In this sense, it is advantageous to initially overestimate ability, which is how we conceptualize overconfidence. It is trivial to show that higher priors raise a person's utility by deluding him into believing that he is more able than he is in truth. More significantly, priors affect the person's task choice. We show that moderately overestimating ability leads the person to undertake the challenging task more often, yielding higher expected output and a higher expected utility (even net of the utility from being deluded about his ability). Intuitively, if individuals tend to avoid the challenging task and, if the mean of the priors are beneath the true ability or exceed it only slightly, an increase in the mean of the priors will lead the person to undertake the challenging task, increasing expected output. Beyond some point, further increases in the mean of the priors make the individual take the challenging task even when its expected output is low, lowering expected output.

We also show that an individual who underestimates the variance of his priors is more willing to choose the challenging task, which raises expected output and utility

given the tendency to take the sure-thing. Consistent with research in psychology, overestimating the precision of the initial signal leads the individual to underweight new information.

Our model identifies conditions under which a rational, time-consistent individual acting in isolation prefers to overestimate both his ability and the precision of his estimate of his ability. A number of other papers have developed explanations for (the first form of) overconfidence under some, but not all, of these assumptions. One class of models shows that overconfidence can evolve by increasing survival probabilities, even though it lowers individual utility. De Long, Shleifer, Summers, and Waldmann (1991) show that overconfident investors, by taking more risky positions with higher returns, may come to dominate asset markets. Waldman (1994) shows that systematic errors, including overconfidence, may be evolutionarily stable. Here overconfidence is second best. Bernardo and Welch (1999) show that overconfidence, while individually harmful, may benefit a population in which herding generates inefficient information aggregation. Again, overconfidence lowers individual utility in these models. Other models generate benefits to overconfidence in the presence of time-inconsistency (Benabou and Tirole 1999a, also see Jovanovic and Stolyarov 1999), or in a game theoretic context (Kyle and Wang 1997; Gervais and Goldstein 2004). Gilboa and Schmeidler (1996) show that a case-based decision maker whose initial aspiration level is high and who occasionally raises his aspiration to exceed his best average performance will converge asymptotically to the expected-utility maximizing decision. Here, overconfident behavior vanishes asymptotically. Like the present paper, Koszegi (2000a,b) considers a Bayesian expected utility maximizers, who cares about his beliefs over ability. Koszegi (2000a) focuses on

information manipulation. We consider the beliefs that maximize utility, showing that moderately overconfident beliefs maximize expected utility.

Our results relate to research in the psychology. This literature is discussed in the next section. Section III develops the model. Section IV discusses possible extensions and robustness. Section V concludes and provides a interpretation of how overconfidence might evolve.

II. Relationship to Literature in Psychology

Research in psychology provides strong evidence of both aspects of overconfidence - overestimating ability and overestimating the accuracy of one's knowledge. It indicates that overconfidence is an important determinant of task choice and identifies advantages of overconfidence (see Oskamp 1965; Alpert and Raiffa 1982; Lichtenstein, Fischhoff, Phillips 1982; and Taylor and Brown 1988; Braumeister (1998) covers many of these topics).[2]

Research on self-efficacy in psychology indicates that perceived self-efficacy is an important determinant of people's choice of tasks (see Bandura 1977, 1986, 1997).[3] Bandura (1986) writes, "People tend to avoid tasks and situations they believe exceed their capabilities, but they undertake and perform assuredly activities they judge themselves capable of handling (p. 393)." Like Smith, Bandura notes that miss-perceptions of self-efficacy are potentially costly,

---

[2] Special issues of the *Journal of Social and Clinical Psychology* (Vol. 8, No. 3, Fall 1989) and the *Personality and Social Psychology Bulletin* (Vol. 21, No. 12, December 1995) investigate a variety of issues relating to overconfidence.

[3] Bandura (1986) defines "perceived self-efficacy" as " people's judgements of their capabilities to organize and execute courses of action required to attain designated types of performances (p. 391)."

> Reasonably accurate appraisals of one's own capabilities is, therefore, of considerable value in successful functioning. Large misjudgements of personal efficacy in either direction have consequences. People who grossly overestimate their capabilities undertake activities that are clearly beyond their reach. As a result they get themselves into considerable difficulties, undermine their credibility, and suffer needless failures…. People who underestimate their capabilities also bear costs, although, as already noted, these are more likely to take self-limiting rather than aversive forms. (p. 393-394).

Bandura also argues for the benefits of overconfidence, stating, "The efficacy judgements that are the most functional are probably those that slightly exceed what one can do at any given time (p. 394)."

Work on depression also emphasizes that estimated ability affects task choice. Beck (1967) identifies "ability, performance, intelligence, health, strength, personal attractiveness, popularity, or financial resources (p. 22)" as areas in which a depressed person might view himself as inadequate[4]. Also emphasizing that depression affects estimates of one's ability, Seligman (1975) says that "Depression is not generalized pessimism, but pessimism specific to the effects of one's own skilled actions (p. 86)." Beck describes how depression affects choice of actions, "the patient seems drawn to activities that are the least demanding for him either in terms of the degree of responsibility or initiative required… (p. 27)." Seligman (1975) writes that "A belief in helplessness may produce apparent intellectual deficits in depressives indirectly, through a motivational impairment (p. 83)." There is some evidence for depressive realism – that nondepressed individuals are overconfident while the assessments of depressed individuals are more realistic (see Alloy and Abramson 1979, and surveys by Alloy and

Abramson 1987, Ruehlman, West, and Pasahow 1985, Dobson and Franche 1989, and Ackermann and DeRubeis 1991). Yet, greater realism on the part of depressed individuals does not seem to be beneficial; Alloy and Abramson (1979) conclude that depressed people are "sadder but wiser."

Taylor and Brown (1988) are among the foremost proponents of the hypothesis that overconfidence is both normal and adaptive[5]. If optimal-overconfidence is anomalous from a rational choice perspective, psychologists have also noted the paradox. Taylor and Brown (1988), write,

> Decades of psychological wisdom have established contact with reality as a hallmark of mental health. In this view, the well-adjusted person is thought to engage in accurate reality testing whereas the individual whose vision is clouded by illusion is regarded as vulnerable to, if not already a victim of, mental illness…. We examine evidence that a set of interrelated positive illusions – namely, unrealistically positive self-evaluations, exaggerated perceptions of control or mastery, and unrealistic optimism – can serve a wide range of cognitive, affective, and social functions (p.193).

Individuals maintain overconfidence in part by disregarding information about ability. In a survey, Shrauger (1975) states that, "Data on variables that involve the cognitive processing of information indicate quite uniformly that evaluative feedback which is inconsistent with one's initial perceptions is not as readily assimilated as that which is consistent (p. 592)." Another survey by Shrauger and Schoeneman (1979) concludes that there is little evidence that people's self-perceptions respond to feedback received by others in natural settings. Self-serving attributions may also lead people to

---

[4] Beck (1967) writes, "Low self-esteem is a characteristic feature of depression (p. 22)." Cuhna (1997) surveys this literature from an economic perspective and shows that depression is associated with lower educational attainment and wages.

attribute negative outcomes to external factors (see Miller and Ross 1975 and Zuckerman 1979).

Our model assumes that people care about their beliefs of their ability, and that low estimates of ability are particularly unpleasant (that utility is concave in perceived ability). In this context, people choose tasks with lower expected output in order to lower the probability of receiving negative information about their ability. The assumption that (in economic terms) people are risk averse in their beliefs over their ability is consistent with a line of research in psychology by Jonathon Brown and Keith Dutton. Brown and Dutton (1995) speculate,

> Failure hurts LSE [low self-esteem] people more than HSE [high self-esteem] people. Cast in the language of an expectancy-value model of behavior this suggests that LSE people place a greater negative value on not attaining a goal than do HSE people. Their behavior in many situations may be guided by this fact. They may become more concerned with protecting the self from the pain of failure than risking success (p. 720).

Dutton and Brown (1997) show that failure on a task affects the emotions of low self esteem individuals more adversely than high self esteem individuals.

This line of work, like the present one, relates this greater sensitivity to adverse outcomes among low self esteem individuals to information processing. Thus, Dutton and Brown [1997] argue that differences in information processing between low and high self esteem individuals may account for the different effects on emotion (utility) among low and high self-esteem individuals. In particular, they argue that the greater adverse effect of negative information on low self esteem individuals arises because high self esteem

---

[5] See Colvin and Block (1994a, 1994b) for a critique, and Taylor and Brown (1994) for a response.

individuals disregard more adverse information than low self esteem individuals by making more self-serving attributions. We show that concavity in beliefs over ability generates an incentive for people to down weight new information, and that this incentive is particularly strong for people who are initially overconfident. One might view this aspect of our analysis as rigorously specifying conditions under which the information processing patterns they observe are optimal.

Interestingly, research on self-handicapping indicate that low-self esteem individuals will make decisions that lower their probability of success in a task in order to reduce the negative information implied by failing (Tice 1991). While aware that self-esteem and perceived self-efficacy may affect "utility" directly, much of the literature in psychology discusses overconfidence assuming that it affects "utility" only through task choice. Our model suggests that utility effects of overconfidence are important for overconfidence to be optimal[6].

We speculate that risk aversion over beliefs about ability may have arisen from matching among mates. People care about their potential mate's ability because it affects their families, but early in life, little information about the ability of potential mates is publicly available. Much information will come from potential mates, which may be partially manipulable, and which will largely depend on beliefs. To the extent that the gains from improving mate quality are greatest at the low end, the benefits from improving beliefs about ability, which will be communicated to mates, will be concave.

---

[6] It would also be possible to obtain optimal overconfidence with higher order interactions between ability and task choice in the production function. Unfortunately, it is unclear *a priori* which way such interactions should enter and unlikely that they will be

This explanation implies that overconfidence will be most important at young ages, when mates are chosen and before information about ability becomes publicly available.

III. The Model

This section develops a simple, single-period model to study the effects of priors on an individual's decisions and outcomes. The benefits to a given course of action will depend on the person's ability, $a \in (0,1)$. At the beginning of life, the individual has some priors over his ability. To ease the analysis, we assume that he believes that his ability is distributed according to a beta distribution with mean μ. The variance depends on the amount of prior information, given by n. The probability density of a is

$f(a) = \frac{\Gamma(n)}{\Gamma(\mu n)\Gamma(n(1-\mu))} a^{\mu n-1}(1-a)^{n(1-\mu)-1}$, with $E[a|\mu,n] = \mu$ and $\text{var}[a|\mu,n] = \frac{\mu(1-\mu)}{n+1}$.[7]

The individual chooses whether or not to undertake a risky task that can either be successful or unsuccessful. Ability increases the probability of success in the risky task. If the person performs the task, the person realizes high output, y_H, with probability a and low output, y_L, with probability $1-a$. (The case of beta-distributed priors and Bernoulli outcomes is particularly tractable, (see for instance, Feller [1968]). Another attractive case is that of normally distributed priors and outcomes. (A previous version of this

---

the same for all tasks, which would imply overconfidence in some areas and under-confidence in others.

[7] The density of a random variable distributed according to a beta is often written as $f(a) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} a^{\alpha-1}(1-a)^{\beta-1}$, with $E[a|\alpha,\beta] = \frac{\alpha}{\alpha+\beta}$ and $\text{var}[a|\alpha,\beta] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$. To ease the analysis, we employ the change of variables $\mu \equiv \frac{\alpha}{\alpha+\beta} = E[a]$ and $n \equiv \alpha+\beta$, implying $\alpha = \mu n$ and $\beta = n(1-\mu)$.

paper, which is available on request, derived similar results in that case.) People encounter a variety of tasks, with varying outcomes. To account for this fact, we assume that the return from success, $r = \frac{y_H}{y_L}$, in the task is randomly distributed with probability density, $f(r)$, which the person knows. Here we assume that the probability of success depends only on ability and not potential returns and that ability does not affect potential returns. This formulation captures the idea that different people will face different tasks, some of which will have higher returns and some of which have worse returns. A more general model might allow the returns to affect the probability and to allow people to choose the optimal task.

If the person undertakes the task, its outcome provides information about his ability. Bayesian updating implies that the posteriors over *a* are also distributed according to a beta distribution with parameters $\mu' = \frac{n\mu + 1}{n+1}$ and $n' = n+1$ if high output is realized and $\mu' = \frac{n\mu}{n+1}$ and $n' = n+1$ if low output is realized.

If the person decides not to perform the task, output is y_0 for certain, where $y_L < y_0$. If the person chooses not to perform the task, no new information about the distribution of his ability is revealed, so that $\mu' = \mu$ and $n' = n$.

The person's utility is a Cobb-Douglas function of his consumption and the expectation of his posteriors over his ability $(1-\theta)\ln y + \theta \ln \mu'$. If the individual takes the risky action, based on his priors, he expects utility of

$$\int_0^1 \tilde{a}\left[(1-\theta)\ln y_H + \theta\ln\left(\frac{n\mu+1}{n+1}\right)\right] + (1-\tilde{a})\left[(1-\theta)\ln y_L + \theta\ln\left(\frac{n\mu}{n+1}\right)\right] f(\tilde{a})d\tilde{a}$$
$$= \mu\left[(1-\theta)\ln y_H + \theta\ln\left(\frac{n\mu+1}{n+1}\right)\right] + (1-\mu)\left[(1-\theta)\ln y_L + \theta\ln\left(\frac{n\mu}{n+1}\right)\right].$$

If the individual does not take the risky action, he has utility of $(1-\theta)\ln y_0 + \theta\ln\mu$ for certain.

Solution of the Model

The solution to the individual's problem involves the choice of a cut-off return r^*. Given his priors, the individual expects his utility to be

$$E[U|\mu,n] = \int_1^{r^*}\left[(1-\theta)\ln y_0 + \theta\ln\mu\right] f(r)dr$$
$$+ \int_{r^*}^{\infty}\left\{(1-\theta)\ln y_L + \mu\left[(1-\theta)\ln r + \theta\ln\left(\frac{n\mu+1}{n+1}\right)\right] + (1-\mu)\theta\ln\left(\frac{n\mu}{n+1}\right)\right\} f(r)dr$$

Differentiating with respect to r^* to obtain the first order condition for the optimal r^* and rearranging implies

$$\ln r^* = \frac{1}{\mu}\ln\left(\frac{y_0}{y_L}\right) + \frac{\theta}{(1-\theta)}\left[\ln\mu - \mu\theta\ln\left(\frac{n\mu+1}{n+1}\right) - (1-\mu)\theta\ln\left(\frac{n\mu}{n+1}\right)\right] > 0$$

It is possible to show that the term in brackets is positive, so $r^* > \ln(y_O/y_L) > 0$.[8]

The individual's expected utility given his actual ability and his priors μ and n is

$$E[U|\mu,n,a] = \int_1^{r^*}\left[(1-\theta)\ln y_0 + \theta\ln\mu\right] f(r)dr$$
$$+ \int_{r^*}^{\infty}\left\{(1-\theta)\ln y_L + a\left[(1-\theta)\ln r + \theta\ln\left(\frac{n\mu+1}{n+1}\right)\right] + (1-a)\theta\ln\left(\frac{n\mu}{n+1}\right)\right\} f(r)dr.$$

---

[8] This result follows from the fact that $\mu\left(\frac{n\mu+1}{n+1}\right) + (1-\mu)\left(\frac{n\mu}{n+1}\right) = \mu$ and from concavity of the natural log function.

This expression differs from $E[U|\mu,n]$ in that the probabilities of success and failure depend on the individual's actual ability, rather than his expectation of it based on his priors. We assess the effect of the individuals priors on his utility using this expression, which conditions on the individual's true ability.

Effect of an Increase in Expected Ability

This section considers the effect of overconfidence on task choice and utility (overall and separately from consumption and ability, which we refer to as "delusion"). In this model, overconfidence arises when the mean of a person's priors over his ability exceed his actual ability ($\mu>a$).[9] We analyze the effect of increasing confidence, as reflected by the mean of a person's priors over his ability, μ, holding ability constant and show that increasing μ raises utility so long as the person is not too over confident (so long as μ is not too high relative to a). This fact, that utility is higher when μ exceeds a somewhat than when $\mu=a$ indicates that moderate overconfidence is optimal.

An increase in μ affects expected utility according to,

$$\frac{\partial E[U|\mu,n,a]}{\partial \mu}=\frac{\theta}{\mu}F\left(r^*\right)+\theta\left[a\frac{n+1}{n\mu+1}+(1-a)\frac{n+1}{n\mu}\right]\left(1-F\left(r^*\right)\right)+\frac{\partial E[U|\mu,n,a]}{\partial r^*}\frac{\partial r^*}{\partial \mu}.$$

The first two terms reflect the effect of higher μ on the individual's utility through his posteriors over his ability (delusion), and are unambiguously positive. The third term gives the effect of μ on the individual's utility through his willingness to perform the challenging task.

[9] Another worthwhile approach would be to specify a distribution of abilities and a distribution of priors and to view people as overconfident when the mean of the distribution of priors exceeds the mean of the distribution of abilities.

Turning to the third term, as shown in the appendix, an increase in μ lowers the threshold for undertaking the task, $\frac{\partial r^*}{\partial \mu} < 0$. The effect of increasing the probability of undertaking the task on utility is,

$$\frac{\partial E[U|\mu,n,a]}{\partial r^*} = \left(1-\frac{a}{\mu}\right)\left[(1-\theta)\ln\left(\frac{c_O}{c_L}\right)+\theta\ln\left(\frac{n+1}{n}\right)\right] f\left(r^*\right)\begin{cases} >0 \text{ if } \mu > a \\ =0 \text{ if } \mu = a \\ <0 \text{ if } \mu < a \end{cases}.$$

This expression gives the effect on utility (combining the consumption and delusion effect) of increases in the threshold for undertaking the risky task. The term in brackets is positive, so raising the threshold for choosing the challenging task, which makes the person less likely to undertake it, raises expected utility when the person initially overestimates his ability ($\mu > a$) and lowers it when the person underestimates his ability ($\mu < a$). Intuitively, a person who correctly estimates his ability (in the sense that μ=a) undertakes the risky task if it is the optimal action, whereas someone who under- (over-) estimates his ability takes the risky action too little (often). Increasing the threshold is neutral for people whose priors reflect their ability, and reduces (raises) utility for people who under- (over-) estimate their ability.

Combining all terms, increasing the person's priors over his ability increases his expected utility so long as his priors understate his ability or do not overstate it too greatly and so long as the effect of beliefs over ability are not too strong (θ is not too high). When the priors overstate his ability too much, the cost from being overly ambitious exceeds the direct utility gain from believing his ability is high, which is the case of Smith's soldiers. This result can be shown by calculating the change in utility

from increases in μ as μ approaches 1. Using the fact that $\lim_{\mu \to \infty} r^* = \frac{c_O}{c_L}$, it is possible to show that

$$\lim_{\mu \to 1} \frac{\partial E[U|\mu,n,a]}{\partial \mu} = \theta\left(\frac{n+1-a}{n}\right) - \frac{c_O}{c_L}(1-a)\left[(1-\theta)\ln\left(\frac{c_O}{c_L}\right) + \theta\ln\left(\frac{n+1}{n}\right)\right] f\left(\frac{c_O}{c_L}\right).$$

The first term is positive, while the second is negative. Thus, so long as θ is not too high (so that the person does not benefit too much from delusion), the combined expression will be negative. Also, the cost of overestimating ability is to get the low consumption instead of the certain consumption. Thus, as $\frac{c_O}{c_L}$ increases (or as the density of tasks where the high payoff just exceeds c_o, $f\left(\frac{c_O}{c_L}\right)$, increases) the cost from greatly overestimating ability increases.

Increasing the mean of the individual's priors also raises his expected utility from consumption. The individual's expected utility from consumption is,

$$E[\ln y|\mu,n,a] = \int_1^{r^*} \ln y_0 \, f(r)dr + \int_{r^*}^{\infty} (\ln y_L + a\ln r) f(r)dr .$$

It is possible to show that,

$$\frac{\partial E[\ln y|\mu,n,a]}{\partial r^*} = \left\{\left(1-\frac{a}{\mu}\right)\ln\left(\frac{c_0}{c_L}\right) - \frac{\theta}{1-\theta}\frac{a}{\mu}\left[\ln\mu - \mu\ln\left(\frac{n\mu+1}{n+1}\right) - (1-\mu)\ln\left(\frac{n\mu}{n+1}\right)\right]\right\} f(r^*)$$

The term in brackets is positive, so when $\mu = a$, an increase in r^*, as would arise if μ were reduced, lowers expected utility from consumption. Only when $\mu >> a$, do increases in μ reduce utility from consumption. Intuitively, in order to avoid risk from learning about his ability, the agent undertakes the challenging task only when its

expected payoff is very high. Slight overestimation of ability offsets his conservativism in undertaking the task and raises his expected utility and expected utility from consumption. When the individual overestimates his ability substantially, he undertakes the task when its expected return is too low. Further increases in his ability lead him to undertake the task too frequently and reduce his expected consumption utility.

Effect of an Increase in the Precision of Priors

This section considers the effect of increasing the precision of the individual's priors, holding the mean of the priors constant. Formally, we consider the effect of increasing *n*, which affects expected utility according to,

$$\begin{aligned}\frac{\partial E[U|\mu,n,a]}{\partial n} &= \theta\left[a\frac{n+1}{n\mu+1}\frac{\mu-1}{(n+1)^2}+(1-a)\frac{n+1}{n\mu}\frac{\mu}{(n+1)^2}\right]\left(1-F\left(r^*\right)\right)+\frac{\partial E[U|\mu,n,a]}{\partial r^*}\frac{\partial r^*}{\partial n}\\ &= \theta\frac{n\mu(\mu-a)+\mu(1-a)}{(n\mu+1)(n+1)n\mu}\left(1-F\left(r^*\right)\right)\\ &\quad +\left(\frac{a}{\mu}-1\right)\left[(1-\theta)\ln\left(\frac{c_O}{c_L}\right)+\theta\ln\left(\frac{n+1}{n}\right)\right]f\left(r^*\right)\frac{\theta}{1-\theta}\frac{1-\mu}{(n\mu+1)(n+1)n\mu}r^*\end{aligned}.$$

When the individual undertakes the task, an increase in *n* leads the person to place more weight on his priors and less weight on the information conveyed by the outcome of the task. Underweighting new information raises utility from ability when the low outcome is realized, but lowers it when the high outcome is realized. Because the individual is risk averse to new information about his ability, the decrease in utility when the low state is realized exceeds the increase when the high state is realized when $\mu = a$, so the individual benefits from downweighting new information when $\mu = a$.

Only when the priors are sufficiently low relative to ability does increasing *n* lower utility – when $\mu < a - \frac{1-a}{n} < a$ the probability that an unbiased signal raises

beliefs is sufficiently high to exceed the benefits from increased exposure to risk. Increasing *n* leads the person to reduce r^* and undertake the task more frequently. When $\mu = a$, changes in r^* are neutral in terms of utility but, as above, they raise consumption utility. Thus, the person benefits from believing that his priors are more precise, in the sense of higher *n* when the mean of his priors equal his true ability.

IV. Extensions and Robustness

This section considers two alternative specifications. First, we consider our assumption of risk aversion in beliefs over ability. This assumption plays an important role in our results. If beliefs over ability do not directly enter the utility function then deviations of expected ability (the mean of priors) from actual ability in either direction will worsen task decisions and lower expected consumption and utility. If ability directly enters the utility function, but linearly, increases in expected ability will raise utility because of delusion, but reduce consumption. In this case, there is a tradeoff between ability utility and consumption utility, which can generate an interior optimal level of overconfidence. Thus, the implication that overconfidence can raise consumption utility depends in risk aversion in posteriors over ability, but the implication that overconfidence can be optimal only depends on beliefs directly affecting utility.

It is also worth considering an extension to a multi-period model. In the single-period model discussed so far, a person who overestimates the precision of his priors does experience a cost in terms of consumption. A person who overestimates the precision of his priors updates his priors more slowly. In a multi-period setting, this slower updating adds a cost in terms of worse task choices and lower consumption. In a multi-period setting, while overestimating the precision of priors can be helpful in

inducing the person to undertake the risky task more frequently, there is an additional cost to overestimating the precision of priors.

V. Conclusions and the Evolution of Overconfidence

Psychologists have found that people are overconfident in their endeavors. Extreme overconfidence and under-confidence imposes considerable costs, but moderate overconfidence appears to be both pervasive and advantageous. Individuals maintain overconfident beliefs in part by biases in information processing – underweighting new information about their ability, especially when it is negative. We have shown that people who care about and are risk averse over their beliefs about their ability and who choose tasks based on their information derive benefits from being overconfident. Specifically, expected output is higher for individuals who moderately overestimate their ability. Expected output is also higher for people who overestimate the precision of their initial information, which implies an underweighting of new information.

If moderate overconfidence raises expected output and expected utility, it is natural to consider how it might be perpetuated and spread in the population. Although we are not aware of studies that study the development and transmission of overconfidence, researchers have argued that childhood experiences are important determinants of self esteem (Dutton and Brown [1997]). It is reasonable to expect that individuals with higher utility and output would tend to have more children, because they have higher fitness and greater ability to support offspring and because they are more attractive to mates. If the information processing strategies that generate overconfidence are transmitted by parents to their children (perhaps in part genetically, as with depression, an extreme form of under-confidence), we would expect the population to

evolve so that the average member of the population will be overconfident. Alternatively, children may acquire these information processing strategies – filtering negative information and dwelling on positive information – by emulating successful individuals in the preceding generation. Indeed, considerable attention (including work on self-help) has been devoted to ways of improving self esteem among children and adults. For all these reasons, we expect the population to evolve toward overconfidence.

Appendix – Proof

This section shows that $\frac{\partial r^*}{\partial \mu} < 0$. We have that

$$\ln r^* = \frac{1}{\mu}\ln\left(\frac{y_0}{y_L}\right) + \frac{\theta}{(1-\theta)}\left[\ln\mu - \mu\theta\ln\left(\frac{n\mu+1}{n+1}\right) - (1-\mu)\theta\ln\left(\frac{n\mu}{n+1}\right)\right] > 0 .$$

So,

$$\begin{aligned}\frac{\partial r^*}{\partial \mu} &= -\frac{r^*}{\mu} + \frac{r^*}{\mu}\frac{\theta}{1-\theta}\left[\frac{1}{\mu} + \ln\left(\frac{n\mu}{n+1}\right) - \ln\left(\frac{n\mu+1}{n+1}\right) - \mu\frac{n+1}{n\mu+1}\frac{n}{n+1} - (1-\mu)\frac{n+1}{n\mu}\frac{n}{n+1}\right] \\ &= -\frac{r^*}{\mu} + \frac{r^*}{\mu}\frac{\theta}{1-\theta}\left[\frac{1}{\mu} + \ln\left(\frac{n\mu}{n+1}\right) - \ln\left(\frac{n\mu+1}{n+1}\right) - \mu\frac{n}{n\mu+1} - (1-\mu)\frac{1}{\mu}\right] \\ &= -\frac{r^*}{\mu} + \frac{r^*}{\mu}\frac{\theta}{1-\theta}\left[\ln\left(\frac{n\mu}{n+1}\right) - \ln\left(\frac{n\mu+1}{n+1}\right) - \frac{n\mu}{n\mu+1} + 1\right]\end{aligned}$$

To sign the expression in brackets, we use a first order Taylor expansion of the logarithm terms. The derivative of the natural logarithm is lowest at the higher value, $\frac{n\mu+1}{n+1}$ than at any other point on the interval $\left[\frac{n\mu}{n+1}, \frac{n\mu+1}{n+1}\right]$, so,

$$\ln\left(\frac{n\mu+1}{n+1}\right) > \ln\left(\frac{n\mu}{n+1}\right) + \frac{n+1}{n\mu+1}\frac{1}{n+1} = \ln\left(\frac{n\mu}{n+1}\right) + \frac{1}{n\mu+1} .$$

Thus,

$$\ln\left(\frac{n\mu+1}{n+1}\right) - \ln\left(\frac{n\mu}{n+1}\right) > \frac{1}{n\mu+1} .$$

Substituting this expression into the one above and simplifying yields $\frac{\partial r^*}{\partial \mu} < -\frac{r^*}{\mu} < 0$.

References

Ackermann, Ruby, and Robert J. DeRubeis. "Is Depressive Realism Real?" *Clinical Psychology Review* 11 (No. 5, 1991): 565-584.

Alloy, Lauren B. and Lyn Y. Abramson. "Judgement of Contingency in Depressed and Nondepressed Students: Sadder but Wiser?" *Journal of Experimental Psychology: General* 108 (No. 4, 1979): 441-485.

Alloy, Lauren B. and Lyn Y. Abramson. "Depressive Realism: Four Theoretical Perspectives. In *Cognitive Processes in Depression*, Lauren B. Alloy and Lyn Y. Abramson, eds. New York: Guilford Press.

Alpert, Marc, and Howard Raiffa. "A Progress Report on the Training of Probability Assessors." in *Judgement Under Uncertainty: Heuristics and Biases,* Daniel Kahneman, Paul Slovic, and Amos Tversky, eds. Cambridge, England: Cambridge University Press. 1982.

Bandura, Albert. "Self-Efficacy: The Exercise of Control." New York: W. H. Freeman and Company. 1997.

Bandura, Albert. *Social Foundations of Thought and Action: A Social Cognitive Theory*. Englewood Cliffs, NJ: Prentice-Hall. 1986.

Benabou, Roland and Jean Tirole. "Self-Confidence: Intrapersonal Strategies." Working Paper, Université des Sciences Sociales, Toulouse. 1999a.

Benabou, Roland and Jean Tirole. "Self-Confidence: Interpersonal Strategies." Working Paper, Université des Sciences Sociales, Toulouse. 1999b.

Beck, Aaron T. *Depression: Clinical Experimental, and Theoretical Aspects.* New York, Evanston, and London: Harper & Row. 1967.

Bernardo, Antonio, and Ivo Welch. "On the Evolution of Overconfidence and Entrepreneurs." Working Paper. 1999.

Braumeister, Roy F. "The Self" in *The Handbook of Social Psychology, Volume I*. Daniel T. Gilbert, Susan T. Fiske, and Gardner Lindzey, eds. New York: McGraw Hill. 1998.

Brown, Jonathon D., and Kieth A. Dutton. "The Thrill of Victory, the Complexity of Defeat: Self-Esteem and People's Emotional Reactions to Success and Failure." *Journal of Personality and Social Psychology* 68 (No. 4, April 1995): 712-22.

Carrillo, Juan D. and Thomas Mariotti. "Strategic Ignorance as a Self-Disciplining Device." *Review of Economic Studies* 67 (No. 3, 2000): 529-44

Colvin, C. Randall, and Jack Block. “Do Positive Illusions Foster Mental Health? An Examination of the Taylor and Brown Formulation.” *Psychological Bulletin* 116 (No. 1, July 1994).

Colvin, C. Randall, and Jack Block. “Positive Illusions and Well-Being Revisited Separating Fiction from Fact.” *Psychological Bulletin* 116 (No. 1, July 1994).

Cunha, A. Cristina. *The Effects of Clinical Depression on Schooling and Wages*. Doctoral Dissertation, The Ohio State University. 1997.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam. “Investor Psychology and Security Market Under- and Over-Reactions.” *Journal of Finance* 53 (No. 6, December 1998): 1839-85.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. “The Survival of Noise Traders in financial Markets.” *Journal of Business* (No. 1, January 1991): 1-19.

Dobson, Keith, and Renée-Louise Franche. “A Conceptual and Empirical Review of the Depressive Realism Hypothesis.” *Canadian Journal of Behavioural Science* 21 (No. 4, October 1989): 419-433.

Elster, Jon. “Introduction.” in *Rational Choice,* Jon Elster, editor. New York: New York University Press. 1986.

Feller, William. *An Introduction to Probability Theory and its Applications.* New York. John Wiley and Sons. 1968.

Gervais, Simon and Itay Goldstein. “Overconfidence and Team Coordination.” Working paper, Duke University. 2004.

Gilboa, Itzhak and David Schmeidler. “Case-Based Optimization.” *Games and Economic Behavior* 15 (1996): 1-26.

Jovanovic, Boyan and Dmitriy Stolyarov. “Ignorance is Bliss.” Working Paper, New York University. 1999.

Koszegi, Botond. “Ego Utility, Overconfidence, and Task Choice.” Working Paper, University of California at Berkeley. 2000a.

Koszegi, Botond. “Ego Utility and Information Acquisition.” Working Paper, University of California at Berkeley. 2000b.

Kyle, Albert S. and F. Albert Wang. “Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test?” *Journal of Finance* 52 (No. 5, December 1997): 2073-90.

Lichtenstein, Sarah, Baruch Fischhoff, and Lawrence D. Phillips. “Callibration Probabilities: The State of the Art to 1980.” in *Judgement Under Uncertainty: Heuristics and Biases*, Daniel Kahneman, Paul Slovic, and Amos Tversky, eds. Cambridge, England: Cambridge University Press. 1982.

Miller, Dale T., and Michael Ross. “Self-Serving Biases in the Attribution of Causality: Fact or Fiction?” *Psychological Bulletin* 82 (No. 2, March 1975): 213-25.

Oskamp, Stuart. “Overconfidence in Case-Study Judgements.” *Journal of Consulting Psychology* 29 (1965): 261-265.

Ruehlman, Linda S., Stephen G. West, and Robert J. Pasahow. “Depression and Evaluative Schemata.” *Journal of Personality* 53 (No. 1, March 1985): 46-92.

Seligman, Martin E. *Helplessness: On Depression, Development, and Death*. San Francisco: W. H. Freeman and Company. 1975.

Shrauger, J. Sidney. “Responses to Evaluation as a Function of Initial Self-Perceptions.” *Psychological Bulletin* 82 (No. 4, July 1975): 581-96.

Shrauger, J. Sidney, and Thomas J. Shoeneman. “Sybolic Interactionst View of Self-Concept: Through the Looking Glass Darkly.” *Psychological Bulletin* 86 (No. 3): 549-573.

Smith, Adam. *An Inquiry into the Nature and Causes of The Wealth of Nations*. Chicago: University of Chicago Press. 1776 [1976].

Taylor, Shelley E. and Jonathon D. Brown. “Illusion and Well-Being: A Social Psychological Perspective on Mental Health.” *Psychological Bulletin* 103: (No. 2): 193-210.

Taylor, Shelley E. and Jonathon D. Brown. “Positive Illusions and Well-Being Revisited Separating Fiction from Fact.” *Psychological Bulletin* 116 (No. 1, July 1994).

Tice, Dianne M. “Esteem Protection or Enhancement? Self-Handicapping Motives and Attributions Differ by Trait Self-Esteem.” *Journal of Personality and Social Psychology* 60 (No. 5, May 1991): 711-725.

Waldman, Michael. “Systematic Errors and the Theory of Natural Selection.” *American Economic Review* 84 (No. 3, June 1994): 482-497.

Zuckerman, Miron. “Attribution of Success and Failure Revisited, or: The Motivational Bias is Alive and Well in Attribution Theory.” *Journal of Personality* 47 (No. 2, June 1979): 245-87.