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Transport Costs and the Size Distribution of a Linearly Arranged System of Cities

Abstract

The question regarding the effects of changing transports costs on the size distribution of cities is an important topic of systems of cities research. The so-called New Economic Geography has already given some answers to this question. One central assumption in this kind of model is a very particular, simplified spatial structure. This contribution investigates the consequences of changing transport costs for a system of cities that are located equidistantly on a straight line. In the case of rising transport costs, the main outcome of this model is worker concentration in the central large cities, while the peripheral regions lose residents.

Keywords: transport costs, agglomeration, urban systems

JEL classification: F12, R12, R13

Transportkosten und die Größenverteilung eines linear angeordneten Städtesystems

Zusammenfassung

Die Auswirkungen von Transportkostenänderungen auf die Größenverteilung von Städtesystemen ist eine wichtige Forschungsfrage. Im Rahmen der sogenannten Neuen Ökonomischen Geographie wurden bereits Antworten gefunden, allerdings nur für eine sehr spezielle, stark vereinfachte Raumstruktur. Der vorliegende Beitrag untersucht die Auswirkungen veränderter Transportkosten auf ein System von Städten, die gleichmäßig auf einer Geraden angeordnet sind. Für den Fall steigender Transportkosten zeigt sich, dass es zur Bevölkerungskonzentration in den großen, im Zentrum des Systems gelegenen Städten kommt, während die peripheren Städte Einwohner verlieren.

Schlagwörter: Transportkosten, Agglomeration, Städtesysteme

JEL-Klassifikation: F12, R12, R13

1 Introduction

One of the original aims of New Economic Geography is to explain how systems of cities can arise and change in a landscape that is, apart from economic aspects, featureless. In chapter 10 of "The Spatial Economy", Fujita, Krugman and Venables (1999) modelled the birth of new cities initiated by external population growth, keeping transport costs constant. In chapter 11 of the same book this modelling approach is applied to the hierarchical orders of consumer preferences, generating hierarchical systems of cities. Tabuchi, Thisse and Zeng (2005, further called TTZ) firstly analysed the effect of changing costs of the transportation of goods on the size distribution of a constant population living in a given system of cities. They expanded Ottaviano, Tabuchi and Thisse's (2002, further called OTT) model to n regions with a very simple spatial geometry. Tabuchi/Thisse (2008) examined the evolution of hierarchical systems of cities in relation to changing transport costs. The spatial structure of the latter approach is the so-called race-track economy, previously applied in the multiregional framework of chapter 6 of Fujita *et al.* (1999).

The aim of this paper is to show how changing transport costs influence the size distribution of a non-hierarchical system of cities with a constant total population, in which the spatial arrangement of the cities allows the effects of heterogeneous distances to be seen. The modelling approach is closest to that of TTZ, but differs from the latter in two aspects: Firstly, I assume a spatial structure with not only one and the same distance between each pair of cities (while TTZ assume only one distance). Secondly, in my model, urban costs are assumed as one general function of urban size for all the cities of the system (while TTZ modelled specific urban costs functions for each city). These changed basic assumptions mean I interpret my main results with regard to the relationships between cities that are located in the centre of the system or at its periphery in terms of a secular time trend or the long-standing impact of transport costs. In contrast, TTZ interpret their results referring to phenomena that have little to do with specific distance aspects (particularly, urbanisation and suburbanisation). Because of the simplicity of the spatial structure of their model, TTZ could solve their model analytically. The model presented here is solvable only by means of numerical simulations.

My main findings are: If the elements of a system of cities are arranged equidistantly on a straight line, changing transport costs have different effects on cities that are closer to the geographical centre of the system than on cities that are located further away, near the 'ends' of the system. For example, in the case of rising costs of the transportation of goods, the mobile population will concentrate in cities around the centre of the system, withdrawing from peripheral locations. Again, falling transportation costs enable the production of goods by skilled labour in remote areas, amplifying the population spread to all regions of the system including peripheral ones. The model has been applied to, amongst others, the increased costs of freight transportation in the Russian Federation as a consequence of the price liberalisation after 1991, since the prices of energy (and hence of transportation services) have multiplied in real terms.[1]

The paper is structured as follows: Section 2 outlines the framework of the general equilibrium model for n regions based on OTT. Section 3 derives equilibrium prices, quantities, wages and indirect utilities. Section 4 presents spatial equilibria resulting from migration processes, which equalise utility differentials between cities. Section 5 is the paper's conclusion.

2 Basic assumptions

The spatial arrangement of the n regions in consideration is shown in fig. 1. Each of them contains one monocentric city with a central business district (CBD), marked by the points in fig. 1. The distance between the CBDs of two adjacent cities is assumed to be one.

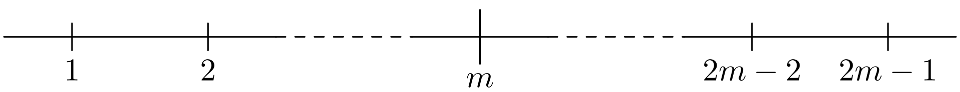

Figure 1: Linear and equidistant formation of $n = 2m - 1$ cities

In addition to the TTZ model, this model is based on the "alternative" approach to modelling agglomeration and trade developed by Ottaviano *et al.* (2002) for two regions, enhanced for urban costs (the OTT model). The factors of production and sectors (or goods, resp.) are characterized by the assumptions of that model.[2]

There are two goods and two factors of production. The factors of production are denoted as A and L. They are assumed to be constant. The A-workers have no qualifications. They are immobile and equally distributed in the space between the

[1] See Kauffmann (2010). Another reason for increasing transport costs over a longer time period could be the shortage of energy resources as a consequence of diminishing supply factors or rising demand (see, e.g., Bräuninger *et al.*, 2005).

[2] The reader will find a detailed explanation of the OTT model in Kauffmann's 2010 work.

cities. They produce the homogenous A-good under constant returns to scale. The transport of the A-good is costless, it is produced and sold under the conditions of perfect competition. The value of one unit of the A-good equals the marginal costs of its production. The A-good is the *numéraire* of the model. The L-workers are highly qualified, they can choose their preferred location in one of the n cities and each of them will migrate if there is any city where the realized indirect utility is higher than in the city where he (or she) actually lives. The L-good is heterogeneous; its production shows increasing returns to scale. It is produced and sold under conditions of monopolistic competition. The transport of the L-good causes costs of τ units of the *numéraire* per unit of itself and of distance. This leads to the matrix of transportation costs $\mathbf{\Delta}$,

$$\mathbf{\Delta} = \begin{pmatrix} 0 & 1 & \dots & m-1 & \dots & n-2 & n-1 \\ 1 & 0 & \dots & m-2 & \dots & n-3 & n-2 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ m-1 & m-2 & \dots & 0 & \dots & m-2 & m-1 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ n-2 & n-3 & \dots & m-2 & \dots & 0 & 1 \\ n-1 & n-2 & \dots & m-1 & \dots & 1 & 0 \end{pmatrix} \tag{1}$$

which is a $n \times n$-Toeplitz-matrix.

The portion of qualified workers living in a specific city of the entire population of qualified workers is represented by the vector $\boldsymbol{\lambda}$. Each variety of the heterogeneous product is produced by a firm that employs ϕ L-workers. The product is supplied in all cities and in the “agricultural” space between them. There are $N = \frac{L}{\phi}$ firms. The variable costs generated by the production of the L-good are assumed to be zero.

Every worker has the quasilinear quadratic utility function

$$U(q_0; q(x), x \in [0, N]) = \alpha \int_0^N q(x)dx - \frac{\beta - \gamma}{2} \int_0^N [q(x)]^2 dx - \frac{\gamma}{2} \Big[\int_0^N q(x)dx \Big]^2 + q_0, \tag{2}$$

where $q(x)$ represents the consumed quantities of the heterogeneous good in a continuous space of varieties reaching from 0 to N, q_0 is the consumed quantity of the *numéraire*, and α, β and γ are positive parameters. If $\beta > \gamma$ the consumer prefers the whole bundle of varieties (love-for-variety).

The L-workers live in cities that form symmetrically around the CBD with a city radius of length $2 \times \frac{\lambda_i}{2n}$. They commute to their working places in the CBD. Urban costs are generated as the sum of costs for using urban space and commuting costs.[3] The same urban cost function $\Theta(\lambda_i)$ is assumed for all cities,

$$\Theta(0) = 0, \qquad \Theta(1) < \infty, \qquad \Theta'(\lambda_i) \geq 0. \qquad \lambda_i \in [0, 1] \tag{3}$$

The budget of every L-worker living in city i is restricted, consequently, to

$$\int_0^N p(x)q(x)dx + q_0 = w_i + \bar{q}_0 - \Theta(\lambda_i), \tag{4}$$

wherein $\bar{q}_0$ represents the initial endowment as a quantity of the *numéraire*. Solving the optimisation task, we get a linear demand function for variety x

$$q(x) = a - bp(x) + c\int_0^N [p(j) - p(x)]dj \tag{5}$$

with parameters

$$a = \frac{\alpha}{\beta + (N-1)\gamma}, \quad b = \frac{1}{\beta + (N-1)\gamma}, \quad c = \frac{\gamma}{(\beta - \gamma)(\beta + (N-1)\gamma)}. \tag{6}$$

The indirect utility function of a worker living in city i is

$$V_i = \frac{a^2 N}{2b} - a\int_0^N p(x)dx + \frac{b + cN}{2}\int_0^N [p(x)]^2 dx - \frac{c}{2}\left[\int_0^N p(x)dx\right]^2 + \bar{q}_0 + w_i - \Theta(\lambda_i). \tag{7}$$

Every city has its own "agricultural" hinterland (where the population of A-workers lives) symmetrically around its line space. A city together with its hinterland is defined as a "region".

3 Determination of equilibrium

The next task is to describe the state of equilibrium of the model. To find the equilibrium prices, we have to determine the derivative of the profit function of a

[3] For further assumptions see Ottaviano *et al.* (2002) p. 430.

representative firm and set it at zero. The operative profit Π of a firm residing in region i amounts to

$$\Pi_i(\boldsymbol{\lambda}) = \sum_j (p_{ij} - \delta_{ij}\tau) q_{ij} \left(\frac{A}{n} + \lambda_j L \right) \tag{8}$$

with distance $\delta_{ij} \in \boldsymbol{\Delta}$, p_{ij} (resp. q_{ij}) the price (resp. quantity) of a variety of the heterogeneous good produced in region i and consumed in region j. The price index for region i is

$$P_i = N(\lambda_i p_{ii} + \sum_{j \neq i} \lambda_j p_{ji}) \;=\; N \sum_{j=1}^{n} \lambda_j p_{ji}. \tag{9}$$

If we replace the demand for one variety in the region of its production q_{ii} in the derivative of eq. (8) with respect to its mill price p_{ii}

$$\frac{d\Pi_i}{dp_{ii}} = \left(q_{ii} + p_{ii} \frac{dq_{ii}}{dp_{ii}} \right) \left(\frac{A}{n} + \lambda_i L \right), \tag{10}$$

by

$$q_{ii} = a - (b + cN) p_{ii} + cP_i \tag{11}$$

we get the equilibrium price p^* of a variety of the homogeneous good both produced and consumed in region i,[4]

$$p^*_{ii} = \frac{a + cP_i}{2(b + cN)}. \tag{12}$$

The derivatives of the profit function with respect to prices in regions $j \neq i$

$$\frac{d\Pi_i}{dp_{ij}} = \left(q_{ij} + p_{ij} \frac{dq_{ij}}{dp_{ij}} - \delta_{ij}\tau \frac{dq_{ij}}{dp_{ij}} \right) \left(\frac{A}{n} + \lambda_j L \right) \tag{13}$$

lead, with

[4] Similarly to Chamberlain's 1933 monopolistic competition model each firm takes the price index P_i as given. Accordingly, the derivative of the firm demand with respect to the mill price, $\frac{dq_{ii}}{dp_{ii}}$, is $-(b + cN)$. This also applies to the derivatives of the demand functions of this firm from other regions.

$$q_{ij} = a - (b + cN)p_{ij} + cP_j, \tag{14}$$

to equilibrium prices

$$p^*_{ij} = \frac{a + cP_j + (b + cN)\delta_{ij}\tau}{2(b + cN)} \qquad \forall\ j \neq i. \tag{15}$$

Analogously we get as equilibrium mill prices in regions j

$$p^*_{jj} = \frac{a + cP_j}{2(b + cN)}. \tag{16}$$

In region i these varieties are sold at prices of

$$p^*_{ji} = \frac{a + cP_i + (b + cN)\delta_{ij}\tau}{2(b + cN)}. \tag{17}$$

With equilibrium prices, the price index of region i will be

$$P_i = \lambda_i N p^*_{ii} + \sum_{j \neq i} \lambda_j N(p^*_{ii} + \frac{1}{2}\delta_{ij}\tau) = Np^*_{ii} + \frac{1}{2}N\tau \sum_j \lambda_j \delta_{ij}. \tag{18}$$

Replacing P_i in eq. (12) with eq. (18) leads to

$$p^*_{ii} = \frac{2a + c\tau N \sum_j \lambda_j \delta_{ij}}{2(2b + cN)} = \frac{2a + c\boldsymbol{\lambda}^T \boldsymbol{\delta}_i \tau N}{2(2b + cN)}, \tag{19}$$

with $\boldsymbol{\delta}_i$ as column i of $\boldsymbol{\Delta}$.[5] From eq. (12) and (17) we get

$$p^*_{ji} = p^*_{ii} + \frac{1}{2}\delta_{ij}\tau. \tag{20}$$

In equilibrium, we get a regional price "landscape". That these different regional prices cannot be balanced by arbitrage is shown in the appendix. Eq. (20) tells us that half of the transport costs are born by firms, and half by consumers.

Our next issue is to show the condition that ensures that the heterogeneous good is allocated to all regions, in other words, that the price of none of the varieties of the L-good reaches its prohibitive limit, or

[5] All vectors are assumed as columns; row vectors are transposed column vectors (e.g., $\boldsymbol{\lambda}^T$).

$$q_{ij} \overset{!}{>} 0. \tag{21}$$

Inserting P_j from eq. (9) in eq. (14), we get, after some computations,[6]

$$q_{ij} = \frac{b + cN}{2b + cN}\Big[a - b\delta_{ij}\tau + cN(\boldsymbol{\lambda}^T \boldsymbol{\delta}_j - \delta_{ij})\frac{\tau}{2}\Big]. \tag{22}$$

In the case of an increase of the freight transport rate τ, the demand for varieties from other regions declines; however, because of the influence of substitution for other varieties on demand, it may rise, too. This result may seem implausible at first glance, but, looking at the distance matrix $\boldsymbol{\Delta}$ we find that, particularly for adjacent columns $(j-1, j)$ far from the central column m, the $\boldsymbol{\lambda}$-weighted sum of $\boldsymbol{\delta}_j$ may exceed one ($\boldsymbol{\lambda}^T \boldsymbol{\delta}_j > \delta_{ij}$). In these cases raising transport costs will strengthen the trade between adjacent regions at the periphery of the system while weakening the exchange of goods between centre and peripheral regions. Conversely, declining transport costs will promote trade relations between central and peripheral regions at the expense of trade between neighbouring regions.

The infimum of (for two regions i and j, at least) prohibitive freight transport rates, suppressing trade between these regions to zero, is denoted by τ_{Trade}. Analogous to OTT, the insertion of eq. (22) into eq. (21) leads to

$$\begin{aligned} a &\overset{!}{>} \Big(2b\delta_{ij} - cN(\boldsymbol{\lambda}^T \boldsymbol{\delta}_j - \delta_{ij})\Big)\ \frac{\tau}{2}, \\ \tau &\overset{!}{<} \frac{2a}{2b\delta_{ij} - cN(\boldsymbol{\lambda}^T \boldsymbol{\delta}_j - \delta_{ij})}, \\ \tau &\overset{!}{<} \frac{2a}{(2b + cN)\delta_{ij} - cN\boldsymbol{\lambda}^T \boldsymbol{\delta}_j}. \end{aligned} \tag{23}$$

Because of the symmetry of the spatial arrangement, $\boldsymbol{\Delta}$ is a symmetric matrix. Therefore, in spatial equilibrium, every distribution $\boldsymbol{\lambda}$ should be symmetric, too. Looking at eq. (1), we quickly can see that, for symmetric $\boldsymbol{\lambda}$,

$$\boldsymbol{\lambda}^T \boldsymbol{\delta}_1 = \boldsymbol{\lambda}^T \boldsymbol{\delta}_n = m - 1 \tag{24}$$

[6] See appendix.

holds.[7] Therefore, for the largest distance $\delta_{1n} = n-1$ we can write eq. (23)[8] as

$$\begin{aligned} \tau &\overset{!}{<} \frac{2a}{(2b+cN)(n-1)-cN(m-1)}, \\ \tau_{\text{Trade}} &= \frac{2a}{(2b+\frac{cN}{2})(n-1)}. \end{aligned} \tag{25}$$

The influence of parameters c and N on the maximum value that the freight transportation rate might have, so that the supply of the heterogeneous good is ensured in all regions, indicates that interregional trade could be regarded as a precondition for product differentiation and preference for variety. The relationship between τ_{Trade} and the spatial extension of the urban system refers to the fact that low transport costs are a precondition for the evolution of large urban systems. Increasing transport costs may bring into question the continuation of human settlements in peripheral regions.

To determine the quantities in equilibrium, we put the equilibrium price p^*_{ii} and the price index P_i from eq. (19) and (18), respectively, into the demand function eq. (11). So doing[9], we get the quantity of the heterogeneous good produced and sold in region i,

$$q^*_{ii} = (b+cN)p^*_{ii} \tag{26}$$

Analogously, the substitution of p^*_{ij} and P_j in eq. (14) leads to

$$q^*_{ij} = (b+cN)(p^*_{ij} - \delta_{ij}\tau) \tag{27}$$

for the quantity of the heterogeneous good produced in city i and sold in a different region j.

It is implied by assumption that the operational profits are completely distributed to the L-workers employed in the firms. This leads to

[7] More generally, one can see that for symmetric $\boldsymbol{\lambda}$ with $(\lambda_{i<j} = 0, \lambda_{i>n-j+1} = 0)$ the relationship $\boldsymbol{\lambda}^T\boldsymbol{\delta}_{j<m} = \boldsymbol{\lambda}^T\boldsymbol{\delta}_{n-j+1} = m-j$ also holds.

[8] In Kauffmann (2010) instead of eq. (25) only the weaker result $\tau_{\text{Trade}} > \frac{2a}{(2b+cN)\delta_{1n}}$ is found. Further results in Kauffmann (2010) are not attenuated.

[9] See appendix.

$$w_i^*(\boldsymbol{\lambda}) = \frac{\Pi_i}{\phi} = \frac{1}{\phi}\sum_j (p_{ij}^* - \delta_{ij}\tau)q_{ij}^*\Big(\frac{A}{n} + \lambda_j L\Big). \tag{28}$$

Substituting q_{ij}^* with eq. (27) and p_{ij}^* with $p_{jj}^* + \delta_{ij}\frac{\tau}{2}$, eq. (28) expands to

$$w_i^*(\boldsymbol{\lambda}) = \frac{(b+cN)N}{L}\sum_{j=1}^{n}\Big(p_{jj} - \delta_{ij}\frac{\tau}{2}\Big)^2\Big(\frac{A}{n} + \lambda_j L\Big), \tag{29}$$

without any possibility for simplification because it contains the whole vector $\boldsymbol{\lambda}$. To determine spatial equilibria, firstly we can replace $\int_0^N p(x)dx$ in the indirect utility function eq. (7) with Np_{ji}.[10] Following this, we introduce eq. (29) into the indirect utility function, and replace the prices with the appropriate formulas.[11]

This leads, finally, to

$$\begin{aligned}
V_i(\boldsymbol{\lambda}) =& \frac{a^2N}{2b} - aN\Big(\frac{a + cN\frac{\tau}{2}\boldsymbol{\lambda}^T\boldsymbol{\delta}_i}{2b+cN} + \frac{\tau}{2}\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\Big) \\
&+ \frac{(b+cN)N}{2(2b+cN)^2}\Big[a^2 + 2a\tau(b+cN)\boldsymbol{\lambda}^T\boldsymbol{\delta}_i + (2b+cN)^2\frac{\tau^2}{4}\boldsymbol{\delta}_i^T\mathrm{diag}(\boldsymbol{\delta}_i)\boldsymbol{\lambda} \\
&\qquad + (3c^2N^2 + 4bcN)\frac{\tau^2}{4}\boldsymbol{\delta}_i^T\boldsymbol{\lambda}\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\Big] \\
&- \frac{cN^2}{2(2b+cN)^2}\Big[a^2 + 2a(b+cN)\tau\boldsymbol{\lambda}^T\boldsymbol{\delta}_i + (2b+cN)^2\frac{\tau^2}{4}\boldsymbol{\delta}_i^T\boldsymbol{\lambda}\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\Big] \\
&+ \frac{(b+cN)N}{L(2b+cN)^2}\Big\{A\Big[(a^2 - (2b+cN)a\tau\frac{1}{n}\boldsymbol{\delta}_i\mathbf{1} + (2b+cN)^2\frac{\tau^2}{4}\frac{1}{n}\boldsymbol{\delta}_i^T\boldsymbol{\delta}_i) \\
&\qquad + \frac{1}{n}acN\tau\boldsymbol{\lambda}^T\boldsymbol{\Delta}\mathbf{1} - (2b+cN)cN\frac{\tau^2}{2}\frac{1}{n}\boldsymbol{\delta}_i^T\boldsymbol{\Delta}\boldsymbol{\lambda} \\
&\qquad + \frac{1}{n}c^2N^2\frac{\tau^2}{4}\mathrm{tr}(\boldsymbol{\Delta}\boldsymbol{\lambda}\boldsymbol{\lambda}^T\boldsymbol{\Delta})\Big] \\
&\quad + L\Big[a^2 - (2b+cN)a\tau\boldsymbol{\lambda}^T\boldsymbol{\delta}_i + (2b+cN)^2\frac{\tau^2}{4}\boldsymbol{\delta}_i^T\mathrm{diag}(\boldsymbol{\delta}_i)\boldsymbol{\lambda} \\
&\qquad + acN\tau\boldsymbol{\lambda}^T\boldsymbol{\Delta}\boldsymbol{\lambda} + c^2N^2\frac{\tau^2}{4}\mathrm{tr}(\mathrm{diag}(\boldsymbol{\lambda})\boldsymbol{\Delta}\boldsymbol{\lambda}\boldsymbol{\lambda}^T\boldsymbol{\Delta}) \\
&\qquad - (2b+cN)cN\frac{\tau^2}{2}\boldsymbol{\delta}_i^T\mathrm{diag}(\boldsymbol{\lambda})\boldsymbol{\Delta}\boldsymbol{\lambda}\Big]\Big\} + \bar{q}_0 - \Theta(\lambda_i).
\end{aligned} \tag{30}$$

[10] This substitution is as a result of the assumption of symmetry of the heterogeneous good: all varieties have the same parameters, α, β, and γ, and hence the same price at one location.

[11] See appendix.

As long as the indirect utilities (or real wages, respectively) are different in some regions, some L-workers will have an incentive to migrate to places where their individual utility is higher. Due to the complexity of the problem we cannot show the result of the process of adjustment to equilibrium analytically. Instead, we derive possible equilibria for specific parameter values by means of numerical simulation. This is the topic of the next section.

4 Migration and spatial equilibrium

It is assumed that L-workers possess complete information about the indirect utility attainable in all cities for the present time and that their choice of a region to live in is determined by utility differentials exclusively and independently from each other. Furthermore, we assume that migration is costless. Every skilled worker is looking for a city where he or she can realise higher utility than in the city where he or she actually lives. Formally, this migration behaviour is described (for one period of time Δt) by

$$\frac{\Delta\lambda_{ji}}{\Delta t} = \begin{cases} \frac{1}{\sum_{k=1}^{n} V_k}(V_i - V_j) & \text{if } \lambda_i > 0,\ \lambda_j > 0 \\ \min\{0, \frac{1}{\sum_{k=1}^{n} V_k}(V_i - V_j)\} & \text{if } \lambda_i > 0,\ \lambda_j = 0 \\ \max\{0, \frac{1}{\sum_{k=1}^{n} V_k}(V_i - V_j)\} & \text{if } \lambda_i = 0,\ \lambda_j > 0 \\ 0 & \text{if } \lambda_i = 0,\ \lambda_j = 0. \end{cases} \tag{31}$$

The system is in spatial equilibrium if the indirect utility prevailing in all inhabited cities ($\lambda > 0$) is equalised,

$$\boldsymbol{V}(\boldsymbol{\lambda}) - \bar{V} \leq \mathbf{0}. \tag{32}$$

Regions that cannot reach the common utility level $\bar{V}$ lose any production of the heterogeneous good, their portion of the L-workforce is zero.

Because of its complexity, eq. (32) cannot be solved analytically. This paper presents some solutions by means of numerical simulations. For this, firstly, the parameter values for a, b, c, N, L and A in eq. (30) are fixed.[12] Secondly, an assumption

[12] For comparability of results the same parameter values as Tabuchi *et al.* (2005) are used: $a = 9$, $b = 1$, $c = 1$, $\phi = 1$, $L = 100$ und $A = 1200$, see p. 435 footnote 12. N results from $\frac{L}{\phi}$.

regarding the size of the system, $n = 2m - 1$, has to be met. Thirdly, an initial distribution $\boldsymbol{\lambda}_0$ is defined. This can be symmetric (for example, uniform, $\lambda_m = 1$ or $\lambda_1 = \lambda_n = \frac{1}{2}$) or asymmetric (for example, $\lambda_{m-i} < \lambda_{m+i}\ \ \forall i < m$ or $\lambda_{i \neq m} = 1$ or $\lambda_1 = \lambda_{n-1} = \frac{1}{2}$). Fourthly, linearising eq. (3) to $\Theta(\lambda_i) = \theta\lambda_i$, we fix the parameter $\theta > 0$. Finally, an ordered set of freight transport rates $\boldsymbol{\tau}$, $0 \leq \tau \leq \tau_{\text{Trade}}$ is fixed. Starting at zero, for each $\tau \in \boldsymbol{\tau}$ the same loop is run: Beginning with $\boldsymbol{\lambda}_0$, utilities $\boldsymbol{V}(\boldsymbol{\lambda})$ are computed by eq. (30). Following eq. (31), the changes of $\lambda_i \in \boldsymbol{\lambda}$ result in

$$\frac{\Delta\lambda_i}{\Delta t} = \sum_{j=1}^{n} \frac{\Delta\lambda_{ji}}{\Delta t} \quad \forall i = 1, \ldots, n. \tag{33}$$

Furthermore, migration is restricted by the identity

$$\sum_{i=1}^{n} \Delta\lambda_i \equiv 0. \tag{34}$$

The loop will end if the equilibrium condition eq. (32), now

$$\boldsymbol{V}(\boldsymbol{\lambda}) - \bar{V} \leq \boldsymbol{\zeta} \tag{35}$$

is fulfilled (the value of the positive threshold ζ has to be small).[13] The equilibrium vectors $\boldsymbol{\lambda}$ can be plotted against the freight transport rates τ. Fig. 2 shows the development of an urban system containing seven cities for $\theta = 100$, the ζ-criterion is 0.001.

Looking at figs. 2–4 on p. 16–17, we consider firstly the case $\tau = 0$: Here the adjustment process always leads to a uniform distribution of $\boldsymbol{\lambda}$. Eq. (30) shows $\boldsymbol{\lambda}$ in all additive terms on the right side only connected with τ; only those constants that are the same for all regions less the urban cost function $\theta\lambda_i$ remain. This means, if the population in a city increases, then the utility in this city will decline, and the migration process will lead the system to a stable equilibrium.

If the transport costs increase slightly, utility declines faster in peripheral regions than in the centre of the system. Hence, some L-workers will move from peripheral cities towards more central regions. The urban costs still stop all the skilled workers from gathering in city m though. In equilibrium, the utility is the same in all cities as

[13] In practice, in run k the routine sets λ_i to zero if the sum of $\Delta\lambda_i$ and λ_i (from run $k-1$) undercuts zero. The loop is stopped if $\max(|\boldsymbol{V}\mathbf{1}_n^T - \mathbf{1}_n\boldsymbol{V}^T|) < \zeta$.

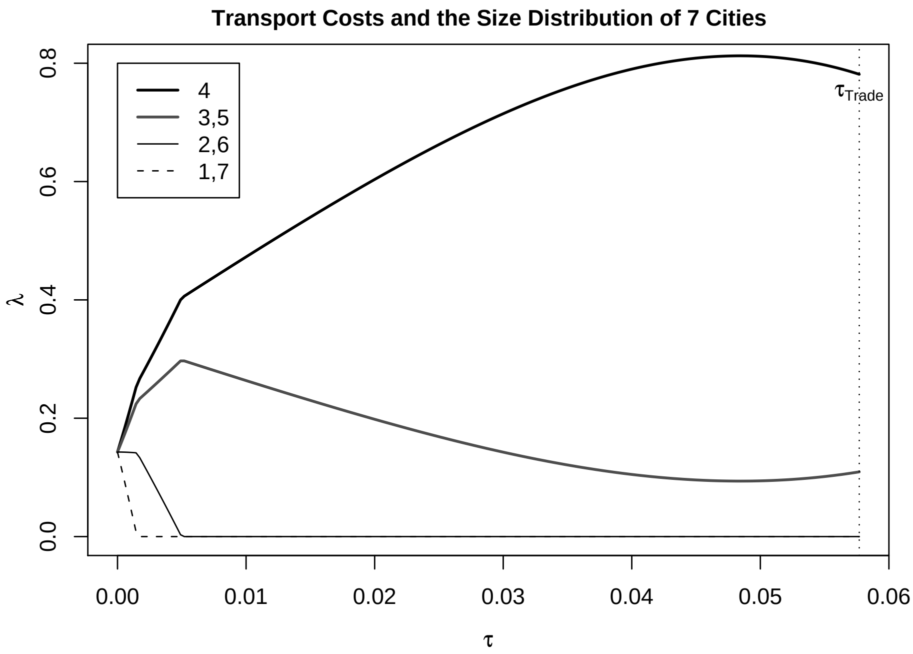

Figure 2: Size relationships in a system of seven cities for different freight transportation rates

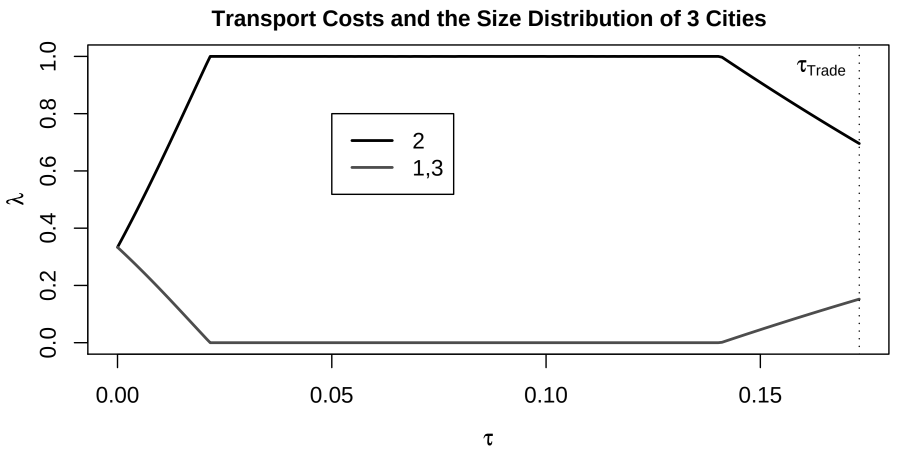

Figure 3: Size relationships in a system of three cities for different freight transportation rates

long as the smallest city still has a population. If the transport costs rise even further, then initially the cities at the fringes of the system will lose their last inhabitants. However, the space between cities is populated by the A-workers, who are immobile. In principle, if transport costs rise, the modelled spatial arrangement leads to spatial concentration if the transport costs are not too large, the urban costs are not too small, and the system is big. In small systems the results are very similar to TTZ: because the geography is small, the urban costs drive some L-workers out of the centre cities (see fig. 3). In large systems, the influence of distance outweighs the effects of urban costs: peripheral cities are rapidly depleted, and there is no dispersion in the case of large transport costs (see fig. 4).

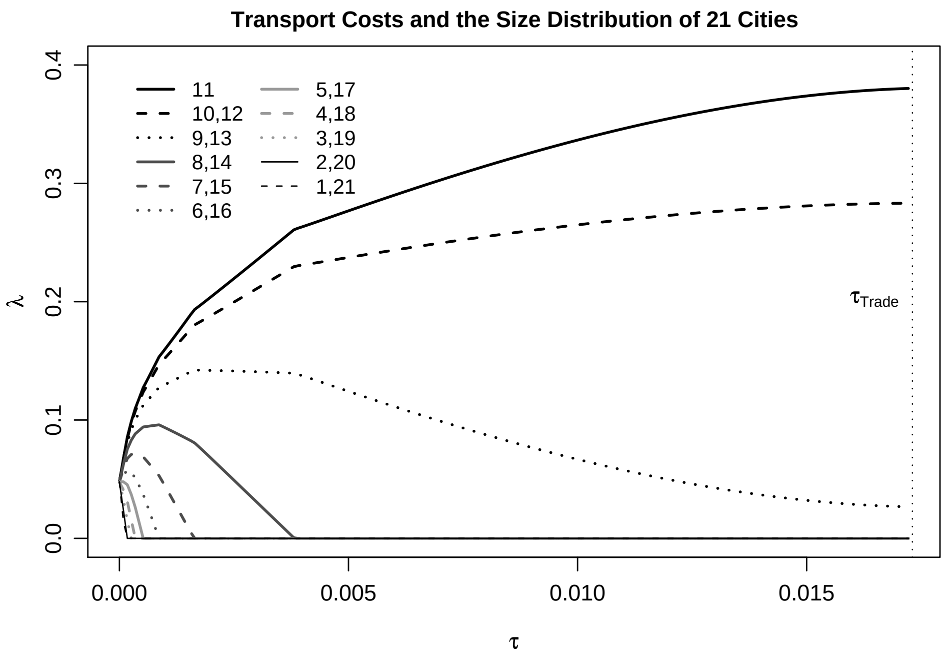

Figure 4: Size relationships in a system of 21 cities for different freight transportation rates

The impact of urban costs is analysed by varying the urban cost parameter θ for fixed $\tau > 0$. Fig. 5 shows the size relationships in a system of five cities. For $\theta = 0$ the migration to city m is unrestricted, any positive transport costs lead to all the L-workers congregating in the central city. This also holds for very small urban costs. The threshold value of θ when some L-workers migrate from city m depends on the values of the other parameters, particularly n and τ. The higher the urban costs, the narrower the limits for urban agglomerations. Urban costs create dispersion.

As long as $\theta > 0$ and $0 < \tau < \tau_{\text{Trade}}$, the central city m will be the largest city of the system. That means, $\lambda_m > \lambda_{i \neq m}$ for positive freight transport rates holds for large

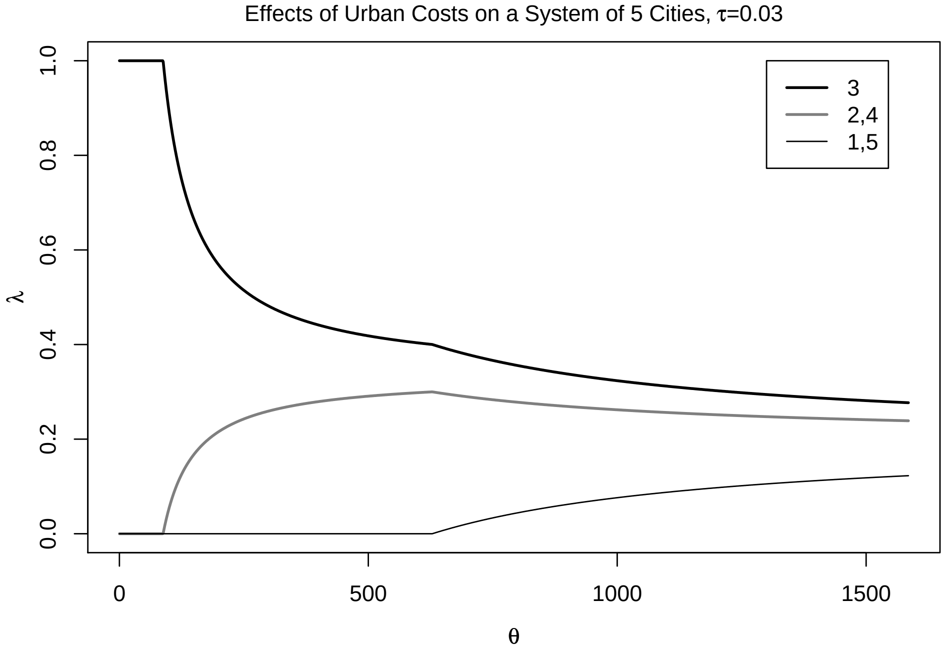

Figure 5: Spatial equilibria at different urban costs

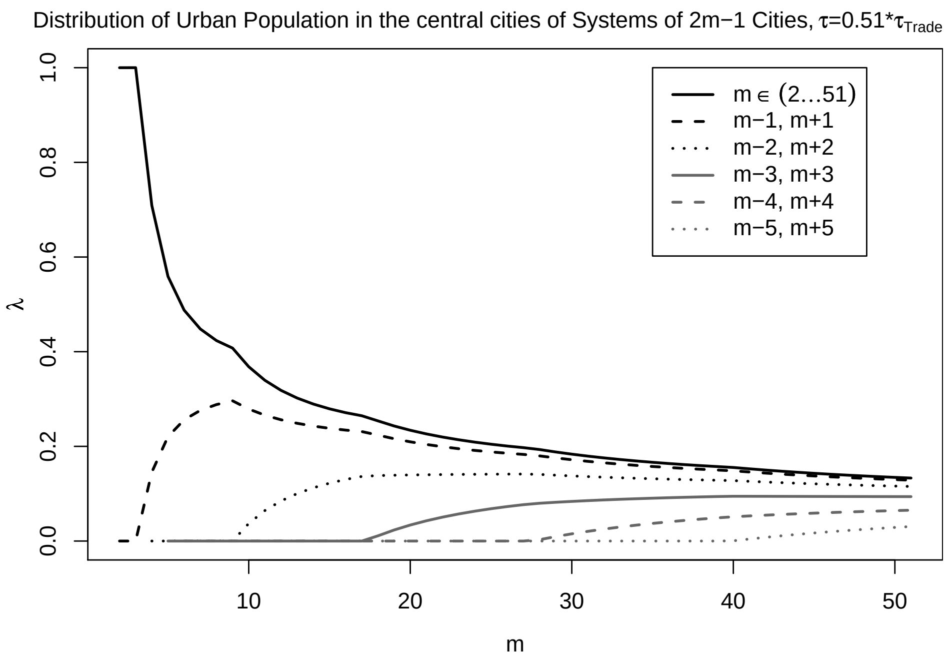

Figure 6: Impact of n on central agglomerations

n, too. Fig. 6 shows this for urban systems from 11 cities to 101 cities ($m = 51$): In a large system of cities the central cities are more dispersed, but this doesn't negate the inhomogeneity of the size distribution. This depends on urban costs: For large urban costs, the distribution of a large system of cities trends to uniformity.

5 Conclusions

The main result of the numeric simulation is the relationship between transport costs and spatial concentration: As long as the so-called "no-black-hole" condition, $\tau < \tau_{\text{Trade}}$, is fulfilled (this is one central assumption in New Economic Geography) and urban costs are not too high (or n is not very small), the population of a linearly arranged system of cities concentrates in central and large cities if transport costs rise.[14] *Vice versa*, declining freight transport costs improve the quality of life in peripheral regions.

The modelled spatial structure should be regarded as a complement to the star-shaped structure in TTZ. That model yields similar results for developed societies that have reached a stage of transport technology that allows the formation of urban systems with large metropolitan areas. However, the TTZ model is based on different assumptions to those of this model, the two sets of results must be interpreted with this in mind. While TTZ analyses the interplay between urban costs (that are assumed as specified functions of the size of cities particularly for each city) and transport costs (that do not have any reference to geographical heterogeneity), the model presented here refers to geography (whereby the metric scaled distance is transformed to an ordinal scaled arrangement) but doesn't consider urban costs as different functions of city size. The TTZ model explains suburbanisation as one possible consequence of the secular declining trend of transport costs in compact, cross-linked spaces. Our model shows the possible outcome of changes in transport costs for cities that are connected by one long transportation line (a river, railway line, or road).

Generally, we should be careful when interpreting the results yielded by highly abstract spatial models with regard to their spatial structure. In modelling systems of cities the assumption of a line shaped structure is more relevant for big countries with sparsely populated regions. Our results predict the withdrawal of skilled production from peripheral to central regions as the result of rising freight transport

[14] This stands in accordance with the empirical results of Kauffmann (2010) for Russia.

costs (or, respectively, a push to the periphery in the case of freight transport becoming cheaper), since geography matters. This could be relevant for the evolution of settlement patterns, particularly in some former socialist countries where transport costs, together with energy costs, have jumped after price liberalisation.

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Appendix

Derivation of eq. (22)

Replacing P_j with $N\sum_{i=1}^{n}\lambda_i p_{ij}$ and p_{ij} with eq. (20) in eq. (14), we get

$$
\begin{aligned}
q_{ij} &= a-(b+cN)(p_{jj}+\delta_{ij}\frac{\tau}{2})+cN\sum_i \lambda_i(p_{jj}+\delta_{ij}\frac{\tau}{2})\\
&= a-bp_{jj}-(b+cN)\delta_{ij}\frac{\tau}{2}+cN\sum_i \lambda_i\delta_{ij}\frac{\tau\{2}{,}\\
&= a-b\,\frac{a+cN\sum_i \lambda_i\delta_{ij}\frac{\tau}{2}}{2b+cN}-(b+cN)\delta_{ij}\frac{\tau}{2}+cN\sum_i \lambda_i\delta_{ij}\frac{\tau}{2},\\
&= \frac{a(2b+cN)-ab}{2b+cN}-\frac{bcN}{2b+cN}\sum_i \lambda_i\delta_{ij}\frac{\tau}{2}-(b+cN)\delta_{ij}\frac{\tau}{2}+cN\sum_i \lambda_i\delta_{ij}\frac{\tau}{2}\\
&= \frac{a(b+cN)}{2b+cN}-\left(\frac{bcN}{2b+cN}-cN\right)\sum_i \lambda_i\delta_{ij}\frac{\tau}{2}-(b+cN)\delta_{ij}\frac{\tau}{2}\\
&= \frac{a(b+cN)}{2b+cN}-\frac{bcN-(2b+cN)cN}{2b+cN}\sum_i \lambda_i\delta_{ij}-(b+cN)\delta_{ij}\frac{\tau}{2}\\
&= \frac{a(b+cN)}{2b+cN}-\frac{b+cN}{2b+cN}\,cN\sum_i \lambda_i\delta_{ij}\frac{\tau}{2}-(b+cN)\delta_{ij}\frac{\tau}{2}\\
&= \frac{b+cN}{2b+cN}\Big[a+cN\sum_i \lambda_i\delta_{ij}\frac{\tau}{2}-(2b+cN)\delta_{ij}\frac{\tau}{2}\Big],\\
&= \frac{b+cN}{2b+cN}\Big[a-b\delta_{ij}\tau+cN(\boldsymbol{\lambda}^T\boldsymbol{\delta}_j-\delta_{ij})\frac{\tau}{2}\Big].
\end{aligned}
$$

5.1 Are there incentives for arbitrage?

As stated in the text, the equilibrium prices of the heterogeneous good specified in eq. (19) and (20) cannot be equalised by means of arbitrage, the proof for this is as follows.

There should be no incentive for somebody to buy one unit of an L-good in city j to sell it for profit in region i, for example. This is the case if, for all $i \neq j$, the non-arbitrage-condition

$$p_{jj}+\delta_{ij}\tau-p_{ji}>0$$

holds. In our model the following is found:

$$\begin{aligned}
p_{jj} + \delta_{ij}\tau - p_{ji} &= \frac{a + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_j\frac{\tau}{2}}{2b + cN} + \delta_{ij}\tau - \left(\frac{a + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\frac{\tau}{2}}{2b + cN} + \delta_{ij}\frac{\tau}{2}\right) \\
&= \frac{(a + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_j) + \delta_{ij}(2b + cN) - (a + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_i)}{2b + cN}\ \frac{\tau}{2} \\
&= \frac{2b\delta_{ij} + cN[\delta_{ij} + \boldsymbol{\lambda}^T(\boldsymbol{\delta}_j - \boldsymbol{\delta}_i)]}{b + cN}\ \frac{\tau}{2}.
\end{aligned}$$

Is it the case that, for any given distribution $\boldsymbol{\lambda}$, the relationship $\delta_{ij} \geq \boldsymbol{\lambda}^T(\boldsymbol{\delta}_i - \boldsymbol{\delta}_j)$ is always valid? Because $\sum_i \lambda_i = 1$, resp. $\boldsymbol{\lambda}^T(\boldsymbol{\delta}_i - \boldsymbol{\delta}_j) \leq \max(\boldsymbol{\delta}_i - \boldsymbol{\delta}_j)$ it is sufficient to show the validity of $\delta_{ij} \geq \delta_{ki} - \delta_{kj}\ \forall\ k$ for $j > i$ (because $\boldsymbol{\Delta}$ is symmetric).

If the columns of $\boldsymbol{\Delta}$ are written as[15]

$$\boldsymbol{\delta}_1 = \begin{pmatrix} |1-1| \\ |2-1| \\ \vdots \\ |n-1| \end{pmatrix}, \boldsymbol{\delta}_2 = \begin{pmatrix} |1-2| \\ |2-2| \\ \vdots \\ |n-2| \end{pmatrix}, \ldots, \boldsymbol{\delta}_n = \begin{pmatrix} |1-n| \\ |2-n| \\ \vdots \\ |n-n| \end{pmatrix}$$

and if

$$\delta_{ij} = j - i \quad \forall \quad j > i,$$

we'll get

$$\delta_{ki} - \delta_{kj} = |k - i| - |k - j|.$$

We have to distinguish between three cases:

1. $k \leq i < j: \quad |k-i| - |k-j| = i - k - j + k = i - j = -\delta_{ij} \quad (< \delta_{ij}),$
2. $i \leq k < j: \quad |k-i| - |k-j| = k - i - j + k = 2k - i - j$ (see below),
3. $i < j \leq k: \quad |k-i| - |k-j| = k - i - k + j = j - i \quad (= \delta_{ij}).$

[15] See eq. (1).

In the second case we find

$$
\begin{aligned}
2k - i - j &\overset{?}{\leq} j - i, \\
2k - j &\overset{?}{\leq} j, \\
2k &\overset{?}{\leq} 2j,
\end{aligned}
$$

commensurately to the presupposition of case 2, $k < j$, hence $|k-i| - |k-j| < \delta_{ij}$, too. At the same time, it is the case that $\max(\boldsymbol{\delta}_i - \boldsymbol{\delta}_j) \leq \delta_{ki} - \delta_{kj} \; \forall \; k$, or

$$\frac{2b}{2b+cN}\delta_{ij} + \frac{cN}{2b+cN}[\delta_{ij} + \boldsymbol{\lambda}^T(\boldsymbol{\delta}_j - \boldsymbol{\delta}_i)] > 0,$$

this means that the no-arbitrage condition is satisfied. Price differentials cannot be equalised by arbitrage. This also holds for the case where some quantity of the heterogeneous good that is produced in a third city k is purchased in region i to sell in region j.

Determination of quantities in equilibrium

Putting p^*_{ii} (eq. 19) and P_i (eq. 18) into eq. (11), we get

$$
\begin{aligned}
q^*_{ii} &= a - (b+cN)p^*_{ii} + cNp^*_{ii} + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\frac{\tau}{2} \\
&= a - bp^*_{ii} + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\frac{\tau}{2} \\
&= \frac{a(2b+cN) - b(a + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\frac{\tau}{2}) + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\frac{\tau}{2}(2b+cN)}{2b+cN} \\
&= \frac{a(b+cN) + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\frac{\tau}{2}(b+cN)}{2b+cN} \\
&= (b+cN)\frac{a + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_i\frac{\tau}{2}}{2b+cN} = (b+cN)p^*_{ii}.
\end{aligned}
$$

This is eq. (26). Eq. (27) is analogously found by replacing p_{ij}^* and P_j in eq. (14) with eq. (15) and eq. (18, with i for j and *vice versa*), respectively:

$$\begin{aligned}
q_{ij}^* &= a - (b+cN)p_{ij}^* + cNp_{jj}^* + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_j\frac{\tau}{2} \\
&= a - (b+cN)(p_{jj}^* + \delta_{ij}\frac{\tau}{2}) + cNp_{jj}^* + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_j\frac{\tau}{2} \\
&= a - b(p_{jj}^* + \delta_{ij}\frac{\tau}{2}) + cN(\boldsymbol{\lambda}^T\boldsymbol{\delta}_j - \delta_{ij})\frac{\tau}{2} \\
&= \frac{a(2b+cN) - b\{a + [cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_j + \delta_{ij}(2b+cN)]\frac{\tau}{2}\} + cN(\boldsymbol{\lambda}^T\boldsymbol{\delta}_i - \delta_{ij})\frac{\tau}{2}(2b+cN)}{2b+cN} \\
&= \frac{a(b+cN) + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_j\frac{\tau}{2}(b+cN) - b\delta_{ij}\frac{\tau}{2}(2b+cN) - cN\delta_{ij}\frac{\tau}{2}(2b+cN)}{2b+cN} \\
&= \frac{a(b+cN) + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_j\frac{\tau}{2}(b+cN) - (b+cN)\delta_{ij}\frac{\tau}{2}(2b+cN)}{2b+cN} \\
&= (b+cN)\Big(\frac{a + cN\boldsymbol{\lambda}^T\boldsymbol{\delta}_j\frac{\tau}{2}}{2b+cN} - \delta_{ij}\frac{\tau}{2}\Big) = (b+cN)(p_{jj}^* - \delta_{ij}\frac{\tau}{2}) \\
&= (b+cN)(p_{ij}^* - \delta_{ij}\tau).
\end{aligned}$$

Determination of the indirect utility

The replacement of $\int_0^N p(x)dx$ in eq. (7) by Np_{ji} yields

$$\begin{aligned}
V_i(\boldsymbol{\lambda}) = \frac{a^2N}{2b} - a\sum_{j=1}^{n}\lambda_j Np_{ji} + \frac{b+cN}{2}\sum_{j=1}^{n}\lambda_j Np_{ji}^2 \\
- \frac{c}{2}\Big(\sum_{j=1}^{n}\lambda_j Np_{ji}\Big)^2 + \bar{q}_0 + w_i - \Theta(\lambda_i).
\end{aligned}$$

Next, we substitute p_{ji} with the equilibrium price from eq. (20):

$$V_i(\boldsymbol{\lambda}) = \frac{a^2N}{2b} - aN\sum_{j=1}^{n}\lambda_j(p_{ii} + \delta_{ij}\frac{\tau}{2}) + \frac{(b+cN)N}{2}\sum_{j=1}^{n}\lambda_j\Big(p_{ii} + \delta_{ij}\frac{\tau}{2}\Big)^2$$
$$- \frac{cN^2}{2}\Big(\sum_{j=1}^{n}\Big(p_{ii} + \delta_{ij}\frac{\tau}{2}\Big)\Big)^2 + \bar{q}_0 + w_i - \Theta(\lambda_i).$$

Replacing w_i with eq. (29) and some sums with vector products, we get

$$V_i(\boldsymbol{\lambda}) = \frac{a^2N}{2b} - aN\Big(p_{ii} + \boldsymbol{\lambda}^T\boldsymbol{\delta}_i\frac{\tau}{2}\Big) + \frac{(b+cN)N}{2}\Big[p_{ii}^2 + \boldsymbol{\lambda}^T\boldsymbol{\delta}_i\tau\Big(p_{ii} + \frac{\tau}{4}\Big)\Big]$$
$$- \frac{cN^2}{2}\Big(p_{ii} + \boldsymbol{\lambda}^T\boldsymbol{\delta}_i\frac{\tau}{2}\Big)^2$$
$$+ \frac{(b+cN)N}{L}\sum_{j=1}^{n}\Big(p_{jj} - \delta_{ij}\frac{\tau}{2}\Big)^2\Big(\frac{A}{n} + \lambda_j L\Big) + \bar{q}_0 - \Theta(\lambda_i).$$

Substitution of p_{ii} and p_{jj} returns

$$V_i(\boldsymbol{\lambda}) = \frac{a^2N}{2b} - aN\sum_{j=1}^{n}\lambda_j\Big(\frac{a + cN\frac{\tau}{2}\boldsymbol{\lambda}^T\boldsymbol{\delta}_i}{2b+cN} + \delta_{ij}\frac{\tau}{2}\Big)$$
$$+ \frac{(b+cN)N}{2}\sum_{j=1}^{n}\lambda_j\Big(\frac{a + cN\frac{\tau}{2}\boldsymbol{\lambda}^T\boldsymbol{\delta}_i}{2b+cN} + \delta_{ij}\frac{\tau}{2}\Big)^2$$
$$- \frac{cN^2}{2}\Big[\sum_{j=1}^{n}\lambda_j\Big(\frac{a + cN\frac{\tau}{2}\boldsymbol{\lambda}^T\boldsymbol{\delta}_i}{2b+cN} + \delta_{ij}\frac{\tau}{2}\Big)\Big]^2$$
$$+ \frac{(b+cN)N}{L}\sum_{j=1}^{n}\Big(\frac{a + cN\frac{\tau}{2}\boldsymbol{\lambda}^T\boldsymbol{\delta}_j}{2b+cN} - \delta_{ij}\frac{\tau}{2}\Big)^2\Big(\frac{A}{n} + \lambda_j L\Big) + \bar{q}_0 - \Theta(\lambda_i).$$

Replacing the sums with vectors and matrices yields eq. (30).