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FINANCING CAPACITY INVESTMENT UNDER DEMAND UNCERTAINTY

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Abstract

Financing capacity investment under demand uncertainty

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This paper studies the interplay between the operational and financial facets of capacity investment. We consider the capacity choice problem of a firm with limited liquidity and whose access to external capital markets is hampered by moral hazard. The firm must therefore not only calibrate its capacity investment and the corresponding funding needs, but also optimize its sourcing of funds. Importantly, the set of available sources of funds is derived endogenously and includes standard financial claims (debt, equity, etc.). We find that when higher demand realizations are more indicative of high effort, debt financing is optimal for any given capacity level. In this case, the optimal capacity is never below the efficient capacity level but sometimes strictly above that level. Further, the optimal capacity level increases with the moral hazard problem's severity and decreases with the firm's internal funds. This runs counter to the newsvendor logic and to the common intuition that by raising the cost of external capital and hence the unit capacity cost, financial market frictions should lower the optimal capacity level. We trace the value of increasing capacity beyond the efficient level to a bonus effect and a demand elicitation effect. Both stem from the risk of unmet demand, which is characteristic of capacity decisions under uncertainty.

Keywords: Capacity, optimal contracts, financial constraints, newsvendor model

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1. Introduction

Capacity investment decisions for new products or markets are often made in a context of considerable demand uncertainty. When calibrating capacity, management must navigate between the Scylla of excess demand's opportunity cost and the Charybdis of excess capacity's setup costs. While companies investing in capacity need to mobilize the necessary human and physical capital, they must also avail of funds to acquire or rent those. When their liquidity reserves are insufficient or when depleting them would involve large opportunity costs, they must tap outside markets for capital. External financing can come from a variety of sources such as bank loans, trade credit, corporate bonds, private equity, public equity markets, etc. Hence companies must not only calibrate their capacity investment and the corresponding amount of funding needed, but also optimize their sourcing of funds.

In this paper, we aim to study the interplay between these operational and financial facets of capacity investment. To this end, we consider the capacity choice problem of a firm with limited liquidity and whose access to external capital markets is hampered by moral hazard. The firm must consider the set of available sources of funds when calibrating its capacity investment. Importantly, this set is derived endogenously and includes standard financial claims (debt, equity, etc.). First, we show that debt financing is optimal for any given capacity level. Next, we show that given optimal debt financing, the optimal capacity is never below the efficient capacity level but sometimes strictly above that level, in which case it increases with the moral hazard problem's severity and decreases with the firm's internal funds.

These results run counter to the newsvendor logic and to the common intuition that by raising the cost of external capital and hence the unit capacity cost, financial market frictions lower investment in capacity. In the same manner, by reducing the need for external funds, leading to lower unit cost, higher internal funds should increase investment in capacity. That logic holds when the financial frictions remain constant or perhaps increase with the level of investment. However, we identify two effects, the *bonus* and *demand elicitation* effects, whereby financial frictions may actually decrease as capacity increases. These effects stem directly from the risk of unmet demand which is characteristic of capacity decisions under demand uncertainty.

Specifically, we consider the problem of a firm with limited liquidity reserves having to make a capacity investment choice under demand uncertainty. We model this situation by means of a standard newsvendor framework, which we augment with two features allowing us to study the effect of the firm's need for funds when access to financial markets involves frictions.

The first feature is that the firm's owner (the entrepreneur) has limited internal funds of his own but is able to raise additional funds from a financial investor, whom we assume to be perfectly competitive. This feature alone, however, causes no tangible departure from the standard newsvendor model. Indeed, absent capital market imperfections, Modigliani and Miller (1958)'s irrelevance theorem applies so that the firm's problem is unaffected by the need to raise funds and the choice of a source of funds is a matter of irrelevance. (Section 3 revisits the theorem in the context of our model.)

We introduce a second feature by assuming that the firm can affect the distribution of demand by taking an action that is non-contractible. Specifically, by exerting a costly effort, the firm can improve the distribution of demand in the sense of the Monotone Likelihood Ratio Property (MLRP). Effort can, for instance, correspond to conducting a sales or marketing campaign, or to improving the product's design. Loosely speaking, MLRP means that higher demand realizations are more indicative of high effort. The non-contractibility of effort, i.e., the fact that its level cannot be specified in an enforceable contract, constitutes a financial market friction that renders the need for outside financing relevant, and the question of the optimal sourcing of funds pertinent.

Given this, the firm must optimize not only capacity but also the source of funds, i.e., the financial contract to offer the investor. Importantly, we follow the optimal contracting approach in which agents optimize over a set of feasible contracts derived *endogenously* from fundamentals, i.e., preferences, physical constraints (e.g., production technology), and constraints on contractibility (i.e., which variables can be used as arguments of an enforceable contract). This is in contrast to existing studies at the interface of finance and operations that assume an exogenously determined feasible set of financial contracts, which typically take the form of debt or, more rarely, of debt and equity.

In our model, simple assumptions about fundamentals ensure that feasible contracts satisfy three conditions. All financial contracts satisfying these conditions constitute the set of feasible sources of finance over which the firm optimizes.

First, we assume that effort apart, all variables (e.g., capacity, funding, costs, revenues, etc.) are contractible except for one: demand. Revenues being contractible, and given the one-to-one mapping between revenues and demand when demand is below capacity, this in fact amounts to assuming that only unmet demand is not contractible. This means that financial contracts specify a capacity, an amount of funds contributed by the investor and the promise of repayments to the investor contingent on revenues. Note that capacity and funding amount are set by the contract, not left for the firm to choose post-contracting. This is also in contrast to much of the literature at the interface of operations and finance.

Second, we assume that the firm is risk-neutral about internal funds above zero but extremely averse to funds falling below zero. This implies that financial contracts must in effect satisfy *limited liability*, i.e., repayments cannot exceed revenues.

Third, we assume that the firm is able to report artificially inflated revenues, and will do so if that leads to lower repayments. Avoiding such manipulation requires that financial contracts be in effect *monotonic*, i.e., that repayments increase with revenues.

The feasible set so-defined includes standard financial claims such as debt, equity, convertible debt, or call warrants as well as combinations of such claims (e.g., debt plus levered equity, etc.). For instance, a debt contract defines a repayment function from the firm to the investor which is equal to the realized revenue when revenues are below the debt's face value, and constant and equal to that face value otherwise. Similarly, an equity stake defines a monotonic repayment function equal to a fixed fraction of the realized revenue. Both repayment functions only depend on revenues, are clearly non-decreasing and respect limited liability.

To begin with, we characterize the optimal source of funds to finance a given capacity. A priori, different sources of funds may have their appeal. For instance, equity financing may be a suitable way to control excessive risk-taking by the firm, a temptation that debt financing might instead exacerbate. However, and following Innes (1990), we show that under our assumptions, using internal funds should be a priority and if external funds are needed, debt financing is preferable to all other external sources of funds. In essence, this is so because when higher demand realizations are more indicative of high effort, repayments to the investor in high demand states are more detrimental to the firm's incentives to exert effort. Debt is then the feasible contract minimizing such repayments as it maximizes repayments in low demand states.

Building on this result, we characterize the firm's optimal capacity investment given that investment is financed optimally. Without financial frictions, i.e., if effort is contractible, the optimal capacity corresponds to the standard newsvendor quantity and yields the maximum profit possible. Interestingly, we find that with financial constraints, the optimal capacity is never below the efficient capacity level but can sometimes *exceed* that level strictly. In the latter case, capacity is shown to increase with the severity of the moral hazard problem as measured by the cost of exerting effort, and to decrease with the firm's internal funds.

We trace the value of increasing capacity beyond its efficient level to a *bonus effect* and a *demand elicitation effect*. These effects stem directly from the risk of unmet demand which is characteristic of capacity decisions under demand uncertainty. As we explain below, both effects result in the firm's payoff net of repayments increasing for demand realizations above the efficient capacity level. This is valuable because when higher demand realizations are more indicative of high effort, payoffs in higher demand states provide the firm with more powerful incentives.

More specifically, we identify that an increase of capacity beyond the efficient level has a *bonus effect*: it allows for an increase in the firm's expected payoff for high demand realizations without decreasing the repayments to the investor. Indeed, rather than distorting capacity away from its efficient level, the firm would prefer a financial contract giving him a higher payoff when demand exceeds capacity. However, such a contract would also decrease the repayments to the investor when demand exceeds capacity and thereby violate the monotonicity condition.

Increasing capacity beyond the efficient level also has a *demand elicitation effect*: it reveals some demand states above the efficient capacity level that would otherwise be non contractible, allowing the firm's payoff to increase more for the highest among those states. Indeed, the increase in expected payoff the extra capacity brings about is not spread equally across demand realizations: Higher realizations bring higher payoffs. The same cannot be achieved via a financial contract because the non-contractibility of demand imposes equal repayments for all demand realizations exceeding capacity. Thus, the firm must distort capacity away from its efficient level to capture the additional incentive value of higher otherwise unmet demands.

In short, when the moral hazard is severe enough, the firm will commit to exerting effort by setting capacity above its efficient level to exploit the incentive power of the *bonus* and *demand elicitation* effects. Moreover, as moral hazard becomes more severe, the firm will increase capacity further to increase both effects' incentive power. Conversely, optimal capacity will decrease with the firm's internal funds.

This paper belongs to the nascent literature on the interplay between operational and financial decisions. This line of research has derived a rich set of implications for how a firm's funding needs affect its capacity and technology choices and for how these in turn impact the firm's financial policy. For instance, Dada and Hu (2008) and Alan and Gaur (2013) consider a newsvendor seeking funding from a bank and find that capacity is set optimally below the efficient level. In Li et al. (2013)'s dynamic inventory model, optimal inventory and financial decisions are found to be myopic and increasing in inventory level and retained earnings. Boyabatli and Toktay (2011) consider a multi-product firm making capacity and (flexible or dedicated) technology choices and show that demand uncertainty's impact on those choices is affected by the firm's need for funds.

These studies capture different aspects of a firm's operations but model its external financing in similar ways: an interest rate is set upfront and the newsvendor then chooses the loan size. In Dada and Hu (2008) the interest rate is set by a monopolist bank. In Alan and Gaur (2013) too but the loan size is capped. In Boyabatli and Toktay (2011), the rate is set by a competitive bank for each technology and loans may be secured or not. Li et al. (2013) focus on short-term debt with an exogenous interest rate.

Two main exogenous restrictions on the set of feasible financial contracts are therefore common to these papers. First, they focus *a priori* on debt financing thus ruling out any other external source of funds such as equity, convertible debt, etc. They only allow *internal equity*, i.e., equity contributed by the newsvendor himself from initial wealth or retained earnings. Second, they assume *a priori* that interest rates are *not* conditional on the loan size chosen by the newsvendor. It is therefore possible that within the context of these models, other financial contracts than debt, such as equity financing for instance, achieve first best or at least improve the outcome's efficiency. Moreover, even restricting the focus to debt financing, the same would probably hold for loan size-dependent interest rates, a point Dada and Hu (2008)'s Proposition 4 makes clearly by showing that the first best is obtained with loan size-dependent interest rates.

Our paper departs from this literature by adopting the optimal contracting approach in which agents optimize over a set of feasible contracts derived *endogenously* from assumptions about fundamentals. These assumptions concern solely preferences, physical constraints, and contractibility. We explore the conditions under which deviations from efficient outcomes arise, once feasible contractual solutions are exhausted.

Our paper also builds on the principal-agent literature with risk-neutrality and limited liability (e.g., Oyer 2000, Poblete and Spulber 2012, Gromb and Martimort 2007) and more specifically that on agency in capacity choice models, notably Dai and Jerath (2013b) and Dai and Jerath (2013a). In particular, Dai and Jerath (2013b) study a capacity choice model in which an employee must be induced to take a demand-enhancing action. They show that capacity in excess of the efficient level may be optimal for a reason akin to the *demand elicitation effect* in our analysis.

The paper proceeds as follows. Section 2 presents the model. Section 3 consider the benchmark case in which effort is contractible. Section 4 establishes the optimality of debt financing when effort is non-contractible. Section 5 studies the optimal capacity level, the optimality of over-investing and the role of internal funds in capacity investment decisions. Section 6 concludes. The Appendix contains all mathematical proofs.

2. A Model of Capacity Investment Financing

2.1. The Newsvendor Model with Financing under Moral Hazard

We consider the problem of a firm with limited liquidity reserves having to make a capacity investment choice under demand uncertainty. We model this situation by means of a standard newsvendor model, which we augment with two features. First, the firm's owner has limited wealth ("internal funds") of his own but is able to raise additional funds from a competitive investor. Second, the owner can affect the distribution of demand by taking a non-contractible action.[1]

[1] We show later that limited wealth *per se* (Section 3), or the non-contractibility of the action *per se* (Section 4) causes no meaningful departure from the standard newsvendor model. However, the combination of both features does.

Newsvendor model Assume universal risk-neutrality and no discounting.[2] Consider a firm whose sole owner, whom we refer to as "the firm", has the following investment project. The firm faces stochastic demand D_1 with distribution $f_1(\cdot)$, cumulative distribution $F_1(\cdot)$, complementary cumulative distribution $\bar{F}_1(\cdot) \equiv 1 - F_1(\cdot)$, and hasard rate $h_1(\cdot) = f_1(\cdot)/\bar{F}_1(\cdot)$. For simplicity, we assume that f_1 is strictly positive over $\mathbb{R}_+$. The firm chooses a capacity $q \in \mathbb{R}_+$ at unit cost $c > 0$ before demand is realized, and eventually receives a revenue $r > c$ per unit sold and salvage value $s < c$ per unsold unit.

Effort choice If the firm does not set up any capacity, i.e., if $q = 0$, then we say that the firms abandons the project. If instead the project is undertaken then after capacity $q > 0$ is installed but before demand is realized, the firm also needs to run the project diligently. This might involve conducting a suitable sales or marketing campaign, investing organizational capital into improving product design and quality, etc. In this case, we say that the firm "works" (exerts effort $e = 1$) which causes it to incur a non-monetary cost $\kappa_1 > 0$. This cost can be thought of as a psychic or physical cost of labor for the owner, or the opportunity cost for the firm of allocating management attention or human capital to the project rather than to other non-modelled valuable projects.[3]

Alternatively, the firm can decide to "shirk" (exert effort $e = 0$), i.e., to be less diligent in selling or marketing the products, to put less organizational resources into product quality and design, or to focus more on other projects. The firm incurs then a smaller non-monetary cost $\kappa_0 > 0$, i.e., with $\Delta\kappa \equiv \kappa_1 - \kappa_0 \geq 0$. In that case, the firm might decide to adjust other aspects of the project to the lower level of effort: the firm may choose to set up cheaper or more expensive capacity, set a different price for the product, etc., which might also affect demand. We assume for simplicity that irrespective of such adjustments, the project's value is always negative if the firm shirks, so that abandoning it ($q = 0$) is preferable.

A case of particular interest is when the firm shirks ($e = 0$) but does not adjust the different features of the project relative to when it works ($e = 1$), i.e., unit price, cost and salvage value remain equal to r, c and s. In this case, shirking does nonetheless affect demand by shifting it to D_0 with distribution f_0. Demand distribution f_0 is less favorable than f_1 in the sense of the Monotone Likelihood Ratio Property (MLRP), i.e., f_1/f_0 is strictly increasing over $\mathbb{R}_+$. Loosely speaking, MLRP means that higher demand realizations are more indicative of high effort.[4] Note also that MLRP implies but is stronger than first-order stochastic dominance and increasing hazard ratios.

[2] The risk-neutrality assumption is both standard and important for the optimality of debt financing in our model. The no-discounting assumption is only for simplicity and can be relaxed easily.

[3] The assumption that no cost is incurred if $q = 0$ is for simplicity. It amounts to assuming that κ_0 is a fixed cost which will allow us to focus on cases where the firm exerts $e = 1$ or sets $q = 0$.

[4] Following a demand realization d, an agent would revise his prior $\nu = \Pr[e = 1]$ to posterior is $\frac{\Pr[e=1 \cap d]}{\Pr[d]} = \frac{\nu f_1(d)}{\nu f_1(d) + (1-\nu) f_0(d)} = \frac{1}{1 + \frac{(1-\nu)}{\nu} \frac{f_0(d)}{f_1(d)}}$ which, by MLRP, is strictly increasing with d.

First-Best For given capacity $q \geq 0$ and effort level $e \in \{0,1\}$, the project's payoff is randomly distributed over $[sq, rq]$ according to density $g_{e,q}$ (implied by density f_e) and is equal to

$$P_{e,q} \equiv sq + (r-s)(D_e \wedge q). \tag{1}$$

Denote the expected profit in the standard newsvendor model with payoff distribution $g_{e,q}$ as

$$\pi_e(q) \equiv \mathbb{E}[P_{e,q}] - cq \tag{2}$$

As per standard arguments, $\pi_e(\cdot)$ is strictly concave over $\mathbb{R}_+$ and the optimal (first-best) capacity is

$$q_e^{FB} \equiv \arg\max_{q \in \mathbb{R}_+} \pi_e(q) = F_e^{-1}\left(\frac{r-c}{r-s}\right) \tag{3}$$

We assume that given r, c and s, the project is viable if $e=1$ but not if $e=0$, in keeping with our assumption that if the firm shirks, the project is not viable irrespective of policy adjustments, i.e.,

$$\max_{q \in \mathbb{R}_+} \pi_1(q) - \kappa_1 > 0 > \max_{q \in \mathbb{R}_+} \pi_0(q) - \kappa_0 \tag{4}$$

so that the first-best optimal outcome is $e=1$ and q_1^{FB}.

Financial contracts The firm has internal funds $W \geq 0$ which might not suffice to fund his (to be determined) desired level of capacity. The firm can however raise funds from a perfectly competitive investor by offering him a financial contract. Such a contract specifies a capacity $q \geq 0$ to be set up, and an investment $I \geq 0$ by the investor against the promise of a repayment from the firm contingent only on variables that are contractible, i.e., they can be specified in an enforceable contract. Note in particular that, in contrast to much of the "operations and finance" literature, capacity and funding amount are set by the financial contract, not left for the firm to decide post-contracting.

We make three simple assumptions about fundamentals ensuring that feasible contracts satisfy three conditions.

First, we assume that all variables (e.g., capacity q, funding I, cost c, revenue $P_{e,q}$, etc.) are contractible except for effort e and demand D_e. (Nonetheless Section 3 below studies contractible effort.) For instance, costs and revenues can typically be retrieved from the firm's books, which can be audited. We think of effort as representing inputs that are less tangible, like whether the company's "best people" are allocated to a project, the intensity of a sales effort, or how much thinking goes into product design. As for demand, revenues $P_{e,q}$ being contractible and given the one-to-one mapping between revenues and demand when demand is below capacity, demand's non-contractibility boils down to assuming that only unmet demand is non-contractible. This assumption seems reasonable as unfilled demands are typically difficult to observe. In our context, this implies that financial contracts cannot distinguish between realizations of demand D_e above capacity q as they all yield the same revenue rq. Hence financial contracts specify a capacity, an amount of funds contributed by the investor and the promise of revenue-contingent repayments $R(\cdot):[sq,rq]\mapsto\mathbb{R}$ such that given payoff $p\in[sq,rq]$, the repayment to the investor is $R(p)$, leaving the firm with net payoff $p-R(p)$.

Second, we assume that the firm is risk-neutral about internal funds above zero but extremely averse to funds falling below zero (e.g., the firm owner's utility is $-\infty$ for negative wealth), a threshold that can be thought of as a subsistence level. This implies that the firm's *limited liability* must be preserved: repayments cannot exceed revenues, i.e., $\forall p\in[sq,rq]$, $R(p)\leq p$.[5,6]

Third, we assume that the firm is able to report artificially inflated revenues by raising funds secretly from a third party. The firm may opportunistically do so to lower its repayment to the investor if the financial contracts specifies a lower repayment for higher (reported) revenues. Avoiding such manipulation requires that financial contracts be in effect *monotonic* in that $R(\cdot)$ must be non-decreasing over $[sq,rq]$. This assumption is standard in the corporate finance literature. In the specific context of the newsvendor model, inflating revenues amounts to using the funds raised secretly to purchase units for r, thereby inflating demand and revenues artificially, before reselling them at the salvage value s.[7]

Any financial contract satisfying these restrictions is considered feasible. Notice that we do not impose that repayments $R(p)$ be positive for all $p\in[sq,rq]$. Indeed, a financial contract can possibly specify that the investor will actually pay the firm, at least for some payoff realizations. Notice also that feasible financial contracts include debt contracts. Indeed a debt contract with face value K corresponds to a repayment function $R(p)=p\wedge K$ which clearly satisfies both the limited liability and the monotonicity conditions.[8] Raising funds by issuing equity is also feasible as a fraction α of equity corresponds to $R(p)=\alpha p$. Obviously, many other contracts (e.g., convertible debt, call warrants, etc.) are also feasible, are as some combinations of contracts (e.g., debt plus levered equity, etc.).

Importantly, the firm optimizes over the entire set of feasible financial contracts that is determined endogenously based on assumptions about fundamentals, i.e., preferences, physical constraints, and contractibility. This is in contrast to studies in which the set of feasible financial

[5] Limited liability is a standard means to ensure agency costs arise despite universal risk-neutrality.

[6] Strictly speaking, this foundation does not imply $R(p)\leq p$ but that the firm's final equity always be non-negative. However, as is standard and as we show latter, if the firm *needs* to raise funds from the investor, it will also find it (at least weakly) optimal to use all the internal funds W for funding its capacity. In that case, the firm's net payoff is $p-R(p)$ and its non-negativity is equivalent to the limited liability condition.

[7] An alternative foundation for monotonic financial contracts may be that the investor could engage in sabotage and reduce the payoff. (See Innes 1990 for a discussion of both foundations.) Strictly speaking, either foundation would not rule out contracts that are non-monotonic, but would imply they are equivalent to monotonic contracts.

[8] We ignore the distinction between interest and principal payments, which is irrelevant in our model.

contracts is exogenous and determined by *a priori* restrictions.

2.2. The Firm's Problem

The firm's problem is to choose capacity $q \geq 0$, the funds I raised from the investor and the contractual repayments $R(\cdot)$ to maximize its expected payoff

$$\max_{q,I,R(\cdot)} \mathbb{E}\left[P_{e,q} - R\left(P_{e,q}\right)\right] + W + I - cq - \kappa_e \tag{5}$$

where the first term is the project's expected payoff net of repayment to the investor.

This choice is made under a number of constraints. First, the investor should accept the contract. Given risk-neutrality and no discounting, this requires that its expected payoff be no less than his investment. Hence, the following investor participation constraint must be satisfied

$$\mathbb{E}\left[R\left(P_{e,q}\right)\right] \geq I \tag{6}$$

Similarly, the firm should also accept the contract, which requires that his expected payoff be no less than his payoff under the status quo when the project is abandoned, i.e., the following firm participation constraint must be satisfied

$$\text{if } q > 0, \qquad \mathbb{E}\left[P_{e,q} - R\left(P_{e,q}\right)\right] + W + I - cq - \kappa_e \geq W \tag{7}$$

Second, taken together, the firm's internal funds and the funds raised from the investor should cover the cost of setting up capacity, i.e., the following funding constraint must be satisfied

$$I + W \geq cq \tag{8}$$

Third, the financial contract must belong to the feasible contract set, i.e., the following limited liability and monotonicity constraints must be satisfied

$$\forall\left(p, p'\right) \in \left[sq, rq\right]^2 \text{ with } p > p', \quad R\left(p\right) \leq p \quad \text{and} \quad R\left(p'\right) \leq R\left(p\right) \tag{9}$$

Finally effort being non-contractible, it cannot be specified as part of the financial contract. Instead, the firm chooses its level based on post-financing incentives. Therefore, the firm must prefer exerting the assumed effort level e rather than the alternative effort level $(1-e)$, i.e., the following incentive compatibility constraint must be satisfied:

if $q > 0$,

$$\mathbb{E}\left[P_{e,q} - R\left(P_{e,q}\right)\right] + W + I - cq - \kappa_e \geq \mathbb{E}\left[P_{(1-e),q} - R\left(P_{(1-e),q}\right)\right] + W + I - cq - \kappa_{(1-e)} \tag{10}$$

The previous problem can be simplified. First, notice that I, the amount raised from the investor, increases the firm's objective (5) and is bounded from above only by the investor's participation constraint (6) which must therefore be binding, i.e.,

$$\mathbb{E}\left[R\left(P_{e,q}\right)\right] = I \tag{11}$$

an expression we can use to eliminate I from the objective and the constraints.

Second, using definition (2) and eliminating constant W, objective (5) can be rewritten as

$$\max_{q,R(\cdot)} \quad \pi_e(q) - \kappa_e \tag{12}$$

Hence the firm's objective is simply to maximize the project's expected profit net of effort cost. This simply reflects the fact that the investor being competitive, his expected profit must equal zero.

After similar simplifications, the firm's participation constraint (7) can be written as

$$\text{if } q > 0, \qquad \pi_e(q) - \kappa_e \geq 0 \tag{13}$$

which given condition (4), cannot hold for $e = 0$ and holds for $e = 1$ only for $q > 0$ such that

$$q \leq \bar{q}_1 \equiv \max\left\{q \text{ s.t. } \pi_1\left(q\right) - \kappa_1 = 0\right\} \tag{14}$$

Note that $\bar{q}_1 > q_1^{FB}$.

This implies that optimization can be restricted to the following: either the firm sets up capacity $q \in (0, \bar{q}_1]$ and exerts $e = 1$, or the firm abandons the project ($q = 0$). In other words, incentive compatibility condition (10) should in fact state that the firm should exert effort $e = 1$ if $q > 0$.

Overall, the firm's problem can be written as follows:

$$\max_{q,R(\cdot)} \pi_1(q) \tag{15}$$

$$s.t.$$

$$q \in (0, \bar{q}_1] \tag{16}$$

$$\mathbb{E}\left[R\left(P_{1,q}\right)\right] \geq \left(cq - W\right)^+ \tag{17}$$

$$\forall \left(p, p'\right) \in \left[sq, rq\right]^2 \text{ with } p > p', \quad R\left(p\right) \leq p \quad \text{and} \quad R\left(p'\right) \leq R\left(p\right) \tag{18}$$

$$\mathbb{E}\left[P_{1,q} - R\left(P_{1,q}\right)\right] - \mathbb{E}\left[P_{0,q} - R\left(P_{0,q}\right)\right] - \Delta\kappa \geq 0 \tag{19}$$

and if the previous problem is not feasible then the firm abandons the project ($q = 0$). (Note that (16) is the firm's participation constraint, (17) the funding constraint, (18) the constraint that the contract be feasible, and (19) the incentive compatibility constraint).

3. Contractible Effort

In this section, we study briefly the situation in which effort is contractible. This is straightforward because in that case, Modigliani and Miller (1958)'s irrelevance theorem holds. Here, the theorem implies that the firm's need for funds and the financial contract employed to raise those funds are a matter of irrelevance. Indeed, when the firm's effort is contractible, the financial contract specifies not only payoff-contingent repayments $R(\cdot)$ but also the effort level. The firm's problem is as before except for incentive compatibility constraint (19) and for the fact that e is now an optimization variable, i.e., the firm maximizes its objective $\pi_1(q)$ by choosing q, $R(\cdot)$ and e.

Since repayments $R(\cdot)$ do not appear in objective (15) and incentive compatibility condition (19) is absent, effort e can be set independently from repayments $R(\cdot)$. This implies that *all* repayment functions $R(\cdot)$ satisfying the remaining constraints (17) and (18) are equivalent, i.e., the firm is indifferent between all feasible financial contracts (i.e., satisfying feasibility constraint (18)) provided they allow raising the needed funds (i.e., satisfy funding constraint (17)).

This is simply the Modigliani-Miller Theorem in the context of our model.

Lemma 1. *When effort is contractible, for a given capacity $q \in (0, \bar{q}_1]$, the firm is indifferent between all financial contracts satisfying funding constraint (17) and feasibility constraint (18).*

The next step is to note that for all $q \in (0, \bar{q}_1]$, the set of such financial contracts is non-empty. Indeed, consider the contract such that $\forall p \in [sq, rq]$, $R(p) = p$. This can be viewed, among others, as a contract selling 100% of the project's equity to the investor.[9] Clearly, this contract is feasible, i.e., satisfies the limited liability and monotonicity conditions in (18). Moreover, it satisfies the funding condition as, by definition of $R(\cdot)$ and $\bar{q}_1$

$$\mathbb{E}[R(P_{1,q})] = \mathbb{E}[P_{1,q}] \geq cq \tag{20}$$

Therefore, all capacities $q \in (0, \bar{q}_1]$ being "fundable" by feasible contracts, the firm's problem boils down to

$$\max_{q \in (0, \bar{q}_1]} \pi_1(q) \tag{21}$$

That is, with contractible effort, the firm's problem amounts to the standard newsvendor problem, hence the following result.

Proposition 1. *When effort is contractible, the firm's optimal decisions are to exert effort $e = 1$ and set capacity at the corresponding first-best level q_1^{FB}.*

[9] Note that although in this case the firm retains no financial interest in the project, the firm is compensated by the investor's investment $I = \mathbb{E}[P_{1,q}]$, which exceeds the funding needs $(cq - W)^+$.

This means that we can simply solve the firm's problem as if internal funds W were sufficient to fund the chosen capacity. In a sense, Lemmas 1 and Proposition 1 justify the use in the literature of the standard newsvendor model, which also provides the first best solution for our problem. Indeed, although the firm might need to raise funds, the model boils down to the standard newsvendor model and so the firm's capacity choice and expected profit also coincide with those of the standard newsvendor model. Absent financial frictions, the need to raise funds has *per se* no impact on the firm's problem.

4. The Optimality of Debt Financing

We now revert to the assumption that effort is non-contractible. Given this, all feasible sources of finance are no longer equivalent. Indeed, effort is now determined by the firm's incentives *after* financing, as reflected in incentive compatibility constraint (19). In other words, effort's non-contractibility causes a friction in the firm's access to funds and so the Modigliani-Miller Theorem no longer holds. This raises the question of the optimal source of funds for the firm for a given capacity choice q and its influence on the firm's choice of capacity.

In the following, we first solve the problem in (15) for a given positive capacity level. Following Innes (1990), we establish that under our assumptions, an optimal (non-unique) repayment function corresponds to a debt contract. We then determine the optimal capacity choice and show that it is always larger or equal than q_1^{FB}, and sometimes strictly larger.

Under our risk-neutrality assumption and contract feasibility conditions, if a project's payoffs satisfy the Monotone Likelihood Ratio Property (MLRP) then an optimal contract is a debt contract (see Innes 1990 or Tirole 2006). The following lemma shows that MLRP extends from demands to payoffs.

Lemma 2. *If f_1/f_0 is strictly increasing over $\mathbb{R}_+$ then for all $q \in \mathbb{R}_+$, $g_{1,q}/g_{0,q}$ is also strictly increasing over $[sq, rq]$.*

The Monotone Likelihood Ratio Property is important because of its implication for the incentive constraints. In short MLRP implies that given an expected amount of repayments, contracts with higher repayments for higher payoffs are more detrimental to the firm's incentives to exert effort. This result is formally stated below.

Lemma 3. *For a given capacity q, consider two feasible financial contracts A and B with respective repayment functions $R_A(\cdot)$ and $R_B(\cdot)$ satisfying the following:*

$$\mathbb{E}[R_A(P_{1,q})] = \mathbb{E}[R_B(P_{1,q})] \tag{22}$$

and $\exists p^* \in (sq, rq)$ such that

$$\forall p \in [sq, p^*), \quad R_A(p) \geq R_B(p) \tag{23}$$

$$\forall p \in (p^*, rq], \quad R_A(p) \leq R_B(p) \tag{24}$$

with strict inequalities for a set of payoffs with positive measure.
Incentive compatibility constraint (19) is strictly more relaxed (i.e., its left-hand side is larger) for financial contract A than for financial contract B.

The intuition for this result is as follows. First, note that conditional on effort $e = 1$, to ensure the firm an expected payoff of \$1 if $p \in (\hat{p}, \hat{p} + dp)$, a financial contract must specify a repayment $R(\hat{p})$ such that $(\hat{p} - R(\hat{p})) f_1(p) dp = 1$. Conditional on effort $e = 0$, this implies an expected payoff of $(\hat{p} - R(\hat{p})) f_0(\hat{p}) dp = f_0(\hat{p}) / f_1(\hat{p})$ for the firm. Under MLRP, the latter expected payoff decreases with $\hat{p}$. Second, compared to contract B, contract A implies a net payoff $(\hat{p} - R(\hat{p}))$ for the firm that is higher for higher payoff realizations $\hat{p}$ and lower for lower payoff realizations. Therefore, holding the firm's expected payoff constant for $e = 1$ (as per condition (22)), it must be that

$$\mathbb{E}[R_A(P_{0,q})] > \mathbb{E}[R_B(P_{0,q})] \tag{25}$$

Finally, and again under condition (22), this implies that incentive compatibility constraint (19) left-hand side is greater for contract A than for contract B.

Thus, Lemmas 2 and 3 imply that higher firm's payoffs for higher realizations of revenues provide stronger incentives to the firm. But a debt contract precisely limits repayments to the investor for higher revenues' realizations by maximising the repayments to the investor for lower realizations. Following Innes (1990), we show next that for a given capacity q, the optimal contract is a debt contract, which we characterize.[10]

Proposition 2. *For a given capacity $q \in (0, \bar{q}_1]$, if the set of contracts satisfying conditions (17), (18) and (19) is non-empty then the firm exerts effort $e = 1$ and an optimal policy for financing the capacity cost cq is to use its internal funds W and fund any short-fall $I(q) = (cq - W)^+$ with debt with face value $K(q)$ equal to the unique solution to*

$$K - (r - s) \int_0^{\left(\frac{K - sq}{r - s}\right)^+} F_1(x)\, dx = (cq - W)^+ \tag{26}$$

- *If $W \geq cq$ then $K = 0$ and the firm does not raise funds.*
- *If $cq > W \geq (c - s)q$ then $K = I(q) \in (0, sq]$ and debt is riskfree.*
- *If $(c - s)q > W$, then $K \in (sq, rq]$ and debt involves default risk.*

[10] Here, the optimal financial contract is generally not unique due to the discrete effort choice our model assumes.

In other words, $R(p) = p \wedge K(q)$ is an optimal repayment function. Equation (26) corresponds to funding condition (17) and investor's binding participation constraint (11), such that $\mathbb{E}[R(P_{1,q})] = (cq - W)^+$.

Note that for $q \in (0, \bar{q}_1]$, equation (26) has a unique solution. Indeed, its left-hand side, equal to $\mathbb{E}[P_{1,q} \wedge K]$, is continuous and strictly increasing in K over $[sq, rq]$. Moreover, for $K = 0$, $\mathbb{E}[P_{1,q} \wedge K]$ is equal to zero which is less than $(cq - W)^+$. Conversely, for $K = rq$, $\mathbb{E}[P_{1,q} \wedge K]$ is equal to $\mathbb{E}[P_{1,q}]$ which from condition (4), is greater than $(cq + \kappa_1)$ for $q \in (0, \bar{q}_1]$. This ensures existence and uniqueness of a solution. Said differently, a demand level $\hat{d}$ always exists such that the firm does not default on his debt with face value $K(q)$ and instead starts earning revenues, where

$$\hat{d}(q) \equiv \left(\frac{K(q) - sq}{r - s}\right)^+ \tag{27}$$

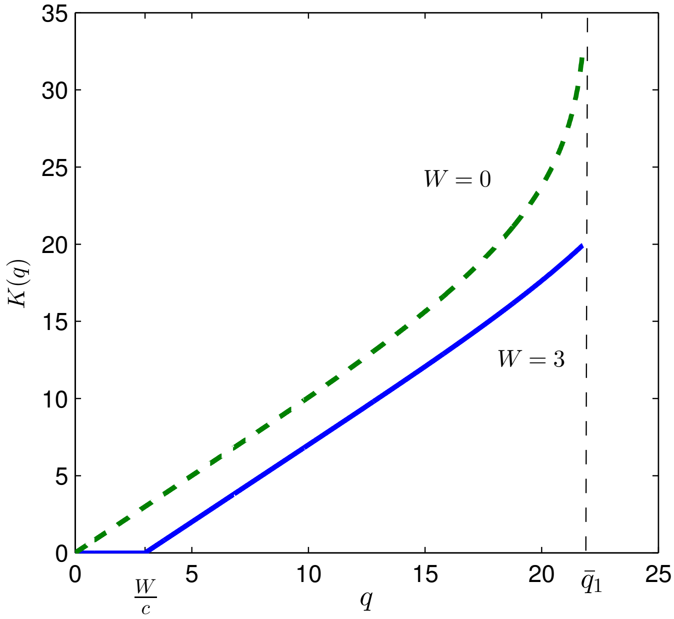
The intuition for the optimality of debt financing is as follows. Consider all feasible contracts allowing the firm to raise a given amount of funds I under the premise that $e = 1$. These contracts all share the same expected repayment which is equal to I. Among those, one and only one is a debt contract. Loosely speaking the debt contract is the contract within that set with the highest possible repayments in low payoff realizations (i.e., $R(p) = p$ for all $p \leq K$ which is as high as the limited liability condition allows) and the lowest repayments for high payoff realizations (i.e., $R(p) = R(K) = K$ for all $p \geq K$ which is as low as the monotonicity condition allows). From Lemma 3, it is therefore the contract providing the most powerful incentives, i.e., for which incentive compatibility constraint (19) is most likely to be satisfied.

The firm needs external funds only for $q > W/c$. In that case, define the debt's default probability is $\delta(q) \equiv Pr(P_{1,q} < K(q)) = F_1(\hat{d}(q))$ and its interest rate is $\rho(q) \equiv K(q)/I(q) - 1 = K(q)/(cq - W) - 1$. Note that given our " no discounting" assumption, interest rates are also a default spreads. For $q \in (W/c, W/(c - s)]$, the firm finances the project with internal funds and risk-free debt, i.e., with default probability $\delta(q) = 0$ and therefore interest rate $\rho(q) = 0$. For $q > W/(c - s)$ instead, the firm must issue risky debt (i.e., $\delta(q) > 0$) which therefore implies a default spread (i.e., $\rho(q) > 0$). As the desired capacity q increases, the following result shows that the debt's face value, interest rate and default probability are all non-decreasing.

PROPOSITION 3. *The optimal debt contract of Proposition 2 has the following properties. For $q \leq W/c$, the firm does not raise external funds and so $K(q) = 0$. Otherwise:*

- *Face value $K(q)$ is a non-decreasing convex function of q.*
- *Interest rate $\rho(q)$ is zero for $q \in (W/c, W/(c - s)]$ and strictly increasing for $q \in (W/(c - s), \bar{q}_1]$.*

Figure 1 **Debt face value K as a function of capacity q**



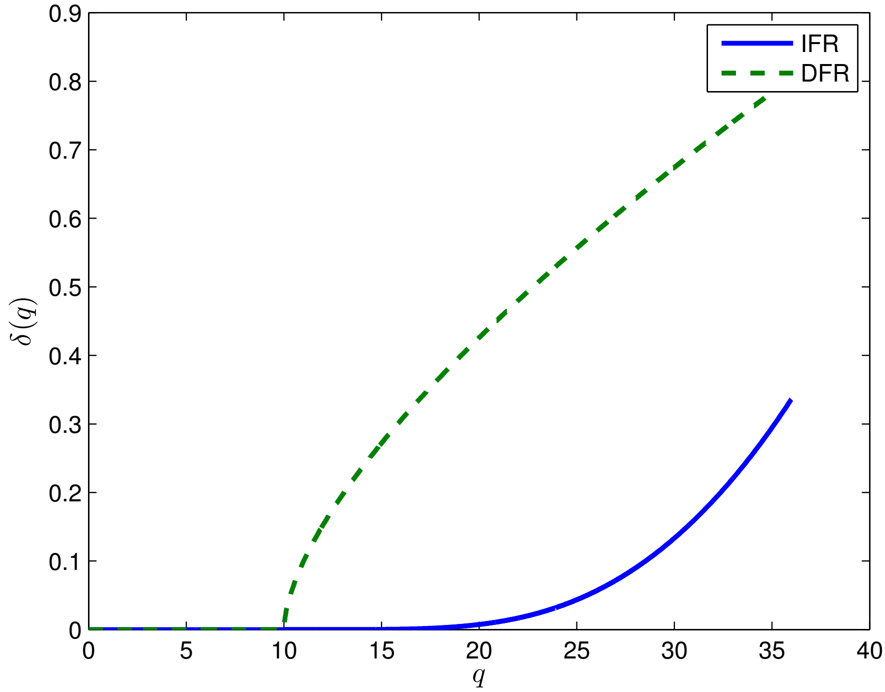
Note. $W = 0$ (dashed line) and $W = 3$ (plain line), $r_1 = 1.4$, $c_1 = 1$, $s_1 = 0.4$, $D_1 \sim Gamma(6, 2)$ and $D_0 \sim Gamma(5, 2)$, $\bar{q}_1 = 21.77$.

- *Default probability $\delta(q)$ is zero for $q \in (W/c, W/(c-s)]$ and strictly increasing for $q \in (W/(c-s), \bar{q}_1]$. Further $\delta(q)$ is convex (resp. concave) in q if and only if demand D_1 has increasing (resp. decreasing) hazard rate $h_1(\cdot)$.*

Proposition 3 is illustrated by Figures 1 and 2, which display the impact of capacity q on face value $K(q)$ and default probability $\delta(q)$, respectively. Figure 1 depicts two face value functions $K(q)$ for $W = 0$ and $W = 3$, respectively and for demands D_1 and D_0 following Gamma distributions with shape and scale parameters equal to $(6, 2)$ and $(5, 2)$, respectively (yielding means equal to 12 and 10, and variances equal to 24 and 20, respectively). Proposition 3 applies because Gamma distributions satisfy MLRP with respect to the shape parameter, holding the scale parameter constant. As the figure shows, the debt face values are increasing convex in capacity. Note that for $W = 3$, $K(q)$ is equal to zero when $q < W/c = 3$ since the firm funds the project with internal funds.

Figure 2 depicts probability of default function $\delta(q)$ for demands with increasing failure rates (IFR) and decreasing failure rates (DFR), respectively. Note that a gamma distribution is IFR if its shape parameter is larger than one, and is DFR otherwise. In particular, Gamma distribution with shape and scale parameters equal to $(.6, 2)$ is DFR. The figure shows, the default probability is increasing convex when D_1 is IFR, and decreasing convex when D_1 is DFR and for $q > W/(c-s) =$

Figure 2 **Default probability function** $\delta(q)$



Note. $W = 5$, $r_1 = 2$, $c_1 = 1$, $s_1 = 0.5$.
Demands with IFR (plain line): $D_1 \sim Gamma(6,2)$ $D_0 \sim Gamma(5,2)$ and
Demands with DFR (dashed line): $D_1 \sim Gamma(.6,20)$, $D_0 \sim Gamma(.2,20)$.

10. When $q \leq 10$, there is either no debt or debt is risk free according to Proposition 2 and so the default probability is zero.

A remark is in order for the case in which the firm's internal funds suffice to finance the first-best optimal capacity, i.e., $W \geq cq_1^{FB}$. In that case, an optimal contract is $R(\cdot) \equiv 0$ (or, as per Proposition 2, $K = 0$ so that condition (26) holds). Clearly, all constraints are satisfied by this contract and the objective is also maximized. Taken with the results in Section 3, this illustrates that limited wealth *per se*, or the non-contractibility of effort *per se* causes no meaningful departure from the standard newsvendor model. Instead, their combination does as we now show.

5. Optimal Investment Decisions

5.1. Optimal Capacity

We have established that if the firm decides to undertake the project (i.e., $q > 0$) then the firm works and an optimal financing arrangement (among others) is for the firm to use its internal funds W, possibly augmented by external debt financing with face value $K(q)$ given by condition (26). We can now determine the optimal capacity choice q^* which solves problem (5).

In particular, for a given capacity q, it may be that even under the optimal financial contract, the firm is still better off exerting low effort. This is determined by incentive compatibility condition (19). Noting that for $e=0,1$,

$$\mathbb{E}\left[P_{e,q}-P_{e,q}\wedge K(q)\right]-\kappa_e=(rq-K)-(r-s)\int_{\hat{d}(q)}^{q}F_e\left(x\right)dx-\kappa_e \tag{28}$$

we can rewrite incentive compatibility constraint (19) as

$$L\left(q,\Delta\kappa\right)\equiv\int_{\hat{d}(q)}^{q}\left(F_0\left(x\right)-F_1\left(x\right)\right)dx-\frac{\Delta\kappa}{(r-s)}\geq 0 \tag{29}$$

Recall that the debt's face value K affects the choice of efforts through the default-threshold for demand, $\hat{d}(q)$. Note further that $L\left(q,\Delta\kappa\right)$ is decreasing in $\Delta\kappa$: as the difference in effort costs increases, shirking becomes more attractive. It is not clear, however, how $L(q,\Delta\kappa)$ varies with q. We can nonetheless define $q_1^{\max}$ as the capacity in $[0,\bar{q}_1]$ for which $L\left(\cdot,\Delta\kappa\right)$ is maximum. This capacity provides the strongest incentive for the firm to exert effort. It can be shown to satisfy the following.

LEMMA 4. *Define* $q_1^{\max}\equiv\arg\max_{q\in[0,\bar{q}_1]}L(\cdot,\Delta\kappa)$. *We have* $q_1^{\max}>q_1^{FB}$.

The intuition for this key result is as follows. Starting from a given capacity q, consider the effect of a marginal increase in capacity, dq, on the firm's payoff and ultimately on incentive compatibility condition (29). There are two countervailing effects. On the one hand, higher capacity means higher payoffs for demand realizations above the initial capacity q. On the other hand, to fund the additional capacity cost $c\cdot dq$, the investor requires a higher face value for his debt. Considering the net effect, the firm's payoff decreases for some demand realizations and increases for others. Importantly, the demand realizations for which the payoff increases are higher than those for which it decreases. Two remarks complete the intuition. First, for $q\in[0,q_1^{FB}]$, the expected dollar increase in firm payoff is non-negative: it is equal to $\frac{\partial\pi_1(q)}{\partial q}\cdot dq\geq 0$, which is strictly positive for $q\in[0,q_1^{FB})$ and equal to zero for $q=q_1^{FB}$. Second, under MLRP, a given payoff in higher demand states provides more powerful incentives than an equivalent payoff in lower demand states. As a consequence, an increase in capacity results in more powerful incentives for the firm, i.e., relaxes incentive compatibility condition (29).

Define

$$S\left(\Delta\kappa\right)\equiv\left\{q\in\left[q_1^{FB},q_1^{\max}\right]\text{ s.t. }L\left(q,\Delta\kappa\right)\geq 0\right\} \tag{30}$$

From the previous analysis, the firm's overall problem boils down to the following. If $S\left(\Delta\kappa\right)\neq\varnothing$ then the optimal capacity choice is $q^*=\inf S\left(\Delta\kappa\right)$. Indeed, objective (15) is strictly decreasing for $q\geq q_1^{FB}$. If instead $S\left(\Delta\kappa\right)=\varnothing$ then there is no way for the firm to both finance a capacity $q\in[q_1^{FB},q_1^{\max}]$ and maintain his own incentives to exert effort $e=1$. In that case, condition (4) implies that the optimal outcome is for the firm to set $q^*=0$ and abandon the project.

We can now characterize the firm's optimal choice of capacity.

Theorem 1. *The firm's optimal choice of capacity q^* and effort level e^* solutions to Problem (15) are as follows. If the firm undertakes the project ($q^* > 0$) then the firm exerts effort $e^* = 1$.*

- *If $W \geq cq_1^{FB}$ then the firm undertakes the project without raising additional funds, and sets up the first-best capacity $q^* = q_1^{FB}$.*
- *If $cq_1^{FB} > W \geq (c-s)\,q_1^{FB}$ then the firm issues debt, undertakes the project, and sets up the first-best capacity $q^* = q_1^{FB}$.*
- *If $(c-s)\,q_1^{FB} > W$ then two thresholds $\Delta\underline{\kappa}$ and $\Delta\bar{\kappa}$ exist such that the optimal outcome is as follows.*

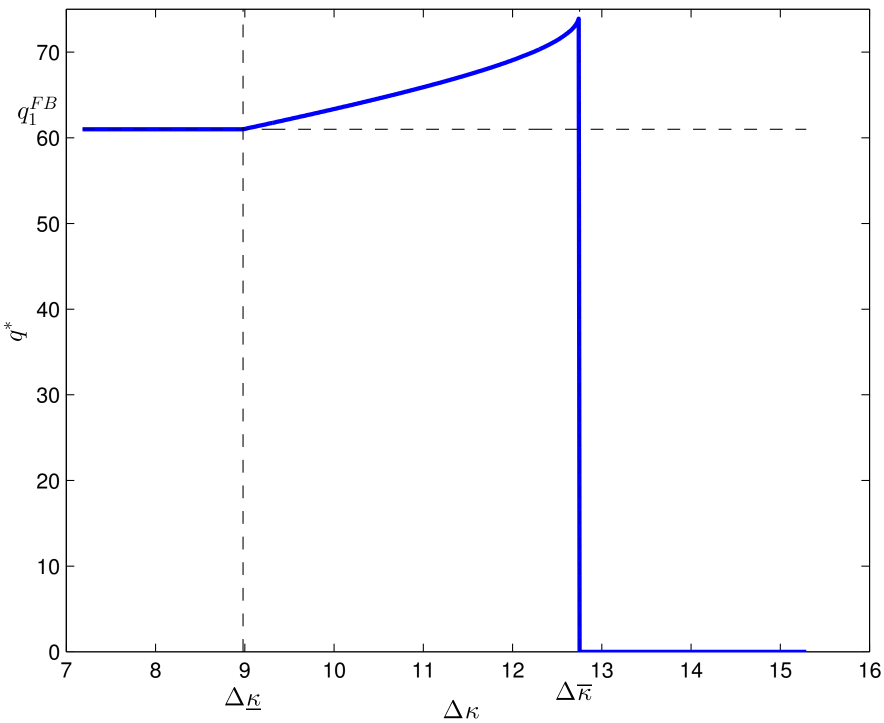
(i) If $0 \leq \Delta\kappa \leq \Delta\underline{\kappa}$ then the firm issues debt, undertakes the project, and sets up the first-best capacity $q^* = q_1^{FB}$.

(ii) If $\Delta\underline{\kappa} < \Delta\kappa \leq \Delta\bar{\kappa}$ then the firm issues debt, undertakes the project, and sets up a capacity level that strictly exceeds the first-best level $q^* > q_1^{FB}$. Further, q^* is monotonically increasing with $\Delta\kappa$.

(iii) If $\Delta\kappa > \Delta\bar{\kappa}$ then the firm abandons the project ($q^* = 0$).

Theorem 1 is illustrated by Figure 3 which displays the impact of effort cost $\Delta\kappa$ on the optimal capacity q^* for parameter values $W = 0$, $r = 11$, $c = 10$, $s = 0$, and for demands D_1 and D_0 following

Figure 3 Optimal Capacity q^* as a function of moral hazard problem's severity $\Delta\kappa$



Note. $W = 0$, $r_1 = 11$, $c_1 = 10$, $s_1 = 0$, $D_1 \sim Gamma(10,10)$ and $D_0 \sim Gamma(8,10)$.

Gamma distributions with shape and scale parameters equal to $(10,10)$ and $(8,10)$, respectively (yielding means equal to 100 and 80, and standard deviations equal to 31.6 and 28.28, respectively). Theorem 1 applies because Gamma distributions satisfy MLRP with respect to the shape parameter, holding the scale parameter constant. As the figure shows, the optimal capacity remains constant and equal to the first-best capacity $q^{FB}=61$ as long as $\Delta\kappa$ is below a first threshold, $\Delta\underline{\kappa}=8.98$. It is then strictly increasing above the efficient capacity level, until $\Delta\kappa$ reaches a second threshold, $\Delta\bar{\kappa}=12.75$, beyond which the firm abandons the project.

The intuition for this result is as follows. If possible, the firm will set up the first-best capacity q_1^{FB} and work ($e^{FB}=1$) to maximize objective (15). If the firm's internal funds are sufficient ($W \geq (c-s)\, q_1^{FB}$), the firm can finance the first-best capacity by combining internal funds and risk-free debt, thus maintaining his incentives to work.[11] Suppose now that the firm's internal funds are below that level. For $\Delta\kappa=0$, incentive compatibility constraint (29) is clearly satisfied for any q and therefore the firm sets up q_1^{FB}. As $\Delta\kappa$ increases, the constraint tightens but the firm sticks to q_1^{FB} until it binds, which occurs for some threshold $\Delta\underline{\kappa}$. Beyond that point, the constraint is violated for q_1^{FB} but holds for some capacities within $(q_1^{FB}, q_1^{\max}]$. Objective (15) being strictly decreasing in q over that interval, the firm will set up the smallest such capacity, i.e., $q^*=\inf S\,(\Delta\kappa)$. As $\Delta\kappa$ increases, the firm must increase capacity q further beyond q_1^{FB} to retain incentives to work, i.e., q^* increases with $\Delta\kappa$. Finally, for some threshold $\Delta\bar{\kappa}$, constraint (29) is binding for $q=q_1^{\max}$. Beyond that point, incentive compatibility cannot be satisfied and the firm must abandon the project.

5.2. Optimal Over-investment

That the project is abandoned if the "effort cost" $\Delta\kappa$ is large enough is hardly surprising, as is the result that the first best obtains if that cost is small enough. Indeed, both results would arise in a model with fixed capacity. The more surprising result is that for intermediate values of $\Delta\kappa$, the firm finds it optimal to set up a capacity in excess of the first best level. Indeed, an immediate consequence of Theorem 1 is that with financing frictions of the type considered in our model, the optimal capacity level is never below that of the standard newsvendor problem, unless the firm abandons the project, and can exceed it strictly. This happens when the firm does not have enough internal funds and the severity of the moral hazard problem is severe enough, as stated below.

Corollary 1. *For $q^*>0$, we have $q^*\geq q_1^{FB}$ where the last inequality is strict if and only if $W<(c-s)q_1^{FB}$ and $\Delta\kappa>\Delta\underline{\kappa}$.*

The role of setting up extra capacity above q_1^{FB} is twofold as may be seen through the following thought experiment. Suppose $q=q^{FB}$ and consider a marginal increase in capacity, dq. By definition

[11] In that case, $K<sq$ so that $\hat{d}=0$. This means that (29) is implied by (4).

of the first best capacity, such an increase creates no value: the increase in expected payoff it generates, must equal the cost it involves, i.e.,

$$\frac{\partial \mathbb{E}\left[P_{1,q}\right]}{\partial q} dq = c \cdot dq.$$

A first remark is that the increase in expected payoff, $c \cdot dq$, is concentrated on demand realizations exceeding q. Why can't the firm replicate this increase via the financial contract rather than by distorting the capacity choice away from its efficient level? Increasing the firm's expected payoff by $c \cdot dq$ for realizations of demand exceeding q (which have probability $\bar{F}_1(q)$ and all yield the same payoff rq) would simply mean reducing $R(rq)$ by $c \cdot dq / \bar{F}_1(q)$. This however would violate the monotonicity constraint as in that case we have

$$\lim_{x \to rq^-} R(x) = K(q) > \lim_{x \to rq^+} R(x) = K(q) - \frac{c \cdot dq}{\bar{F}_1(q)}.$$

This shows that the first role of the extra capacity is a *bonus effect*: it allows for an increase in the firm's expected payoff for high demand realizations without a commensurate reduction in the repayment to the investor for the same realizations which would violate the monotonicity condition.

However, even in the absence of the monotonicity condition on the financial contract, the firm may increase capacity above the efficient level. This is because the increase in expected payoff the extra capacity brings about is not spread equally across demand realizations above q. Instead, the payoff increases by $(r-s)(x-q) + s \cdot dq$ for demand realization $x \in (q, q+dq)$ and by a higher $r \cdot dq$ for $x \geq q + dq$. Again, why can't the firm replicate this via the financial contract? Absent the monotonicity condition, the firm could increase his expected payoff for those demand realizations. However, the increase in payoff would have the be the same across all of them, as the financial contract cannot distinguish between demand realizations due to demand's non-contractibility.

This shows that the second role of the extra capacity is a *demand elicitation effect*: it is also a way to elicit some demand realizations above q that would otherwise not be contractible, and to increase payoff more for higher demand realizations (i.e., $x \geq q + dq$) than for lower ones (i.e., $x \in (q, q+dq)$). That elicitation is valuable because under MLRP, payments in the higher payoff states provide more powerful incentives.

In summary, when the financial friction are high enough, first best cannot be achieved. In this case, the monotonicty condition and the non-contractibility of demand can induce the firm to improve the sensitivity of his payoff to effort through the *bonus* and *demand elicitation* effects, respectively, which requires increasing capacity above its efficient level so as to capture otherwise unmet demands.

5.3. The Impact of Internal Funds on Investment

The bonus and demand elicitation effects have further implications for the impact of internal funds on capacity levels. When the moral hazard problem's severity is low enough, the firm sets capacity q^* to q_1^{FB}, independently on its internal funds. For high level of financial frictions, however, the optimal capacity level can be *decreasing* with the firm's internal funds, as stated by the following result.

PROPOSITION 4. *Two non-negative thresholds* $\underline{W}$ *and* $\overline{W}$ *exist such that*

(i) If $W < \underline{W}$ *then the project is abandoned* ($q^* = 0$).

(ii) If $W \in [\underline{W}, \overline{W})$ *then capacity exceeds the first best* ($q^* > q_1^{FB}$) *and decreases with internal funds* W.

(iii) If $W \geq \overline{W}$ *then the first-best capacity is set up* $q^* = q_1^{FB}$.

The result states that if internal funds are low enough, the firm simply must abandon the project as the firm cannot at the same time raise sufficient funds and maintain incentives. For intermediate levels of internal funds, the firm can do both but at the cost of distorting capacity sufficiently beyond the efficient level to benefit from sufficiently powerful *bonus* and *demand elicitation* effects. As internal funds increase within that range, the need for external funds decreases, as does the debt's face value which also relaxes the incentive compatibility constraint. As a result, the bonus and demand elicitation effect required to maintain incentives need not be as powerful and the firm can lower capacity accordingly. The firm sets then capacity at the first-best level only when internal funds are large enough.

Figure 4 depicts the impact of firm internal funds W on optimal capacity q^* for the settings of Figure 3. Internal funds W varies while $\Delta\kappa$ is kept equal to 13.9. In this case, the firm starts raising additional funds to run the project when $W = \underline{W} = 4.2$. The installed optimal capacity decreases then in internal funds until $W = \overline{W} = 35.3$, at which point the firm sets capacity $q^* = q^{FB} = 61$.

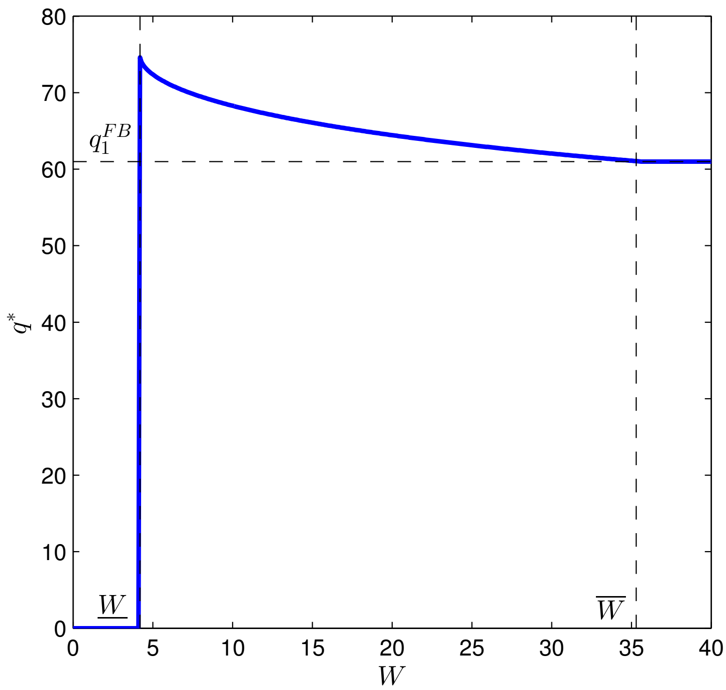
6. Conclusion

In this paper, we have considered the interaction between a firm's capacity choice problem and its decision of how to finance that capacity. The question is relevant only when access to financial markets is hampered by frictions, which we model by way of a moral hazard problem. In this context, the firm must account for the set of available sources of funds when calibrating its capacity investment. Our main findings are as follows. First, we show that debt financing is optimal for any given capacity level. Second, we show that given optimal debt financing, the optimal capacity is never below the efficient level but sometimes strictly above that level. Third we show that in the latter case, the optimal capacity level increases with the moral hazard problem's severity and decreases with the firm's internal funds. These results run counter to the newsvendor logic and to the common intuition that by raising the cost of external capital and hence the unit capacity cost, financial market frictions lead to lower investment in capacity. We trace the value of increasing capacity beyond the efficient level to a *bonus effect* and a *demand elicitation effect* both related to financial contract feasibility constraints and the risk of unmet demands.

Our framework can be extended in many directions. We have restricted our analysis to the natural assumption that higher demand realizations are more indicative of high effort. This need not be the case. For instance, efforts could affect (reduce) the variance of demand. In this case, the MLRP would not hold and equity could provide a better source of funding than debt. Further, we focus on financial frictions stemming from moral hazard. Other types of frictions, however, are possible such as asymmetric information.

Finally, an important aspect of our analysis is that it follows the optimal contracting approach in which the set of available sources of finance is derived endogenously from assumptions about fundamentals, i.e., preferences, physical constraints, and contractibility. This is in contrast to most of the nascent literature at the intersection of operations and finance which assumes exogenously specified sets of feasible contracts. Thus our paper opens a new direction to handle problems at the intersection between finance and operations, which consists in optimizing on not only the

Figure 4 Optimal Capacity q^* as a function of internal funds W



Note. $\Delta\kappa = 13.9$, $r_1 = 11$, $c_1 = 10$, $s_1 = 0$, $D_1 \sim Gamma(10, 10)$ and $D_0 \sim Gamma(8, 10)$.

operational aspects, but also on the financial contract.

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Appendix A: Proof of Lemma 2

The distributions of payoff are

$$\begin{cases} g_{e,q}(x) = f_e\left(P_{e,q}^{-1}(x)\right) & \text{for } x \in [sq, rq) \\ g_{e,q}(rq) = \bar{F}_e(q) \end{cases} \tag{31}$$

Hence $g_{1,q}/g_{0,q}$ is increasing over $[sq, rq)$ because f_0 and f_1 satisfy MLRP.

Moreover, we have

$$\begin{aligned}
\frac{g_{1,q}(rq)}{g_{0,q}(rq)} &= \frac{\bar{F}_1(q)}{\bar{F}_0(q)} && (32)\\
&= \frac{\int_q^{+\infty} f_1(x)\,dx}{\int_q^{+\infty} f_0(x)\,dx} && (33)\\
&= \frac{\int_q^{+\infty} \frac{f_1(x)}{f_0(x)} f_0(x)\,dx}{\int_q^{+\infty} f_0(x)\,dx} && (34)\\
&> \frac{\int_q^{+\infty} \frac{f_1(q)}{f_0(q)} f_0(x)\,dx}{\int_q^{+\infty} f_0(x)\,dx} \quad \text{by MLRP} && (35)\\
&> \frac{f_1(q)}{f_0(q)} && (36)\\
&> \frac{f_1(x)}{f_0(x)} \quad \text{for all } x \in [0, q) \text{ by MLRP} && (37)\\
&> \frac{g_{1,q}(p)}{g_{0,q}(p)} \quad \text{for all } p \in [sq, rq) && (38)
\end{aligned}$$

■

Appendix B: Proof of Lemma 3

We want to show that the left-hand side of incentive compatibility constraint (19) is weakly larger for contract A than for contract B, i.e.,

$$\mathbb{E}[P_{1,q} - R_A(P_{1,q})] - \mathbb{E}[P_{0,q} - R_A(P_{0,q})] - \Delta\kappa > \mathbb{E}[P_{1,q} - R_B(P_{1,q})] - \mathbb{E}[P_{0,q} - R_B(P_{0,q})] - \Delta\kappa.$$

After simplification and using condition (22), this amounts to showing

$$\mathbb{E}[R_A(P_{0,q})] > \mathbb{E}[R_B(P_{0,q})] \tag{39}$$

Therefore, we have

$$\begin{aligned}
\mathbb{E}[R_A(P_{0,q})] - \mathbb{E}[R_B(P_{0,q})] &= \int_{sq}^{p^*} (R_A(p) - R_B(p))\, g_{0,q}(p)\,dp - \int_{p^*}^{rq} (R_B(p) - R_A(p))\, g_{0,q}(p)\,dp \\
&= \int_{sq}^{p^*} (R_A(p) - R_B(p)) \frac{g_{0,q}(p)}{g_{1,q}(p)} g_{1,q}(p)\,dp && (40)\\
&\quad - \int_{p^*}^{rq} (R_B(p) - R_A(p)) \frac{g_{0,q}(p)}{g_{1,q}(p)} g_{1,q}(p)\,dp \\
&> \frac{g_{0,q}(p^*)}{g_{1,q}(p^*)} \int_{sq}^{p^*} (R_A(p) - R_B(p))\, g_{1,q}(p)\,dp && (41)
\end{aligned}$$

$$-\frac{g_{0,q}(p^*)}{g_{1,q}(p^*)}\int_{p^*}^{rq}\left(R_B(p)-R_A(p)\right)g_{1,q}(p)\,dp$$

$$> \frac{g_{0,q}(p^*)}{g_{1,q}(p^*)}\left(\mathbb{E}\left[R_A(P_{1,q})\right]-\mathbb{E}\left[R_B(P_{1,q})\right]\right) \tag{42}$$

$$> 0 \tag{43}$$

■

Appendix C: Proof of Proposition 2

Step 1. We adapt the analysis in Innes (1990) to show that we can restrict to debt contracts without loss of generality.

For a given capacity $q \in (0, \bar{q}_1]$, the firm's problem is to find a contract $R(\cdot)$ satisfying constraints (17), (18) and (19). Consider such a contract, $R(\cdot)$, assuming one exists, and consider the debt contract with face value $K \in (sq, rq)$ defined as the unique solution to

$$\mathbb{E}\left[P_{1,q} \wedge K\right] = \mathbb{E}\left[R(P_{1,q})\right] \tag{44}$$

Note that by construction, the debt contract satisfies (17), because contract $R(\cdot)$ does. Also, like all debt contract, it satisfies feasibility condition (18).

Notice that feasibility condition (18) implies that $\exists p^* \in (sq, rq)$ such that $\forall p \leq p^*,\ R(p) \leq p \wedge K$ and $\forall p \geq p^*,\ R(p) \geq p \wedge K$. Hence, the conditions of Lemma 3, implying that incentive compatibility constraint (19) is strictly tighter for contract $R(\cdot)$ than for the debt contract. In that sense, $R(\cdot)$ is dominated by the debt contract.

Step 2. Define function h as

$$h(K) \equiv K - (r-s)\int_0^{\left(\frac{K-sq}{r-s}\right)^+} F_1(x)\,dx - (cq-W)^+ \tag{45}$$

Expression (26) is written as $h(K) = 0$. Note that $h(K)$ is strictly increasing in K as $h'(K) = 1$ for $K \leq sq$ and $h'(K) = 1 - F_1\left(\frac{K-sq}{r-s}\right)$ for $K \geq sq$.

If $W \geq (c-s)q$ then $(cq-W)^+ \leq sq$ and

$$h\left((cq-W)^+\right) = (cq-W)^+ - (r-s)\int_0^0 F_1(x)\,dx - (cq-W)^+ = 0 \tag{46}$$

Therefore $K(q) = (cq-W)^+ \in [0, sq]$. Debt is riskfree because $K(q) \leq sq$ while $P_{1,q} \geq sq$.

Conversely, if $W < (c-s)q$ then $(cq-W)^+ > sq$ and so

$$h(sq) = sq - (r-s)\int_0^0 F_1(x)\,dx - (cq-W)^+ < 0 \tag{47}$$

Moreover

$$h(rq) = rq - (r-s)\int_0^q F_1(x)\,dx - (cq-W)^+ \tag{48}$$

$$= \left[\pi_1(q) - \kappa_1\right] + \left[\kappa(1) + cq - (cq-W)^+\right] > 0 \tag{49}$$

Indeed, the first bracket term non-negative over $[0, \bar{q}_1]$ and the second one is strictly positive because $\kappa_1 > 0$.

Therefore $K(q) \in (sq, rq)$. Moreover, debt is risky because $K(q) > sq$ and $P_{1,q} \geq sq$. ■

Appendix D: Derivatives of $K(q)$ and $\hat{d}(q)$

The following lemma provides the derivatives of $K(q)$ and $\hat{d}(q)$ with respect of q.

LEMMA 5. *If $W \geq (c-s)\,q$ then*

$$\frac{\partial \hat{d}\,(q)}{\partial q} = 0 \quad and \quad \frac{\partial K\,(q)}{\partial q} = c.$$

If $W < (c-s)\,q$ then

$$\frac{\partial \hat{d}\,(q)}{\partial q} = \frac{\bar{F}_1\,(q_1^{FB})}{\bar{F}_1\left(\hat{d}\,(q)\right)} \quad and \quad \frac{\partial K\,(q)}{\partial q} = \frac{c-s}{\bar{F}_1\left(\hat{d}(q)\right)} + s.$$

Proof: *Case 1.* $W \geq (c-s)\,q$

In that case, equation (26) implies $K\,(q) \leq sq$ and $\hat{d}\,(q) = 0$. Therefore $\frac{\partial \hat{d}(q)}{\partial q} = 0$, and taking the first-order derivative of condition (26) with respect to q we deduce yields

$$\frac{\partial K\,(q)}{\partial q} = c$$

Case 2. $W < (c-s)\,q$

In that case, equation (26) implies $K\,(q) > sq$ and $\hat{d}\,(q) = \left(\frac{K(q)-sq}{r-s}\right)$.

Taking the first-order derivative of condition (26) with respect to q yields

$$\frac{\partial K\,(q)}{\partial q} - (r-s)\,\frac{\partial \hat{d}\,(q)}{\partial q} F_1\left(\frac{K-sq}{r-s}\right) = c \tag{50}$$

Since

$$\frac{\partial \hat{d}\,(q)}{\partial q} = \frac{\frac{\partial K(q)}{\partial q} - s}{r-s}$$

this expression can be rewritten as

$$s + (r-s)\,\frac{\partial \hat{d}\,(q)}{\partial q} - (r-s)\,\frac{\partial \hat{d}\,(q)}{\partial q} F_1\left(\hat{d}\,(q)\right) = c \tag{51}$$

or

$$\frac{\partial \hat{d}\,(q)}{\partial q} = \frac{c-s}{r-s}\,\frac{1}{1-F_1\left(\hat{d}\,(q)\right)} = \left(1 - \frac{r-c}{r-s}\right)\frac{1}{1-F_1\left(\hat{d}\,(q)\right)} = \frac{\bar{F}_1\,(q_1^{FB})}{\bar{F}_1\left(\hat{d}\,(q)\right)} \tag{52}$$

and the corresponding derivative of $K(q)$ is obtained from (50). ■

Appendix E: Proof of Proposition 3

Lemma 5 (see Appendix D) implies that $\hat{d}(q)$ is non-decreasing in q. Further $\bar{F}_1(\cdot)$ is non-increasing so that the derivative of $\hat{d}(q)$ in Lemma 5 is also non-decreasing and $\hat{d}(q)$ is convex in q. It follows from (27) that $K(q)$ is also non-decreasing convex in q, which yields the first part of the result.

Further the derivative of interest rate $\rho(q)$ is equal to, for $(c-s)q > W$,

$$\frac{\partial \rho(q)}{\partial q}(q) = \frac{1}{cq-W}\frac{\partial K(q)}{\partial q} - \frac{c}{(cq-W)^2}K(q). \tag{53}$$

But from (26), we have

$$K = (r-s)\int_0^{\left(\frac{K-sq}{r-s}\right)^+} F_1\,(x)\,dx + cq - W$$

$$\begin{aligned} &\leq F_1(\hat{d}(q))(r-s)\hat{d}(q)+cq-W \\ &= F_1(\hat{d}(q))(K-sq)+cq-W, \end{aligned}$$

where the last equality holds from (27). By rearranging the terms, we deduce

$$K \leq \frac{1}{\bar{F}_1(\hat{d}(q))}\left(cq-W-sqF_1(\hat{d}(q))\right)$$

which with (54) implies

$$\begin{aligned} \frac{\partial\rho(q)}{\partial q} &\geq \frac{1}{cq-W}\frac{\partial K(q)}{\partial q} - \frac{c}{\bar{F}_1(\hat{d}(q))}\left(\frac{1}{cq-W}-\frac{sqF_1(\hat{d}(q))}{(cq-W)^2}\right) \\ &= \frac{1}{cq-W}\left(\frac{c-s}{\bar{F}_1\left(\hat{d}(q)\right)}+s\right) - \frac{c}{\bar{F}_1(\hat{d}(q))}\left(\frac{1}{cq-W}-\frac{sqF_1(\hat{d}(q))}{(cq-W)^2}\right) \\ &= \frac{csF_1\left(\hat{d}(q)\right)}{(cq-W)\bar{F}_1\left(\hat{d}(q)\right)} \geq 0 \end{aligned}$$

where the first equality holds from Lemma 5.

Finally, the derivative of $\delta(q) \equiv F_1\left(\hat{d}(q)\right)$ is equal to

$$\begin{aligned} \frac{\partial\delta(q)}{\partial q} &= f_1\left(\hat{d}(q)\right)\frac{\partial\delta(q)}{\partial q} \\ &= f_1\left(\hat{d}(q)\right)\frac{\bar{F}_1\left(q_1^{FB}\right)}{\bar{F}_1\left(\hat{d}\left(q\right)\right)} = h_1\left(\hat{d}(q)\right)\bar{F}_1\left(q_1^{FB}\right) \end{aligned}$$

where the second equality holds from Lemma 5. Since $\left(\hat{d}\left(q\right)\right)$ is non-decreasing in q, $\partial\delta(q)/\partial q$ is non-negative and non-decreasing (resp. non-increasing) when $h_1(\cdot)$ is increasing (res. decreasing). ■

Appendix F: Proof of Lemma 4

We have

$$\frac{\partial L\left(q,\Delta\kappa\right)}{\partial q} = \left(F_0\left(q\right)-F_1\left(q\right)\right) - \frac{\partial\hat{d}\left(q\right)}{\partial q}\left(F_0\left(\hat{d}\left(q\right)\right)-F_1\left(\hat{d}\left(q\right)\right)\right) \tag{54}$$

Case 1. $W \geq (c-s)\,q$

In that case $\hat{d}\left(q\right)=0$ from Lemma 5 (see Appendix D), which implies

$$\frac{\partial L\left(q,\Delta\kappa\right)}{\partial q} = \left(F_0\left(q\right)-F_1\left(q\right)\right) > 0 \tag{55}$$

This means that $L\left(q,\Delta\kappa\right)$ is increasing in q and that $q_1^{\max}=\bar{q}_1>q_1^{FB}$.

Case 2. $W < (c-s)\,q$

In that case, $\partial\hat{d}\left(q\right)/\partial q = \bar{F}_1\left(q_1^{FB}\right)/\bar{F}_1\left(\hat{d}\left(q\right)\right)$ from Lemma 5.

Hence, for $W<(c-s)\,q_1^{FB}$, we can rewrite condition (54) as

$$\begin{aligned} \frac{\partial L\left(q,\Delta\kappa\right)}{\partial q} &= \left(F_0\left(q\right)-F_1\left(q\right)\right) - \frac{\bar{F}_1\left(q_1^{FB}\right)}{\bar{F}_1\left(\hat{d}\left(q\right)\right)}\left(F_0\left(\hat{d}\left(q\right)\right)-F_1\left(\hat{d}\left(q\right)\right)\right) && (56) \\ &= \left(\bar{F}_1\left(q\right)-\bar{F}_0\left(q\right)\right) - \frac{\bar{F}_1\left(q_1^{FB}\right)}{\bar{F}_1\left(\hat{d}\left(q\right)\right)}\left(\bar{F}_1\left(\hat{d}\left(q\right)\right)-\bar{F}_0\left(\hat{d}\left(q\right)\right)\right) && (57) \end{aligned}$$

For $q \leq q_1^{FB}$, we have $\bar{F}_1(q) \geq \bar{F}_1(q_1^{FB})$ and so

$$\frac{\partial L(q,\Delta\kappa)}{\partial q} \geq \left(\bar{F}_1(q) - \bar{F}_0(q)\right) - \frac{\bar{F}_1(q)}{\bar{F}_1\left(\hat{d}(q)\right)}\left(\bar{F}_1\left(\hat{d}(q)\right) - \bar{F}_0\left(\hat{d}(q)\right)\right) \tag{58}$$

$$\geq \bar{F}_1(q)\left[\left(1 - \frac{\bar{F}_0(q)}{\bar{F}_1(q)}\right) - \left(1 - \frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\right)\right] \tag{59}$$

$$= \bar{F}_1(q)\left[\frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)} - \frac{\bar{F}_0(q)}{\bar{F}_1(q)}\right] \tag{60}$$

Note now that $\bar{F}_0/\bar{F}_1$ is decreasing. Indeed,

$$\frac{d}{dx}\frac{\bar{F}_0(x)}{\bar{F}_1(x)} = \frac{-f_0(x)\bar{F}_1(x) + f_1(x)\bar{F}_0(x)}{\left(\bar{F}_1(x)\right)^2} < 0 \tag{61}$$

which is negative since MLRP implies that D_1 stochastically dominates D_0 according to the hazard rate order[12].

Next, note that for all $q < \bar{q}_1$, $\hat{d}(q) < q$. This implies that for all $q \leq q_1^{FB}$, $\partial L(q,\Delta\kappa)/\partial q > 0$, which in turn implies that $q_1^{\max} > q_1^{FB}$. ■

Appendix G: Proof of Theorem 1

Recall that constraint (4) implies an upper bound on $\Delta\kappa$, i.e., $\pi_1(q_1^{FB}) - \pi_0(q_0^{FB}) > \Delta\kappa$. If $\pi_1(q_1^{FB}) - \pi_0(q_0^{FB}) < (r-s)L(q_1^{FB},0)$, then for all values of $\Delta\kappa$ such that (4) holds, we have $L(q_1^{FB},\Delta\kappa) > 0$ so that $q^* = q_1^{FB}$ for all W and we have thresholds $\Delta\underline{\kappa} = \Delta\bar{\kappa} = 0$.

Assume now that $\pi_1(q_1^{FB}) - \pi_0(q_0^{FB}) < (r-s)L(q_1^{FB},0)$, which corresponds to the non-degenerate case. If $W \geq (c-s)q_1^{FB}$ then $\hat{d}(q_1^{FB}) = 0$. In that case, condition (4) implies $L(q_1^{FB},\Delta\kappa) > 0$ and so $q^* = \inf S(\Delta\kappa) = q_1^{FB}$.

For all q, $L(q,0) > 0$ and for $\Delta\kappa$ large enough $L(q,\Delta\kappa) < 0$. Moreover, $L(q,\Delta\kappa)$ is continuous and strictly decreasing with $\Delta\kappa$. Therefore, we can define $\Delta\underline{\kappa}$ and $\Delta\bar{\kappa}$ by $L(q_1^{FB},\Delta\underline{\kappa}) = 0$ and $L(\hat{q}_1,\Delta\bar{\kappa}) = 0$.

For $\Delta\kappa \leq \Delta\underline{\kappa}$, $L(q_1^{FB},\Delta\kappa) \geq 0$ and so $q^* = \inf S(\Delta\kappa) = q_1^{FB}$.

For $\Delta\underline{\kappa} < \Delta\kappa \leq \Delta\bar{\kappa}$, $L(q_1^{FB},\Delta\kappa) < 0$ and $L(\hat{q}_1,\Delta\kappa) \geq 0$ so $q^* = \inf S(\Delta\kappa) > q_1^{FB}$ and increases with $\Delta\kappa$.

For $\Delta\kappa > \Delta\bar{\kappa}$, $S(\Delta\kappa) = \varnothing$ and $q^* = 0$. ■

Appendix H: Proof of Proposition 4

Consider the left-hand side of incentive compatibility constraint (29) as a function $L(W,q,\Delta\kappa)$. We have

$$L\left(0,q_1^{FB},0\right) = \int_{\hat{d}\left(q_1^{FB}\right)}^{q_1^{FB}} \left(F_0(x) - F_1(x)\right)dx > 0 \tag{63}$$

Recall that constraint (4) implies an upper bound on $\Delta\kappa$, i.e., $\pi_1(q_1^{FB}) - \pi_0(q_0^{FB}) > \Delta\kappa$.

[12] To see this and for completeness, note that

$$f_1(x)\bar{F}_0(x) = f_0(x)\int_x^{+\infty}\left(\frac{f_1(x)}{f_0(x)}/\frac{f_1(t)}{f_0(t)}\right)f_1(t)\,dt \leq f_0(x)\int_x^{+\infty} f_1(t)\,dt = f_0(x)\bar{F}_1(x) \tag{62}$$

where the inequality holds since under MLRP, for all $t > x$, we have $f_1(x)/f_0(x) < f_1(t)/f_0(t)$.

Case 1: $\pi_1\left(q_1^{FB}\right)-\pi_0\left(q_0^{FB}\right)<(r-s)\,L\left(0,q_1^{FB},0\right)$. In that case, for all values of $\Delta\kappa$ such that (4) holds we have $L\left(0,q_1^{FB},\Delta\kappa\right)>0$. Since $\partial L\left(W,q,\Delta\kappa\right)/\partial W\geq 0$, this implies $L\left(W,q_1^{FB},\Delta\kappa\right)\geq 0$ and so $q^*=q_1^{FB}$ for all W, i.e. $\underline{W}=\overline{W}=0$.

Case 2: $\pi_1\left(q_1^{FB}\right)-\pi_0\left(q_0^{FB}\right)>(r-s)\,L\left(0,q_1^{FB},0\right)$. In that case, values of $\Delta\kappa$ such that condition (4) holds exist such that

$$L\left(0,q_1^{FB},\Delta\kappa\right)=\int_{\hat{d}\left(q_1^{FB}\right)}^{q_1^{FB}}\left(F_0\left(x\right)-F_1\left(x\right)\right)dx-\frac{\Delta\kappa}{(r-s)}<0 \tag{64}$$

Since L is continuous and $\partial L\left(0,q,\Delta\kappa\right)/\partial\Delta\kappa<0$, we can define $\Delta\kappa_0$ as the unique solution to $L\left(0,q_1^{FB},\Delta\kappa\right)=0$.

For all $\Delta\kappa\leq\Delta\kappa_0$, $L\left(0,q_1^{FB},\Delta\kappa\right)\geq 0$ which given $\partial L\left(W,q,\Delta\kappa\right)/\partial W\geq 0$ implies $L\left(W,q_1^{FB},\Delta\kappa\right)\geq 0$ for all W so that $\underline{W}=\overline{W}=0$. Now consider $\Delta\kappa>\Delta\kappa_0$, which implies $L\left(0,q_1^{FB},\Delta\kappa\right)<0$. Note that for $W\geq(c-s)\,q_1^{FB}$ we have $\hat{d}\left(q_1^{FB}\right)=0$ which implies

$$\begin{aligned}
L\left((c-s)\,q_1^{FB},q_1^{FB},\Delta\kappa\right) &= \int_0^{q_1^{FB}}\left(F_0\left(x\right)-F_1\left(x\right)\right)dx-\frac{\Delta\kappa}{(r-s)} && (65)\\
&= \frac{\pi_1\left(q_1^{FB}\right)-\pi_0\left(q_1^{FB}\right)-\Delta\kappa}{(r-s)} && (66)\\
&> \frac{\pi_1\left(q_1^{FB}\right)-\pi_0\left(q_0^{FB}\right)-\Delta\kappa}{(r-s)}>0 && (67)\\
& && (68)
\end{aligned}$$

the latter inequality being implied by condition (4).

Note that $q_1^{\max}\left(W\right)$ is a function of W. If $L\left(0,q_1^{\max}(0),\Delta\kappa\right)\geq 0$ then let $\underline{W}=0$. Otherwise define $\underline{W}$ as the smallest W solution to $L\left(W,q_1^{\max}(W),\Delta\kappa\right)=0$. We have for all $W<\underline{W}$ (if any), $L\left(W,q_1^{\max}(W),\Delta\kappa\right)<0$ and so $L\left(W,q,\Delta\kappa\right)<0$ for all $q\leq\bar{q}_1$. Hence the project is abandoned ($q^*=0$). Define $\bar{W}$ as the unique solution to $L\left(W,q_1^{FB},\Delta\kappa\right)=0$. Since $L\left(W,q_1^{FB},\Delta\kappa\right)$ is non-decreasing in W, we have for all $W\geq\bar{W}$, $L\left(W,q_1^{FB},\Delta\kappa\right)\geq 0$ and so $q^*=q_1^{FB}$.

For $W=\underline{W}$, $q^*=q_1^{\max}(\underline{W})$ and for all $W\in[\underline{W},\bar{W})$

$$q^*(W)=\inf\left\{q\in\left[q_1^{FB},q_1^{\max}(W)\right]\ \text{s.t. } L\left(W,q,\Delta\kappa\right)\geq 0\right\} \tag{69}$$

By definition, $q_1^{\max}(W)\leq\bar{q}_1$ for all W. Hence, the previous condition can be rewritten as

$$q^*(W)=\inf\left\{q\in\left[q_1^{FB},\bar{q}_1\right]\ \text{s.t. } L\left(W,q,\Delta\kappa\right)\geq 0\right\} \tag{70}$$

Since for all q, $\partial L\left(W,q,\Delta\kappa\right)/\partial W\geq 0$, $q^*(W)$ decreases with W over $[\underline{W},\bar{W})$.

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