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Interaction of Contracts in a Heterogeneous Life Insurance Portfolio

Dissertation zur Erlangung des akademischen Grades eines Doktors der Wirtschaftswissenschaften (Dr. rer. pol.) an der Fakultät für Mathematik und Wirtschaftswissenschaften der Universität Ulm

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Overview of Research Papers

Research papers included in this dissertation

1. Eckert, J., Graf, S., and Kling, A. (2020): A measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio. *European Actuarial Journal: DOI: 10.1007/s13385-020-00225-2.*

2. Eckert, J., Graf, S., Kling, A., and Ruß, J. (2020): Cross Subsidizing Effects between existing and new policyholders in traditional life insurance *Working Paper, Ulm University.*

3. Eckert, J. (2020): Werden bei einer Differenzierung der Überschussbeteiligung in der Lebensversicherung Neukunden zu Lasten des Bestands bevorzugt? *Working Paper, Ulm University.*

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Research Context and Summary of Research Papers

1 Field of Research

Pricing, hedging and risk management of insurance policies are important for life insurance companies, especially in the current situation an enduring low interest rate period prevailing in the financial market. Existing academic literature (see for example Grosen and Jørgensen (2000), Bacinello (2003), Bauer et al. (2005), Graf et al. (2011) and Gatzert and Martin (2014)) on this field of research typically assumes a homogeneous insurance portfolio, i.e. a portfolio with identical policies. However, insurance portfolios consist of many different types of contracts, i.e. they are heterogeneous in many aspects. This cumulative thesis contributes to the field of research on heterogeneous insurance portfolios and further focuses on the interdependence between different cohorts of insurance contracts whose returns are linked to the same reference portfolio.

Due to the enduring low interest rate period, the technical rate of interest for the calculation of local GAAP reserves has decreased in practically every European country. Previously granted long-term guarantees were not priced adequately by many life insurance companies and therefore additional reserves have had to be set up. This problem occurs especially in the insurance sector in countries where products with interest rate guarantees are traditionally very popular, such as the German insurance market. Additionally, given many current regulatory regimes, e.g. within Solvency II in Europe[1], market consistent valuation of life insurance liabilities is nowadays required which on the one hand makes the value and risk of existing liabilities more transparent. On the other hand, these regulatory standards increase the pressure to develop innovative products which reduce the risk of the insurance companies.

As a consequence, new products with different kinds of guarantees, different types of surplus participation schemes as well as various investment strategies were and are currently developed by many insurance companies. For example, while traditional participating products in many countries have been generally equipped with a guaranteed interest rate that has to be granted on an annual basis, the new designed variations of participating products provide only a guaranteed

[1]See for example Eling et al. (2007) which gives an overview of the solvency regulation development.

benefit at contract's maturity (cf. Reuß et al. (2015) and Reuß et al. (2016)) and lower or remove the annual return guarantees instead. As a result, an insurance portfolio considered as a whole nowadays consists of contracts with different guaranteed interest rates and different product designs, for example in a typical German life insurance portfolio annual return guarantees range from 0 to 4 percent. For the pricing of insurance policies, this heterogeneity of the insurance portfolio must be taken into account.

Insurance contracts of the same insurance portfolio depend on each other. Their premiums for example are invested in the same reference portfolio and the investment returns of this joint asset pool are shared between all policyholders. To stabilize the investment return local GAAP reserves are typically build up which have not yet been allocated to a specific insurance contract and thus belong to all contracts of the considered insurance portfolio. The research papers of this thesis deal with new questions that arise from this context.

There are only few academic papers which consider heterogeneous portfolios in their analyses so far. For example, Hieber et al. (2019) theoretically show the existence of parameter combinations which allow integrating individually fairly priced insurance contracts in an existing insurance portfolio without any transfer of wealth. Cross subsidization effects between different cohorts of contracts in a heterogeneous portfolio are treated in even less papers: Døskeland and Nordahl (2008) analyse intergenerational cross subsidization effects of guaranteed pension contracts written at different times. The focus of Yi et al. (2019) which consider collective pensions without an investment guarantee by the plan sponsor is to show that reserves are helpful to stabilize the benefits of later generations. The cross subsidization effects on the risk-return profiles of different generations is analysed by Hieber et al. (2015). Wieland (2017) focuses on the interactions between new business and existing business from the insurance company's point of view.

These papers however do not apply a clear method for assessing the cross subsidizing effects in their analyses. Hence, this thesis introduces a well-defined measure for ex ante expected and ex post realized cross subsidizing effects and therefore extends the research on heterogeneous insurance portfolios.

2 Motivation and Objectives

Fair valuation is often discussed based on a single contract or from the viewpoint of a homogeneous portfolio. In other words, only the parameters of the considered insurance contract, such as premium payments and payments to the policyholders, are taken into account in the valuation. The interaction to other contracts in the portfolio is neglected. Therefore, fair calculation in a heterogeneous insurance portfolio is addressed in this thesis where fair valuation

of a contract means that this contract neither benefits from nor subsidizes the other contracts in the heterogeneous insurance portfolio.

This thesis also aims to thoroughly analyse the interdependence between different contracts that means we analyse whether there are contracts which benefit from or subsidize other contracts in the portfolio. For assessing this issue, both the future and the past should be considered, i.e. it should be determined how much one contract or one collective of contracts will benefit in expectation (ex ante) or has already benefited (ex post) from other contracts in the insurance portfolio so far. To answer these questions, we first need a well-defined measure to systematically analyse the interaction in a heterogeneous insurance portfolio. This issue lead us to the following research question:

Research Question 1: How can you measure the interaction of contracts in a heterogeneous life insurance portfolio? How can you systematically analyse cross-subsidizing effects?

Traditional life insurance with surplus participation is still a major part of business of life insurance companies and therefore an important part of the old-age provision of many people in Europe, and especially in Germany[2]. These products typically consist of two parts: a guaranteed benefit based on a guaranteed interest rate and a surplus participation on the basis of the returns of a joint asset pool. The financial guarantees of new contracts have been steadily reduced.

Hence, there is an ongoing discussion whether traditional life insurance business is still attractive for new policyholders entering the already existing portfolio or if their contributions are only applied to finance the (now too high) guarantees of the existing portfolio. This question is very relevant for decisions on retirement savings and cannot be answered straightforward. On the one hand, newly issued contracts have a lower guaranteed interest rate than the already existing portfolio and hence in certain scenarios might subsidize old guarantees. On the other hand, the new business benefits from high returns of the existing asset portfolio that has been built up with previous policyholders' premiums. Hence, this issue leads to the second research question:

Research Question 2: Which cross-subsidizing effects are between different cohorts of traditional participating life insurance products? Do new policyholders benefit from or subsidize the existing portfolio? Under which circumstances is there a benefit for or subsidization from new business?

[2] At the end of 2019, the proportion of traditional life insurance with surplus participation in the German market is more than 58% of the old-age provisions contracts in force (cf. statistical publication of the German Insurance Association (GDV): https://www.gdv.de/resource/blob/60734/ba7317759252abad585896e4fe700507/bav—lv-in-zahlen-2020—download-data.pdf, accessed 01.08.2020)

To increase the attractiveness of participating life insurance products for new policyholders, some companies offer the possibility of an increased surplus participation as compensation for somehow reduced guarantees. Hence, there is a differentiation of the surplus participation between the existing policyholders who are invested in the traditional participating insurance product and the new business invested in the product with reduced guarantees.

To determine the "spread", which indicates the differentiation of surplus participation between existing business and new business, we apply the method described in the report of the "Deutsche Aktuarvereinigung e.V." (cf. DAV (2017)): The surplus participation of two insurance contracts should be differentiated such that they are of "equal value" despite of different guarantee levels. "Value equivalence" in this setting means that applying a stochastic simulation the expected value of the discounted benefits adjusted by the premium amount is equal for both product designs. However, it is not clear whether each contract must be considered on a stand-alone basis or if the interaction with an existing insurance portfolio shall be taken into account. This leads to the third research question:

Research Question 3: What effects do a collective, i.e. taking into account the existing business, and an individual consideration have on the determination of an appropriate surplus differentiation? What are the effects of a differentiation of surplus participation on the attractiveness of existing policyholders, new policyholders and the shareholders? Are new policyholders preferred at the expense of existing policyholders due to the differentiation of the surplus participation in life insurance?

The interdependence between contracts in a heterogeneous life insurance portfolio has a strong influence on the pricing of life insurance policies. Hence, insurance companies will be faced with these topics and they have to deal with the aforementioned research questions. The three papers included in this dissertation contribute to the literature about heterogeneous life insurance portfolios and significantly improve the modelling, measuring and quantification of the interaction of contracts in a heterogeneous life insurance portfolio.

3 Summary of Research Papers

Research Paper 1: A measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio

This paper is joint work with Stefan Graf as well as Alexander Kling and it has been published in the *European Actuarial Journal* in 2020. The paper systematically analyses the interdependence between different contracts, provides a framework and answers the question how the interdependence can be measured.

The interactions of contracts in a heterogeneous life insurance portfolio are often quite complex and the question whether certain contracts benefit from others or implicitly sponsor others is not clear in most cases. Therefore, we start with a short motivating example showing how two contracts with different interest rate guarantees interact in a portfolio. We assume that one contract has a guaranteed maturity benefit and the other contract has a year-by-year guarantee. Both contracts are fairly priced on an individual basis. To demonstrate the interaction of the contracts, we increase the guaranteed rate of the first contract which may lead to a situation where this contract is on an individual basis not fair any more. Quite naturally, the other contract has to pay a price for this.

Afterwards, a methodology for analysing and measuring the interaction of different contracts in a heterogeneous insurance portfolio is presented: the concept of the so-called "collective bonus", respectively "collective malus". The collective bonus indicates how much one contract or one collective of contracts benefits in expectation (ex ante) or has already benefited (ex post) from other contracts in the insurance portfolio so far. The collective malus accordingly determines how much a contract subsidizes or has already subsidized other contracts. Based on risk-neutral valuation, the collective bonus is defined as the difference of the contract's returns within the insurance portfolio and a theoretical investment of the contract's premiums in the capital market. An interpretation of a collective bonus is therefore given by a previous or expected outperformance as compared to the reference portfolio and a collective malus by a past or expected underperformance of the insurance contract.

It is also shown that the market value of assets and liabilities equal each other if and only if the sum of the collective boni (collective mali) of all policyholders and the shareholders is zero. This provides an alternative way to calculate the collective bonus and indicates the connection between the introduced concept and an insurance company's market value balance sheet. Moreover, the concept of the collective bonus provides a method to check, whether a contract is calculated fairly in a heterogeneous insurance portfolio, i.e. the contract neither benefits from nor subsidizes the other contracts of the insurance portfolio.

In the last section of the paper the concept of the collective bonus is applied to the motivating example and two already existing examples from the academic literature: Hieber et al. (2019) and Døskeland and Nordahl (2008). Ex ante and ex post calculations of the collective bonus are illustrated. In Hieber et al. (2019) a new insurance contract is added to an existing insurance portfolio whereby the interaction based on the default risk of the insurance company. Then it is checked whether the newly added insurance contract benefits from the existing portfolio and have a collective bonus or the newly added contract subsidize the existing portfolio and have a collective malus. The influence of the new contract and different product designs of the newly added contract on the collective bonus of the existing business as well as the collective bonus of the new contract are also analysed.

Døskeland and Nordahl (2008) analyse intergenerational effects of guaranteed pension contracts which share the same asset pool and interact due to the surplus participation on the earnings of the company and a common bonus reserve. The authors deal with a portfolio of 80 generations of identical pension contracts that are written at different times. The older generations leave parts of their reserves for future generations of contracts. Thus the expected discounted payoff at maturity is for younger generations greater than the paid single premium and for the older generations smaller than the single premium. There are intergenerational cross subsidization effects from older generations to younger generations. With the concept of the collective bonus, the amount of subsidization is measured and the results of the paper are verified. It is also shown how sensitive intergenerational effects are with respect to the amount of reserves that were built up in the past.

In summary, in this paper we analyse the interaction of contracts in a heterogeneous insurance portfolio and provide a methodology to answer the question of subsidization and beneficiation with a well-defined measure. Hence, this paper answers the first part of the research questions. Moreover, we are able to verify the results of two original papers from the academic literature and measure the amount of subsidization by the means of the collective bonus. Since the focus of this paper is the introduction of a corresponding methodology of the collective bonus, the examples and model frameworks are kept rather simple. In the following papers the concept is applied to more realistic models.

Research Paper 2: Cross Subsidizing Effects between existing and new policyholders in traditional life insurance

This paper is joint work with Stefan Graf, Alexander Kling and Jochen Ruß. In this work, we apply the concept of the collective bonus presented in *"A measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio"* to consider the interaction between new policyholders, the existing policyholders of the insurance portfolio and the shareholders.

On the one hand, low guarantees of new contracts help to reduce the average guaranteed rate of interest for the insurance company. On the other hand, new policyholders benefit from high yields of the existing asset portfolio that was build up by previous policyholders and the shareholders. Therefore, the question whether new policyholders benefit from or subsidize the existing portfolio is not at all clear. We explain effects resulting from a common surplus participation and provide an answer, under which circumstances a benefit for or subsidization from new business exists.

A stochastic balance sheet and a very detailed cash flow projection model with a strong focus on German market practice are applied. The model based on Burkhart et al. (2017) is extended

to the case of a heterogeneous insurance portfolio and a different interest rate model is used in order to allow for negative interest rates. In contrast to Burkhart et al. (2017) who apply this model to analyse the risk sharing between policyholders and shareholders in a Solvency II setting, we apply the model to deal with the question whether new policyholders benefit from or subsidize the existing portfolio. To answer this question two different cohorts of traditional participating endowment policies are considered: One cohort represents the existing business with a high guaranteed interest rate and the other cohort represents the new business with a lower annual return guarantee, whereby both cohorts are equipped with an additional surplus participation.

We determine both, the ex ante collective bonus and the ex post collective bonus for the new business, existing business and the shareholders vice versa. We conclude that the high guaranteed interest rate of the existing business has in general to be granted independent of the development of the reference portfolio. Given the low interest rate environment, it results in a high ex ante collective bonus for the existing business. The new business benefits from existing reserves, because the asset valuation reserves and so-called free reserve for bonuses and rebates (*fRfB*) are used to provide a rather high surplus participation. A higher *fRfB* and higher asset valuation reserves at the inception time of the new business increase the ex ante collective bonus.

To better understand who benefits especially from the existing reserves, the existing business and the new business are then considered separately. The existing business (in expectation) would be better off if the insurance company did not take on any new business, mainly because the existing business would not need to share existing reserves with the new policyholders. The shareholders benefit from the introduction of new business, since the new business reduces the average guarantees to be paid. The new business would be worse off, if they were without the existing business, mainly because in this case there were no existing reserves to be additionally provided to the new clients. The shareholders would benefit from the absence of the existing business, because the high old guarantees were gone.

By increasing the guaranteed interest rate of the existing business, the expected subsidization from the new business and the shareholders to the existing portfolio increases. The new business and the shareholders subsidize the contractually agreed guarantee of the existing business. The effect of a change in the guaranteed rate of interest of new business is also analysed. While the ex ante collective bonus of the existing business is only marginally affected by the change of the guarantee of new business, the ex ante collective bonus of new business increases sharply by the guaranteed rate and the ex ante collective bonus of the shareholders decreases accordingly.

Moreover, our paper treats the case with more cohorts of new business and a higher volume of new business contracts in one cohort. In both cases, the ex ante collective bonus of existing business and new business decreases and the ex ante collective bonus of the shareholders

increases. While the shareholders benefit from new business, the existing reserves have to be shared between more contracts by increasing the volume of new business.

This paper analyses the interaction of traditional participating endowment policies. On the one hand, the low guarantees of new contracts help to reduce the average guaranteed rate of interest for the insurance company. On the other hand, new policyholders benefit from high yields of an already existing asset portfolio build up by previous policyholders and the shareholders. Whether new policyholders take advantage of or subsidize the existing portfolio partly depends on the performance of the capital market, the volume of the existing reserves and the pricing of the new business. In summary, this paper gives an overview of and analyses factors, which have a major influence on the attractiveness of traditional participating life insurance products for existing and new policyholders, and hence answers the second part of the research questions.

Research Paper 3: Werden bei einer Differenzierung der Überschussbeteiligung in der Lebensversicherung Neukunden zu Lasten des Bestands bevorzugt?

This research paper is a single author paper. The focus of the paper is on the German insurance market. Hence, the paper is written in German. Based on the concept of the collective bonus of *"A measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio"* as well as the asset liability model and the findings of *"Cross Subsidizing Effects between existing and new policyholders in traditional life insurance"*, the objective of the third paper is to analyse the effects of a differentiation of the surplus participation between existing and new business. Furthermore, the question of a potential disadvantage of the existing business due to the surplus differentiation is treated.

In 2004 the German supervisor, the "Bundesanstalt für Finanzdienstleistungsaufsicht (BaFin)", prohibited the differentiation of surplus participation in the case of a pooled capital investment (cf. VerBaFin (2004)). In 2017 the "Deutsche Aktuarvereinigung e.V. (DAV)" argues that the pronouncement of the "BaFin" cannot be applied in the current situation due to significantly different conditions[3] and therefore a differentiation of surplus participation can be justified (cf. DAV (2017)). Many insurance companies follow the report of the DAV and currently allow for a differentiation of the surplus participation.

In this paper, the interaction of twenty different cohorts of traditional participating endowment policies is considered. Cohorts 1 to 19 represent the existing and cohort 20 represents the new

[3]For the DAV, different conditions mean that there was no opportunity to choose between products with different kind of guarantees in 2004 and the maximum actuarial interest rate was approximate equal for all cohorts of insurance contracts in Germany at that time.

business. Different alternatives for the pricing of the new business are analysed, whereby only two parameters, the guaranteed interest rate of the insurance contract and the declaration of the surplus participation, are modified. In accordance with the maximum interest rate in Germany, the guaranteed interest rate of the first new business strategy is set to 0.9% p.a. and there is no differentiation of the surplus participation. For the other strategies, the guaranteed interest rate is reduced to 0% p.a. and based on DAV (2017) two possibilities for determining the "spread", which indicates the differentiation of surplus participation between existing business and new business, are considered: a so-called collective determination, i.e. taking into account the existing business, and an individual determination, i.e. without assuming any existing business such that only the new business and the shareholders interact. The effects of the differentiation of surplus participation are analysed afterwards.

The expected profit for the shareholders is greater for new business with a lower guaranteed interest rate and higher surplus participation than in the situation with a higher guaranteed interest rate and lower surplus participation for the cohort of new business. The shareholders must provide less capital in poor capital market scenarios in case of a lower guaranteed interest rate. Our analysis also shows that the expected attractiveness of the existing business is reduced for a new business strategy with a reduced guarantee and increased surplus participation as compared to a new business strategy without a reduction in the guarantee and without differentiation of the surplus participation. If the asset portfolio performs well, the surplus participation of the existing business is slightly lower in the situation with surplus differentiation, because the new business receives a higher participation. If the asset portfolio performs badly, the existing business just receives in the case with and without surplus differentiation the guaranteed allocation.

In this paper, two different possibilities to compensate the decrease in the expected attractiveness of the existing portfolio due to the surplus differentiation are presented: The reduction of the guarantee for new business allows for a more risky investment, from which the existing policyholders also benefit. Another possibility is to increase the policyholders' share of the company's investment result. By applying these methods, the expected attractiveness of the existing business and the expected profit of the shareholders will increase as a result of a new business strategy with surplus differentiation and reduced guarantee compared with a new business strategy without differentiation and without reduction of the guarantee.

The case where no new business is assumed is also treated. The results are in line with *"Cross Subsidizing Effects between existing and new policyholders in traditional life insurance"*: The reserves built up by the existing business and the shareholders are shared between a smaller number of cohorts in the situation without new business. Assuming that all management rules remain unchanged the ex ante collective bonus and thus the expected attractiveness of the existing business are higher than in the situation with new business. The expected profit of

the shareholders decreases compared to all situations with new business. The shareholders are subsidized in expectation by new business.

Moreover, the paper includes some sensitivity analysis: A different number of new business contracts is considered and the corresponding "collective spread" as well as the "individual spread" are determined. While the "collective spread" raises by increasing the number of new business contracts, the "individual spread" is independent of the new business volume. In an additional sensitivity analysis the new business is fairly priced on a stand-alone basis instead of applying the currently valid maximum interest rate. The results of this sensitivity analysis are in line with the base case.

In summary, the third paper analyses the effects of a differentiation of surplus participation on existing and new business as well as on the profitability of the shareholders and answers the third part of the research questions.

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Research Papers

1 A measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio

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A measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio

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Abstract

Because of the long-term nature of life insurance policies including interest rate guarantees and the current low interest rate environment, the fair valuation of insurance contracts is of particular interest. Fair valuation is often discussed on a single contract basis or from the viewpoint of a homogenous portfolio, i.e. a portfolio with identical policies. However, insurance portfolios are heterogeneous, i.e. consist of many different contracts. These contracts interact, e.g. because they share reserves, profits and the risk of default of the insurance company. In this paper, we introduce a methodology how interactions within heterogeneous insurance portfolios can be measured and provide some sample analyses showing how different contracts may subsidize each other. This methodology also allows for a check, whether a contract is fairly calculated in a heterogeneous portfolio.

Keywords
Life insurance; Heterogeneous portfolio; Participating contracts; Risk-neutral valuation; Interaction of contracts.

1. Introduction

The current situation on the financial markets, in particular the enduring low interest rate period, has different consequences for insurers: Many companies realize that their previously issued long-term guarantees were not priced adequately and have to set up additional reserves. This problem occurs primarily in the insurance sector in countries where products with interest rate guarantees are traditionally very popular, for example the German insurance market. Hence, pricing and risk management of these insurance portfolios is currently very challenging. Additionally, in many regulatory environments, e.g. under Solvency II in Europe, market consistent valuation of life insurance liabilities is nowadays required which makes the value and risk of existing liabilities more transparent.

As a consequence, new products with different kinds of guarantees were and are currently developed by many insurance companies. The technical rate of interest for the calculation of local GAAP reserves has decreased in practically every European country. Whereas traditional participating products in many countries have been generally equipped with a guaranteed interest rate on an annual basis so far, re-designed variations of participating products only provide some guaranteed benefit at contract's maturity (cf. Reuß et al. (2015) and Reuß et al. (2016)) and no or lower annual return guarantees. Therefore, insurance portfolios consist of many different contracts and the assumption of a homogeneous insurance portfolio, which is often implicitly made in the literature – especially for the pricing, hedging and risk management of insurance policies (cf. Grosen and Jørgensen (2000); Bacinello (2003); Bauer et al. (2005); Graf et al. (2011); Gatzert and Martin (2014)) – becomes questionable. As an example, in a typical German life insurance portfolio annual return guarantees range from 0% to 4%. There is a lot of interdependence between these different insurance contracts, e.g. sharing the same bonus reserve, having the same credit risk of the insurance company and, in particular, investing the premium in the same reference portfolio and thus sharing investment returns of a joint asset pool.

In the present paper, we provide a framework to analyse such interdependence between different contracts. We give an opportunity to check, whether a contract is actually calculated fairly in a heterogeneous insurance portfolio meaning that we answer the question, whether there are contracts which benefit from or subsidize other contracts in the portfolio. To answer this question we introduce the concept of the so-called 'collective bonus', respectively 'collective malus'. The collective bonus describes how much one contract or one collective of contracts benefits in expectation (ex ante) or has already benefited (ex post) from other contracts in the insurance portfolio so far. The collective malus accordingly indicates how much a contract subsidizes or has already subsidized other contracts.

The paper is organised as follows: In Section 2, we motivate the concept of the collective bonus

by means of a simple example. In Section 3, the theoretical framework of the collective bonus is introduced: Based on a general model for the insurance company's assets and liabilities explained in Section 3.1, we define the collective bonus in Section 3.2 and further present an alternative opportunity to calculate the collective bonus. After that, we show how the concept of collective bonus relates to a simplified balance sheet in Section 3.3. In Section 4, the concept of the collective bonus is applied to our motivating example and two more examples from the academic literature (cf. Døskeland and Nordahl (2008) and Hieber et al. (2019)). Finally, Section 5 concludes.

2. Motivating example

We start with a fairly simple example to illustrate the idea of the methodology proposed later. In a first step, we assume that the insurer's portfolio only consists of single premium insurance contracts. The contracts are concluded at time $t_0 \in \mathbb{N}$ and have a term of $T \in \mathbb{N}$ years. At time t_0, the policyholder pays a single premium P_{t_0} that is invested in some reference portfolio $(F_t)_{t \geq t_0}$. The contracts have a guaranteed maturity benefit $P_{t_0} \cdot e^{g_1 \cdot T}$ given by accumulating the single premium P_{t_0} by an annual interest rate g_1 until maturity. Besides that, the policyholder annually receives the maximum of the annually guaranteed minimum return g_2 and a share η of the 1-year performance $\frac{F_t}{F_{t-1}}$ on the reference portfolio. Thus, at time $t = t_0 + T$, the insurance benefit is given by

$$L_{t_0+T} = P_{t_0} \cdot \max\left(e^{g_1 \cdot T}; \prod_{i=t_0+1}^{t_0+T} \max\left(e^{g_2}; \eta \cdot \frac{F_i}{F_{i-1}}\right)\right). \tag{1}$$

Remark.

(i) If $g_2 \geq g_1$ the contract comes with a year-by-year interest rate guarantee and equation (1) *can be simplified to an insurance contract e.g. studied by Miltersen and Persson (2003) with $L_{t_0+T} = P_{t_0} \cdot \prod_{i=t_0+1}^{t_0+T} \max\left(e^{g_2}; \eta \cdot \frac{F_i}{F_{i-1}}\right)$.*

(ii) For $g_2 \to -\infty$ equation (1) *is equal to $L_{t_0+T} = P_{t_0} \cdot \max\left(e^{g_1 \cdot T}; \eta^T \cdot \frac{F_{t_0+T}}{F_{t_0}}\right)$, i.e. an insurance contract with a so called point-to-point guarantee at maturity (cf. Brys and de Varenne (1997) and Grosen and Jørgensen (2002)).*

At time $t_0 + T$, the insurance benefit L_{t_0+T} is paid out to the policyholder and the shareholder receives the remaining difference between the value of the assets and the policyholder account, i.e.

$$E_{t_0+T} := P_{t_0} \cdot \frac{F_{t_0+T}}{F_{t_0}} - L_{t_0+T}.$$

We consider a heterogeneous insurance portfolio with two contracts A and B. The insurance benefits of both contracts are defined by formula (1) however assuming different parameters. Both contracts are concluded at time 0 ($t_0^A = t_0^B = 0$) and have a duration of ten years ($T^A = T^B = 10$). The single premium of contract A and B is equal to one ($P_{t_0}^A = P_{t_0}^B = 1$). Insurance contract A is a contract with a guaranteed maturity benefit ($g_A := g_1 \in \mathbb{R}$) and an annual minimum return of 0% ($g_2 = 0$) which ensures that there is at least no reduction of the policyholder account on an annual basis. For contract B, we model a year-by-year guarantee, i.e. $g_1 = g_2$ in equation (1) and for ease of notation set $g_B := g_1 = g_2$. The guaranteed interest rate of contract A is given by $g_A = 2.00\%$ and the guaranteed interest rate of contract B is given by $g_B = 1.00\%$.

Assuming the capital market model presented in Section 4.1 (interest rates following a Vasiçek model and the risky asset following a generalization of the Black-Scholes model), the 'fair' share of the 1-year return on the reference portfolio of contract A is equal to 84.34 percent ($\eta^A = 84.34\%$) and of contract B is equal to 83.24 percent ($\eta^B = 83.24\%$) if both contracts are fairly priced individually[1]. In an insurance portfolio, which consists of both, contract A and contract B simultaneously, we do not necessarily need fair individual pricing but assume that the insurance portfolio as a whole should be priced fairly, i.e. we require

$$E_{\mathcal{Q}}\left[\frac{B_{t_0}}{B_{t_0+T}} \cdot \left(L_{t_0+T}^A + L_{t_0+T}^B\right)\middle|\mathcal{F}_{t_0}\right] = P_{t_0}^A + P_{t_0}^B. \tag{2}$$

To demonstrate the interaction of the contracts, we increase the guaranteed rate of the first contract g_A which may lead to situation where contract A is on an individual basis not fair any more, i.e. $E_{\mathcal{Q}}\left[B_{10}^{-1} \cdot L_{10}^A \middle| \mathcal{F}_0\right] > P_0^A$. The discounted payoff of contract A is now in expectation greater than the single premium paid. Due to the assumption that the whole insurance portfolio should be priced fairly (cf. equation (2)), the policyholder of contract B has to pay a price for this. In the terminology of this paper, we call this additional benefit for the policyholder of contract A collective bonus and the price for policyholder B collective malus. In the next section, we provide the theoretical framework of the concept of the collective bonus and afterwards revisit this example in Section 4.2.

3. The concept of the collective bonus

In this section, we describe how a collective bonus can be quantified in the context of a simplified model of the insurance company. Applying the model described in Section 3.1 the collective

[1] We call a contract fair, if $V_{t_0} = E_{\mathcal{Q}}\left[\frac{B_{t_0}}{B_{t_0+T}} \cdot L_{t_0+T} \middle| \mathcal{F}_{t_0}\right] = L_{t_0} (= P_{t_0})$ using the risk-neutral pricing measure $\mathcal{Q}$ where $(B_t)_{t\geq 0}$ denotes the risk-free asset applied as the discount factor in our setting and further introduced in Section 3.

bonus is defined in Section 3.2.

3.1. Model setup

The insurance company's balance sheet consists of the market value of assets on the one side and the market value of liabilities on the other side. The market value of liabilities is further decomposed into the best estimate of liabilities which 'belongs' to the policyholder as well as the present value of future profits, cf. Table 1. In a first step, we do not require that the total value of assets and liabilities equal each other, i.e. assets and liabilities may be defined independent of each other. In Section 3.3 we will address, what equality of assets and liabilities means for the collective bonus and vice versa.

ASSETS	**LIABILITIES**
market value of asset A_t	present value of future profits $PVFP_t$
	best estimate of liabilities BE_t

Table 1.: Simplified balance sheet of the insurance company.

We assume the existence of a risk-neutral pricing measure $\mathcal{Q}$ on the measurable space $(\Omega, \mathcal{F})$ equipped with filtration $\mathbb{F} = (\mathcal{F}_t)_{t\in\mathbb{N}}$ in what follows. The market value of assets at time $t \in \mathbb{R}$ is denoted by A_t. The best estimate of liabilities (BE_t) is defined as the present value of future expected cash flows payable to the policyholders under the pricing measure $\mathcal{Q}$. For example, these cash flows can be premiums from the policyholder or benefit payments to the policyholder. Further, the expected present value of any future cash flows to or from the insurance company's shareholders is denoted as the present value of future profits ($PVFP_t$)[2].

Best Estimate of Liabilities

For a definition of the collective bonus, we consider the insurance company at time t^*. We assume that the insurer's portfolio consists of N potentially different contracts[3]. The contract $i \in \{1, \ldots, N\}$ was concluded at time $t_0^i \in \mathbb{N}$, $t_0^i \leq t^*$ and has a term of $T^i \in \mathbb{N}$ years.

[2]Definition BE: cf. Article 77 of the European Union (2009); definition PVFP: cf. Principle 6 of the CFO Forum (2009).

[3]Any new business after time t^* is not considered here.

We discretize the term of the contract in full years and assume annual premium payments. A premium of contract $i \in \{1, \dots, N\}$ at time t is denoted by $P_t^i > 0$. Each premium is paid at the beginning of the year. We neglect any charges, surrender, mortality and solvency capital requirements. We assume that at time t^* the contracts $i \in N_1$, $N_1 \subseteq \{1, \dots, N\}$ had already matured. Thus at time t^*, the insurance portfolio consists of the contracts $i \in N_2$, $N_2 := \{1, \dots, N\} \setminus N_1$ (cf. Figure 1).

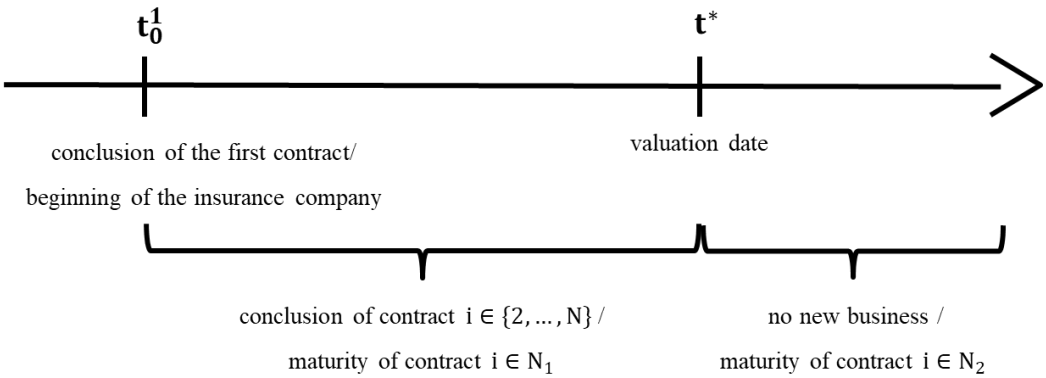

Figure 1.: Duration of the different contracts and valuation date.

The policyholder account L_t^i at time $t = t_0^i$ is equal to the first premium payment, i.e. we consider the policyholder account immediately after premium payment and set $L_{t_0^i}^i = P_{t_0^i}^i$. On an annual basis e.g. at time $t \in \mathbb{N}$, a contractually arranged credit f_t^i, $i \in \{1, \dots, N\}$ is added to each policyholder account. Note the annual credit f would typically not coincide with the insurance company's asset return. The policyholder account $i \in \{1, \dots, N\}$ at time t is thus given by

$$L_t^i = \begin{cases} \sum_{l=t_0^i}^{t} P_l^i \cdot \prod_{j=l}^{t-1} (1 + f_{j+1}^i), & t \in \{t_0^i, \dots, t_0^i + T^i - 1\} \\ \sum_{l=t_0^i}^{t_0^i+T^i-1} P_l^i \cdot \prod_{j=l}^{t_0^i+T^i-1} (1 + f_{j+1}^i), & t = t_0^i + T^i \\ 0, & t < t_0^i \text{ and } t > t_0^i + T^i. \end{cases} \tag{3}$$

At time $t_0^i + T^i$, the maturity benefit $L_{t_0^i+T^i}^i$ is paid out to the policyholder. The considered time t is immediately before the payment to the policyholder. After the payment, the value of the account is zero.

Using the pricing measure $\mathcal{Q}$ and the risk-free asset $(B_t)_{t \geq 0}$ with $B_0 = 1$, which serves as the discount factor in the risk-neutral framework, the best estimate of liabilities of the contract

$i \in N_2$ at time t^* is defined as

$$BE^i_{t^*} = E_{\mathcal{Q}}\left[\frac{B_{t^*}}{B_{t^i_0+T^i}} \cdot L^i_{t^i_0+T^i} - \sum_{j=t^*+1}^{t^i_0+T^i-1} \frac{B_{t^*}}{B_j} \cdot P^i_j\right]. \tag{4}$$

The best estimate of liabilities at time t^* of all contracts is the sum of the best estimate of each contract. Note that the insurance portfolio only consists of the contracts $i \in N_2$ at time t^*, hence we obtain

$$BE_{t^*} = \sum_{i \in N_2} BE^i_{t^*}.$$

Present Value of Future Profits

Next, we consider the present value of future profits $PVFP_{t^*}$ at time t^*. The cash flows of the shareholder in the period $(j-1, j)$ are denoted by X_j. If X_j is positive the shareholder receives a dividend payment. If X_j is negative the shareholder has to make a payment to cover the losses of the insurance company. In line with the valuation of liabilities, cash flows to or from the shareholder are considered directly before the considered time t^*. Thus,

$$PVFP_{t^*} = E_{\mathcal{Q}}\left[\sum_{j=t^*+1}^{T} \frac{B_{t^*}}{B_j} \cdot X_j\right],$$

where T is the expiration of the last contract of the insurance company.

Market Value of Assets

Finally, we consider the development of the market value of assets, again after potential premium payments. The premiums of all contracts are invested in the same reference portfolio $(F_t)_{t \geq t^1_0}$. We do not specify the concrete assets in the portfolio, but we assume that the net market return of the considered reference portfolio in year t is denoted by $\frac{F_t}{F_{t-1}}$. At time $t = t^1_0$ the first premium $P_{t^1_0}$ is paid. We assume that there is no initial payment by the shareholder and thus $A_{t^1_0} = P_{t^1_0}$. The market value of assets at time t^* can be derived by its previous value by subtracting insurance benefits paid out, accounting for the performance of the reference portfolio, adding premium payments and paying out dividends to the shareholders/receiving payments from the shareholder, i.e.

$$A_{t^*} = \left(A_{t^*-1} - \sum_{i \in N_1} L^i_{t^i_0+T^i} \cdot \mathbb{1}_{\{t^*-1=t^i_0+T^i\}}\right) \cdot \frac{F_{t^*}}{F_{t^*-1}} + \sum_{i \in N_2} P^i_{t^*} - X_{t^*}. \tag{5}$$

The simplified balance sheet of the insurance company (cf. Table 1) based on the model at time t^* is summarized in Table 2.

ASSETS	LIABILITIES
A_{t^*}	$PVFP_{t^*} = E_{\mathcal{Q}}\left[\sum_{j=t^*+1}^{T} \frac{B_{t^*}}{B_j} \cdot X_j\right]$ $BE_{t^*} = \sum_{i \in N_2} E_{\mathcal{Q}}\left[\frac{B_{t^*}}{B_{t_0^i+T^i}} \cdot L^i_{t_0^i+T^i} - \sum_{j=t^*+1}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_j} \cdot P^i_j\right]$

Table 2.: Simplified balance sheet of the insurance company based on the model setup.

3.2. Definition of the collective bonus

This section will first define the collective bonus and then explain the relationship between the collective bonus and the best estimate of liabilities. The main idea and motivation of the collective bonus is to quantify, whether an insurance contract has earned (ex post) or in expectation will earn (ex ante) more than an investment in the capital market.

Definition. *Assume the model setup of Section 3.1. The collective bonus of contract i at time t^* is given by*

$$CB^i_{t^*} := \begin{cases} \underbrace{E_{\mathcal{Q}}\left[\sum_{j=t^*}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_{j+1}} \cdot L^i_j \cdot \left(1 + f^i_{j+1} - \frac{F_{j+1}}{F_j}\right)\right]}_{\text{ex ante}} + \underbrace{L^i_{t^*} - \sum_{j=t_0^i}^{t^*} P^i_j \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}}}_{\text{ex post}}, & t^* < t_0^i + T^i \\ \left(L^i_{t_0^i+T^i} - \sum_{j=t_0^i}^{t_0^i+T^i-1} P^i_j \cdot \prod_{k=j+1}^{t_0^i+T^i} \frac{F_k}{F_{k-1}}\right) \cdot \frac{F_{t^*}}{F_{t_0^i+T^i}}, & t^* \geq t_0^i + T^i. \end{cases} \tag{6}$$

The first part of the definition, $E_{\mathcal{Q}}\left[\sum_{j=t^*}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_{j+1}} \cdot L^i_j \cdot \left(1 + f^i_{j+1} - \frac{F_{j+1}}{F_j}\right)\right]$, is the sum of the expected difference between the investment in the reference portfolio and an investment in the insurance contract with an annually credited return f. This part is called *ex ante collective bonus.*

The second part, $L^i_{t^*} - \sum_{j=t_0^i}^{t^*} P^i_j \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}}$, takes account of how much the policyholders already participated in the insurance contract compared to an investment in the development of the capital market before t^*. We call this the *ex post collective bonus.* If $t^* \geq t_0^i + T^i$, the collective bonus only consists of the ex post collective bonus.

The collective bonus of all N contracts at time t^* is defined by CB_{t^*}

$$CB_{t^*} = \sum_{i=1}^{N} CB_{t^*}^i.$$

Next, we study the relation between the collective bonus and the best estimate of liabilities of in-force contracts:

Proposition 1. *Assume the model setup of Section 3.1. The sum of the collective boni of in-force contracts $i \in N_2$ is equal to the difference of the best estimate of liabilities and the premium payments before time t^* invested in the reference portfolio $(F_t)_{t \geq t_0^1}$:*

$$BE_{t^*} - \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} = \sum_{i \in N_2} CB_{t^*}^i.$$

Proof. cf. Appendix A.

A consequence of *Proposition 1* is the following possibility to calculate the ex ante collective bonus of contract $i \in N_2$:

$$CB_{t^*}^i(\text{ex ante}) := E_{\mathcal{Q}} \left[\sum_{j=t^*}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_{j+1}} \cdot L_j^i \cdot \left(1 + f_{j+1}^i - \frac{F_{j+1}}{F_j}\right) \right] = BE_{t^*}^i - L_{t^*}^i, \tag{7}$$

since

$$\begin{aligned}
BE_{t^*}^i &\overset{\text{Proposition 1}}{=} CB_{t^*}^i + \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} \\
&= E_{\mathcal{Q}} \left[\sum_{j=t^*}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_{j+1}} \cdot L_j^i \cdot \left(1 + f_{j+1}^i - \frac{F_{j+1}}{F_j}\right) \right] + L_{t^*}^i - \sum_{j=t_0^i}^{t^*} P_j^i \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} + \sum_{j=t_0^i}^{t^*} P_j^i \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} \\
&= E_{\mathcal{Q}} \left[\sum_{j=t^*}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_{j+1}} \cdot L_j^i \cdot \left(1 + f_{j+1}^i - \frac{F_{j+1}}{F_j}\right) \right] + L_{t^*}^i.
\end{aligned}$$

Hence, the ex ante collective bonus is easily obtained as a difference of best estimate of liabilities and the current value of the policyholder account.

3.3. Relation between the simplified balance sheet and the collective bonus

In this section, we show how the concept of the collective bonus relates to the simplified balance sheet introduced in Table 1, in particular focussing on the identity

$$A_{t^*} = PVFP_{t^*} + BE_{t^*}.$$

For this, we introduce the 'collective bonus of the shareholder'. The present value of future profits $PVFP_{t^*}$ represents all future gains or losses for the shareholder and thus, the ex ante collective bonus of the shareholder. For the ex post collective bonus we assume that all previous cash flows X_j, $j \leq t^*$, had been invested in the reference portfolio $(F_t)_{t \geq t_0^1}$. This results in the collective bonus of the shareholder $CB_{t^*}^{sh}$ as

$$CB_{t^*}^{sh} = PVFP_{t^*} + \sum_{j=t_0^1+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}}.$$

Proposition 2. *Assume the model setup of Section 3.1. The market value of assets is equal to the sum of present value of future profits and best estimate of liabilities if and only if the sum of the collective boni is zero, i.e.*

$$A_{t^*} = BE_{t^*} + PVFP_{t^*} \Leftrightarrow CB_{t^*} + CB_{t^*}^{sh} = 0.$$

Proof. cf. Appendix B.

Applying *Proposition 2*, the concept of the collective bonus provides an opportunity to check if the simplified balance sheet is actually balanced. If it is and additionally one contract has a collective bonus greater than zero another contract or the shareholder has to compensate for this by a respective collective malus. Thus, the concept of the collective bonus provides a measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio.

Remark.
The model presented in Section 3.1 can be extended by e.g. mortality, surrender, charges and still, the propositions of this section can be shown to hold by similar means.

4. Applications

In this section, the concept of the collective bonus is applied on our motivating example (cf. Section 4.2) and two existing models from the academic literature, Hieber et al. (2019) in

Section 4.3 as well as Døskeland and Nordahl (2008) in Section 4.4. Before we start with the examples, we describe the capital market model applied in all of the following examples.

In Hieber et al. (2019), a new insurance contract is added to an existing insurance portfolio. In Section 4.3 we will answer the question, whether this added insurance contract receives a collective bonus (and thus benefits from the existing portfolio) or a collective malus (and thus subsidizes the existing portfolio). Also, we show how this new contract influences the collective bonus of already existing products within the portfolio. Intergenerational effects of guaranteed pension contracts are analysed in Section 4.4 using the model described by Døskeland and Nordahl (2008). For both examples, we use the notation of Section 3. In the appendix, we include a glossary with a comparison of our notation and the notion used by the respective original papers.

4.1. Capital market model

For the application of the concept of the collective bonus, the same capital market model is applied in all three cases we study in the following (i.e. the motivating example, Hieber et al. (2019) and Døskeland and Nordahl (2008)): We consider a financial market on a filtered probability space $(\Omega, \mathcal{F}, \mathcal{Q}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0})$ There are two sources of uncertainty, interest rates and stocks. The short rate process $(r_t)_{t>0}$ follows a Vasiçek model (cf. Vasiçek (1977)) and the stock price $(S_t)_{t>0}$ follows a generalization of the Black-Scholes model (cf. Black and Scholes (1973)):

$$dr_t^{\mathcal{Q}} = \kappa \cdot \left[\theta - r_t^{\mathcal{Q}}\right] \cdot dt + \sigma^r \cdot dW_t^{(1)}$$

$$\frac{dS_t^{\mathcal{Q}}}{S_t^{\mathcal{Q}}} = r_t^{\mathcal{Q}} \cdot dt + \rho \cdot \sigma^S \cdot dW_t^{(1)} + \sqrt{1-\rho^2} \cdot \sigma^S \cdot dW_t^{(2)}$$

where $W_t^{(1)}$ and $W_t^{(2)}$ are independent standard Wiener processes. In line with Hieber et al. (2019) we assume a parameter setting as follows: The mean revision speed is $\kappa = 0.3$, the current short rate $r_0 = 1.15\%$, the expected short rate $\theta = 4.2\%$, the short rate's volatility $\sigma^r = 0.015$. The volatility of the stocks is given by $\sigma^S = 0.2$, S_0 is equal to 1 and an instantaneous correlation of $\rho = 0.15$ is used. The risk free asset is given by $B_t = \exp\left(\int_0^t r_u du\right)$ and serves as the discount factor in the risk-neutral framework. The model calibration of Hieber et al. (2019) is applied on all of the following three applications of the collective bonus.

4.2. Motivating example

We now revisit the contracts from Section 2. If contract A and contract B are priced fairly on a collective basis, according to equation (2), fair parameter combinations of both contracts depend on each other. If, for example, we increase the guarantee g_A of contract A this must be compensated by a change in the guarantee of contract B. The consequence would be as follows: The policyholder of contract A gets a collective bonus and the policyholder of contract B gets a collective malus.

Using a Monte-Carlo simulation with 100,000 scenarios and a time horizon of ten years, we calculate pairs of guaranteed rates (g_A, g_B), such that equation (2) holds. In Figure 2, the required fair guaranteed rate of contract A is plotted against the corresponding guaranteed rate of contract B.

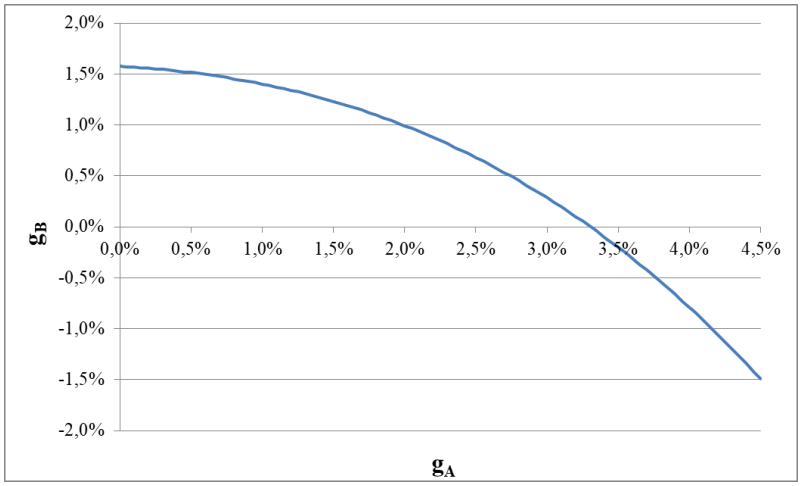

Figure 2.: Pairs of guaranteed rates g_A and g_B.

For example, one pair of 'fair' guaranteed rates (g_A, g_B) is given by (2.00%; 1.00%). In this particular case, both, contract A and contract B are also priced fairly from an individual point of view (cf. Section 2). If in contrast, g_A is roughly 3.30%, the corresponding fair guaranteed rate of contract B is 0%. In other words, contract B then has a money-back guarantee.

Next, using the model and notation of Section 3, we calculate the collective bonus of contract A and contract B for different pairs of guarantees from Figure 2. The collective boni are calculated at contract's inception, i.e. $t^* = 0$. Thus, the collective boni derived only consist of ex ante collective boni. In Table 3, some guaranteed rates g_A, g_B and the appropriate collective boni of contract A, expressed as percentage of the single premium paid, are shown. Given that the contracts are priced fairly on a collective basis, this means that the collective bonus of contract B, CB_0^B is given by the same amount, however with opposite sign. We therefore refrain from showing CB_0^B in this table.

$\mathbf{g_A}$	0.00%	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%	3.50%	4.00%
$\mathbf{CB_0^A}$	-4.84%	-4.33%	-3.38%	-1.95%	0.00%	2.47%	5.50%	9.07%	13.17%
$\mathbf{g_B}$	1.59%	1.52%	1.41%	1.24%	1.00%	0.68%	0.28%	-0.21%	-0.80%

Table 3.: Ex ante collective bonus of contract A.

If for example the guaranteed interest rate of contract A is equal to 3.00% and the corresponding year-by-year guarantee of contract B is about 0.28%, contract B in expectation subsidizes contract A with 5.50% of its single premium. The fact, that the sum of the collective boni is zero, if equation (2) holds, is a direct consequence of *Proposition 1.* In this example, for $g_A > 2.00\%$, contract A always receives a collective bonus whereas (naturally) contract B always receives a collective malus. The policyholder of contract B in expectation subsidizes the policyholder of contract A. For $g_A < 2.00\%$, the policyholder of contract B benefits from the policyholder of contract A.

4.3. Collective bonus of an insurance contract added to an existing insurance portfolio

In this section we consider a portfolio of different insurance policies with different starting and maturity points in time as modelled in Hieber et al. (2019). The insurance portfolio of the company consists of N single premium contracts. Each of the contracts has an annually guaranteed minimum return g^i, $i = 1, 2, \ldots, N$. The duration of the contract is T^i, $i = 1, \ldots, N$. Initially, the insurer provides some equity $E_0 \geq 0$. At the beginning of each contract, a single premium $P^i_{t_0^i} > 0$ is paid. At the end of each year, the maximum of the annual guaranteed return g^i and a share $\alpha^i \in [0, 1]$ of the annual return on the considered reference portfolio F is credited to the policyholder account leading to

$$L^i_{t+1} = L^i_t \cdot \max\left(\exp\left(g^i\right), \left(\frac{F_{t+1}}{F_t}\right)^{\alpha^i(t+1)}\right); L^i_{t_0^i} = P^i_{t_0^i},$$

for $t \in \{t_0^i+1, \ldots, t_0^i+T^i\}$. Furthermore, the policyholder receives a terminal bonus at maturity. The terminal bonus is given by

$$TB_{t_0^i+T^i} = \beta^i \cdot \max\left(P^i_{t_0^i} \cdot \frac{F_{t_0^i+T^i}}{F_{t_0^i}} - L_{t_0^i+T^i}; 0\right).$$

The factor β^i describes the share of the terminal bonus which is assigned to each single contract.

The reference portfolio F consists of a share λ of the risk-free asset $(B_t)_{t>0}$ and a share $(1-\lambda)$ of the risky asset $(S_t)_{t>0}$ (cf. Section 4.1). Consistent with Hieber et al. (2019) we choose $\lambda = 90\%$.

Interaction between the different policies results from the default risk of the insurance company. The insurance company defaults if the market value of assets is lower than the total liabilities, i.e.

$$A_t < \sum_{i=1}^{N} L_t^i = L_t.$$

The time of default is defined as

$$\tau = \inf\left\{t \in \{1,\dots,T\} \middle| A_t < \sum_{i=1}^{N} L_t^i\right\} \text{ with } \inf\{\emptyset\} = \infty$$

where T is the expiration of the last contract of the insurance company.

In case of default, the assets A_τ are liquidated and distributed to the policyholders. Each policyholder receives a share proportional to the size of her account at time τ by

$$\frac{L_\tau^i}{L_\tau} \cdot A_\tau.$$

The insurance portfolio consists of the following contracts at time $t^* = 0$ (cf. Table 4). Consistent to Hieber et al. (2019), 01/01/2015 is defined as our valuation date t^*.

$\mathbf{i}$	$\mathbf{e^{g^i} - 1}$	**start** $(\mathbf{t_0^i})$	**maturity** $(\mathbf{t_0^i + T^i})$	$\mathbf{L_0^i}$	$\beta^{\mathbf{i}}$	$\alpha^{\mathbf{i}}$	$\mathbf{P^i_{t_0^i}}$
1	1.75%	01/01/2014	31/12/2034	8,000	0.30	0.90	7,681
2	2.25%	01/01/2010	31/12/2029	17,000	0.20	0.90	13,969
3	2.75%	01/01/2005	31/12/2024	13,000	0.20	0.90	8,773
4	3.25%	01/01/2001	31/12/2023	14,000	0.20	0.90	7,871
5	4.00%	01/01/1995	31/12/2023	20,000	0.10	0.90	7,512
6	3.50%	01/01/1990	31/12/2022	18,000	0.10	0.90	4,987

Table 4.: The existing insurance portfolio.

The sum of the liabilities at time 0 amounts to $\sum_{i=1}^{6} L_0^i = 90,000$. Furthermore, Hieber et al. (2019) assume that the insurance company's equity is $18,000$ at time $t^* = 0$ and the initial market value of the assets is $90,000 + 18,000 = 108,000$. All parameters from Table 4 are from Hieber et al. (2019). To calculate the ex post collective bonus, we need the single premium and the past evolution of returns of the reference portfolio F. In contrast to Hieber et al. (2019),

who do not consider the past, we assume the past evolution of returns of the reference portfolio F to be the average net return of the German insurance companies (cf. Appendix C).

i	**asset evolution**	$\mathbf{L_0^i}$	$\mathbf{P_{t_0^i}^i \cdot \prod_{k=t_0^i+1}^{t^*} \frac{F_k}{F_{k-1}}}$	**ex post collective bonus**
1	$\frac{F_0}{F_{-1}} = 1.0463$	8,000	8,036	-36
2	$\frac{F_0}{F_{-5}} = 1.2438$	17,000	17,375	-375
3	$\frac{F_0}{F_{-10}} = 1.5479$	13,000	13,581	-581
4	$\frac{F_0}{F_{-14}} = 1.8949$	14,000	14,914	-914
5	$\frac{F_0}{F_{-19}} = 2.9206$	20,000	20,091	-91
6	$\frac{F_0}{F_{-25}} = 4.1482$	18,000	20,688	-2,688

Table 5.: Ex post collective bonus of the existing insurance portfolio.

Table 5 shows the asset evolution of each contract from its beginning to time zero, the value of the contract at time zero, the value of the investment of the single premium in the reference portfolio with net market return $\frac{F_t}{F_{t-1}}$ and the ex post collective bonus, which can be calculated as the difference between the value of the contract and the investment in the reference portfolio. All contracts show a negative ex post collective bonus (i.e. a collective malus). This shows that an investment in the reference portfolio is better than an investment in the insurance contract. The shareholder has apparently benefited from the existing insurance portfolio ($CB_0^{sh}(\text{ex post}) = 4,685$).

In a next step, we consider the ex ante collective bonus of each contract (equation (7)) at time $t^* = 0$ by

$$CB_{t^*}^i(\text{ex ante}) := E_{\mathcal{Q}}\left[\sum_{j=t^*}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_{j+1}} \cdot L_j^i \cdot \left(1 + f_{j+1}^i - \frac{F_{j+1}}{F_j}\right)\right] = BE_{t^*}^i - L_{t^*}^i$$

and show the results in Table 6.

CB_0^1(ex ante)	CB_0^2(ex ante)	CB_0^3(ex ante)	CB_0^4(ex ante)	CB_0^5(ex ante)	CB_0^6(ex ante)
140.00	688.73	706.92	1,020.00	2,299.70	1,226.67

Table 6.: Ex ante collective bonus of each contract of the insurance portfolio.

All contracts show an ex ante collective bonus. The shareholder has to compensate this and has an ex ante collective malus $PVFP_0 = -6,082.02$. In expectation, the shareholder subsidizes the policyholders. This result is in line with Hieber et al. (2019): The interaction between the different policies results from the default risk of the insurance company. Hieber et al. (2019) create a situation in which it is possible that the insurance company defaults.

The collective bonus is the sum of the ex ante and the ex post part and shown in Table 7:

i	**1**	**2**	**3**	**4**	**5**	**6**	**shareholder**
$\mathbf{CB_0^i}$**(ex ante)**	140.00	688.73	706.92	1,020.00	2,299.70	1,226.67	-6,082.02
$\mathbf{CB_0^i}$**(ex post)**	-36.00	-375.00	-581.00	-914.00	-91.00	-2,688.00	4,685
$\mathbf{CB_0^i}$	104.00	313.73	125.92	106.00	2,208.70	-1,461.33	1,397.02

Table 7.: Collective bonus of each contract of the insurance portfolio and the shareholder.

Contracts 1 to 5 have a collective bonus and only contract 6 has a collective malus. The reason is the high ex post collective malus of this contract. Contract 1 to 4 and especially contract 5 – the contract with the highest annual return guarantee – in expectation are subsidized by contract 6 and the shareholder. Every contract has an ex ante collective bonus and an ex post collective malus.

Now, we add the following two contracts which are individually fairly priced to the already existing portfolio. As described in Hieber et al. (2019), the shareholders also increase their equity when a new contract is added:

Contract A: Initial equity $E_0^A = 756$, $t_0^A = t^* = 0$, time to maturity $T^A = 20$, end date 31/12/2034 and $(\alpha^A, \beta^A, P_{t_0^A}^A, g^A) = (0.80, 0.20, 8000, 2.60\%)$.

Contract B: Initial equity $E_0^B = 756$, $t_0^B = t^* = 0$, time to maturity $T^B = 20$, end date 31/12/2034 and $(\alpha^B, \beta^B, P_{t_0^B}^B, g^B) = (0.90, 0.20, 8000, 1.55\%)$.

The parameters of both contracts are from Hieber et al. (2019). In Table 8 we show the ex ante collective bonus, the ex post collective bonus and the collective bonus of each existing contract as well as the newly added contract under the assumption that contract A is added to the existing insurance portfolio. In brackets, we show changes of the respective numbers compared to the situation before the new business was written. In Table 9 we show the influence on the collective bonus of each existing contract as well as the newly contract B under the assumption that contract B is added.

If contract A is added, the ex ante collective bonus of each contract of the existing portfolio is reduced (cf. Table 8) and contract A has an ex ante collective bonus of 99.10. The existing insurance portfolio subsidizes the new contract. As in Hieber et al. (2019), the reason is the

i	**A**	**1**	**2**	**3**	**4**	**5**	**6**	**PVFP$_0$**
CB$_0^i$	99.10	137.39	642.09	626.07	933.61	2,166.72	1,110.50	-5,715.47
(ex ante)	(99.10)	(-2.61)	(-46.63)	(-80.85)	(-86.40)	(-132.98)	(-116.17)	(366.55)
CB$_0^i$	0.00	-36.0	-375.0	-581.0	-914.0	-91.0	-2,688.0	4,685.0
(ex post)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
CB$_0^i$	99.10	101.39	267.09	45.07	19.61	2,075.72	-1,577.50	-1,030.47
	(99.10)	(-2.61)	(-46.63)	(-80.85)	(-86.40)	(-132.98)	(-116.17)	(366.55)

Table 8.: Changes of the collective boni due to the added contract A.

i	**B**	**1**	**2**	**3**	**4**	**5**	**6**	**PVFP$_0$**
CB$_0^i$	-0.09	140.09	690.95	708.32	1,022.01	2,300.28	1,228.58	-6,090.12
(ex ante)	(-0.09)	(0.09)	(2.22)	(1.40)	(2.00)	(0.58)	(1.92)	(-8.10)
CB$_0^i$	0.00	-36.0	-375.0	-581.0	-914.0	-91.0	-2,688.0	4,685.0
(ex post)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
CB$_0^i$	-0.09	104.09	315.95	127.32	108.01	2,209.28	-1,459.42	-1,405.12
	(-0.09)	(0.09)	(2.22)	(1.40)	(2.00)	(0.58)	(1.92)	(-8.10)

Table 9.: Changes of the collective boni due to the added contract B.

high guarantee and the small initial equity of contract A. There is of course no change of the ex post collective boni. If contract B is added (cf. Table 9), the ex ante collective boni of the contracts of the existing portfolio are almost the same and the ex ante collective bonus of contract B is nearly zero. Thus, contract B seems to be somewhat 'fairer' for the contracts 1 to 6 and the shareholder than contract A. We see that with the concept of the collective bonus, it is not only possible to verify one main result of Hieber et al. (2019), it is also possible to show and measure the collective effects in expectation (ex ante) and in the past (ex post) within the given life insurance portfolio.

4.4. Intergenerational effects of guaranteed pension contracts

In this section, we analyse the interaction of an insurance portfolio with different generations of contracts sharing their reserves. For this example, we use the model described by Døskeland and Nordahl (2008).

In this paper, 80 generations of identical pension contracts are considered, i.e. at time $t_0^i = i$, $i \in \{1, \ldots, 80\}$ a contract is sold. All contracts are single premium contracts and have the same term to maturity, i.e. $T^i = T^j$, $i, j \in \{1, \ldots, 80\}$. In the following example, we use

$T^i = 20$. Every contract has the same contractually agreed guarantee. All payments of the policyholders are invested into the reference portfolio F. Investment returns are distributed between policyholder accounts, the shareholder, and some bonus reserve. This bonus reserve does not belong to a specific account. Every policyholder gets a share of this bonus reserve at maturity of their contract. We refrain from a detailed explanation of the development of liabilities, the bonus reserve and equity. For more details, we refer to the original paper of Døskeland and Nordahl (2008).

One result of the paper is presented in Figure 3. This figure shows the so-called 'expected annual risk-adjusted return' of the insurance contract for each generation. The stochastic projection (number of scenarios: 100,000; time horizon: 100 years) starts at the beginning of the first contract.

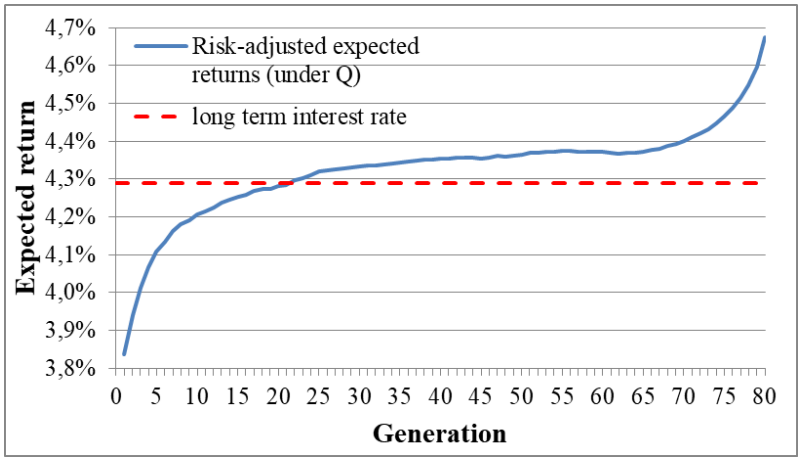

Figure 3.: Risk-adjusted expected return of different generations.

To calculate the risk-adjusted expected return, the expected return at maturity under the risk-neutral measure $\mathcal{Q}$ $\left(E_{\mathcal{Q}}\left[L^i_{t_0^i+T^i}\right]\right)$ and the single premium $\left(P^i_{t_0^i}\right)$ of each contract are considered and aligned by:

$$\text{risk-adjusted expected return}^i = \frac{1}{T^i} \cdot \ln\left(\frac{E_{\mathcal{Q}}\left[L^i_{t_0^i+T^i}\right]}{P^i_{t_0^i}}\right).$$

The abscissa of Figure 3 describes the different generations of pension contracts. The ordinate shows the risk-adjusted expected return. The focus of their analysis is on the generations between 20 and 60, the going-concern phase, because the first generations have to build up the bonus reserve and the last generations benefit from the build-down period.

The younger generations have a risk-adjusted expected return above the long term interest rate of $e^\theta = 4.29\%$ (e.g. generation 60: 4.37%) while the risk-adjusted expected return of earlier generations is less than 4.29% (e.g. generation 10: 4.21%). This implies an expected

discounted payoff at maturity greater than the single premium for the younger generations and an expected discounted payoff smaller than the single premium for the older generations and suggests a cross-subsidization from older generations to younger generations.

Applying the concept of the collective bonus, this result of the original paper can be verified and, in particular, the amount of cross-subsidization can be made transparent. The collective bonus of a specific generation is calculated as its present value at the beginning of the contract of this generation but given the information at time $t = t_0^1$. Thus, no information about the asset development between $t = t_0^1$ and the beginning of the contract is taken into account. In Figure 4, the collective bonus (collective malus, resp.) of the different generations of contracts is presented.

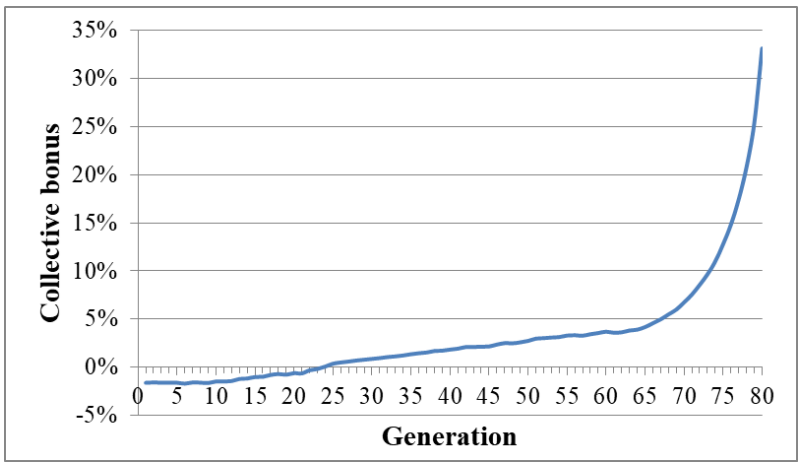

Figure 4.: Collective bonus of different generations.

If we compare the collective bonus of generation 10 ($CB_{10}^{10} = -1.45\%$) with the collective bonus of generation 60 ($CB_{60}^{60} = 3.71\%$), we see that the older generation is equipped with a collective malus while the younger generation profits from a collective bonus. The shareholder has a collective malus of -2.56 showing that the shareholder in expectation subsidizes the policyholders.

Finally, we want to illustrate, how the volume of the bonus reserve influences the collective bonus. We exemplarily consider the collective bonus of generation 40 and reduce the bonus reserve at outset t_0^{40} by 20 percent (case 1) and increase it by 20% (case 2). In Table 10 the collective bonus of the simulation above (base case), case 1 and case 2 are shown:

	base case	**case 1: -20%**	**case 2: +20%**
$\mathbf{CB_{40}^{40}}$	0.18%	-0.06%	0.54%

Table 10.: The influence of the bonus reserve on the collective bonus of generation 40.

For a 20% lower initial bonus reserve, generation 40 is equipped with a collective malus. A relatively slight reduction of initial reserves is already enough to turn generation 40 into a 'sponsoring generation'. In case 2, where the initial bonus reserve is 20% higher than in the base case, the collective bonus of generation 40 is about 0.54% which again shows the direct influence of the bonus reserve on the collective bonus. This shows how sensitive intergenerational effects are with respect to the amount of reserves that were built up in the past.

5. Conclusion

There are a number of interdependencies and interactions between different contracts in a heterogeneous insurance portfolio, e.g. different policies sharing the same asset pool, surplus participation of insurance contracts on the earnings of the company, default risk of the company, etc. The interactions are often quite complex and the question whether certain contracts benefit from others or implicitly sponsor others is not clear in most cases.

We depicted these interactions and questions with a short motivating example showing how two contracts with different interest rate guarantees can have interactions in the portfolio. Then we provided a methodology to analyse and measure the interaction of different contracts in a heterogeneous insurance portfolio by introducing the concept of the collective bonus / collective malus. This allows us to identify contracts who subsidize other contracts (contracts with a collective malus) and contracts that benefit from others (contracts with a collective bonus). We distinguish between subsidies and interactions that already happened in the past (ex post collective bonus / malus) and the expected value of interactions that happen in the future (ex ante collective bonus / malus). The definition and interpretation of the collective bonus is the difference of the contract's returns within the insurance portfolio and a theoretical investment of the contract's premiums in the capital market. A collective bonus therefore can be interpreted as a past or expected outperformance, a collective malus can be interpreted as a past or expected underperformance.

With this rather intuitive definition, we are able to show that the market value of assets and liabilities equal each other if and only if the sum of the collective boni / mali of all stakeholders (i.e. all policyholders and the shareholder) is zero. This provides us alternative ways how the collective bonus / malus can be calculated and how it corresponds to an insurance company's market value balance sheet. After introducing our theoretical concept, we show how the concept of the collective bonus can be applied to our motivating example and two models from the existing literature, by means of Hieber et al. (2019) as well as Døskeland and Nordahl (2008). For all examples mentioned, we use the same capital market model in order to illustrate both, ex ante and ex post calculations of the collective bonus.

In Hieber et al. (2019), a new insurance contract is added to an existing insurance portfolio. We analyse the question, whether this newly added insurance contract receives a collective bonus (and thus benefits from the existing portfolio) or a collective malus (and thus subsidizes the existing portfolio). Also, we show how this new contract influences the collective bonus of already existing products within the portfolio. As Hieber et al. (2019), we show how different product designs of the newly added contract can lead to significantly different implications for the collective bonus of the newly added contract and the existing portfolio, even though all newly added contracts are priced fairly individually. Following Døskeland and Nordahl (2008), we analyse intergenerational effects of guaranteed pension contracts. In this paper, the authors deal with a portfolio of identical contracts that are written at different times. Maturing contracts leave (parts) of their reserves for future contracts which creates intergenerational subsidization effects. We are able to verify the results of the original paper and measure the amount of subsidization by means of the collective bonus.

All examples of this paper are rather simple and therefore perfectly suitable to illustrate the methodology of the collective bonus and verify results of already existing papers. In future research, this concept should now be applied to more realistic models. In particular, we see a lot of research questions for insurance companies still selling traditional business with surplus participation for the policyholders. Here, low guarantees of new contracts certainly help to reduce the average guaranteed rate of interest for the insurance company. At the same time, however, new policyholders profit from high yields of an already existing asset portfolio build up by previous policyholders. Therefore, the question whether new policyholders benefit from or subsidize the existing portfolio is not at all clear and can be analysed applying the concept of collective bonus.

Appendix

A. Proof of *Proposition 1*

Proposition 1. *Assume the model setup of Section 3.1. The sum of the collective boni of in-force contracts $i \in N_2$ is equal to the difference of the best estimate of liabilities and the premium payments before time t^* invested in the reference portfolio $(F_t)_{t \geq t_0^1}$:*

$$BE_{t^*} - \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} = \sum_{i \in N_2} CB_{t^*}^i.$$

Proof.
To follow the proof easier the main transformations are marked in bold.

First, we consider the definition of L_t^i. If $t \in \{t_0^i, \ldots, t_0^i + T^i - 1\}$, from equation (3) follows:

$$\begin{aligned}
L_t^i &= \sum_{l=t_0^i}^{t} P_l^i \cdot \prod_{j=l}^{t-1} \left(1 + f_{j+1}^i\right) \\
&= P_t^i + \sum_{l=t_0^i}^{\mathbf{t-1}} P_l^i \cdot \prod_{j=l}^{t-1} \left(1 + f_{j+1}^i\right) \\
&= P_t^i + \left(1 + f_t^i\right) \cdot \left(\sum_{l=t_0^i}^{t-1} P_l^i \cdot \prod_{j=l}^{\mathbf{t-2}} \left(1 + f_{j+1}^i\right) \right) \\
&= P_t^i + \left(1 + f_t^i\right) \cdot L_{t-1}^i.
\end{aligned}$$

For $t = t_0^i + T^i$ we analogously get (there is no premium payment at $t_0^i + T^i$):

$$\begin{aligned}
L_{t_0^i+T^i}^i &= \sum_{l=t_0^i}^{t_0^i+T^i-1} P_l^i \cdot \prod_{j=l}^{t_0^i+T^i-1} \left(1 + f_{j+1}^i\right) = \left(1 + f_{t_0^i+T^i}^i\right) \cdot \left(\sum_{l=t_0^i}^{t_0^i+T^i-1} P_l^i \cdot \prod_{j=l}^{\mathbf{t_0^i+T^i-2}} \left(1 + f_{j+1}^i\right) \right) \\
&= \left(1 + f_{t_0^i+T^i}^i\right) \cdot L_{t_0^i+T^i-1}^i.
\end{aligned}$$

With the definition of the best estimate of liabilities we get:

$$BE_{t^*} = \sum_{i \in N_2} BE_{t^*}^i \overset{\text{Equation (4)}}{=} \sum_{i \in N_2} E_{\mathcal{Q}} \left[\frac{B_{t^*}}{B_{t_0^i+T^i}} \cdot L_{t_0^i+T^i}^i - \sum_{j=t^*+1}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_j} \cdot P_j^i \right].$$

Next we use the alternative calculation of the policyholder account (cf. above):

$$\begin{aligned}
BE_{t^*} &= \sum_{i \in N_2} E_{\mathcal{Q}} \left[\frac{B_{t^*}}{B_{t_0^i+T^i}} \cdot \mathbf{L}_{\mathbf{t_0^i+T^i-1}}^{\mathbf{i}} \cdot \left(\mathbf{1} + \mathbf{f}_{\mathbf{t_0^i+T^i}}^{\mathbf{i}}\right) - \sum_{j=t^*+1}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_j} \cdot P_j^i \right] \\
&= \sum_{i \in N_2} E_{\mathcal{Q}} \left[\frac{B_{t^*}}{B_{t_0^i+T^i}} \cdot L_{t_0^i+T^i-1}^i \cdot \left(1 + f_{t_0^i+T^i} - \frac{\mathbf{F}_{\mathbf{t_0^i+T^i}}}{\mathbf{F}_{\mathbf{t_0^i+T^i-1}}} + \frac{\mathbf{F}_{\mathbf{t_0^i+T^i}}}{\mathbf{F}_{\mathbf{t_0^i+T^i-1}}} \right) - \sum_{j=t^*+1}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_j} \cdot P_j^i \right] \\
&= \sum_{i \in N_2} E_{\mathcal{Q}} \left[\frac{B_{t^*}}{B_{t_0^i+T^i}} \cdot L_{t_0^i+T^i-1}^i \cdot \left(1 + f_{t_0^i+T^i}^i - \frac{F_{t_0^i+T^i}}{F_{t_0^i+T^i-1}} \right) \right] \\
&\quad + E_{\mathcal{Q}} \left[\frac{B_{t^*}}{B_{t_0^i+T^i}} \cdot L_{t_0^i+T^i-1}^i \cdot \frac{F_{t_0^i+T^i}}{F_{t_0^i+T^i-1}} \right] - E_{\mathcal{Q}} \left[\sum_{j=t^*+1}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_j} \cdot P_j^i \right].
\end{aligned}$$

Next, we apply the tower property:

$$
\begin{aligned}
BE_{t^*} = &\sum_{i\in N_2} E_{\mathcal{Q}}\left[\frac{B_{t^*}}{B_{t_0^i+T^i}}\cdot L^i_{t_0^i+T^i-1}\cdot\left(1+f^i_{t_0^i+T^i}-\frac{F_{t_0^i+T^i}}{F_{t_0^i+T^i-1}}\right)\right]\\
&+E_{\mathcal{Q}}\left[L^i_{t_0^i+T^i-1}\cdot\frac{B_{t^*}\cdot\mathbf{E}_{\mathcal{Q}}\left[\mathbf{B}^{-1}_{\mathbf{t_0^i+T^i}}\cdot\mathbf{F}_{\mathbf{t_0^i+T^i}}\middle|\mathcal{F}_{\mathbf{t_0^i+T^i-1}}\right]}{F_{t_0^i+T^i-1}}\right]-E_{\mathcal{Q}}\left[\sum_{j=t^*+1}^{t_0^i+T^i-1}\frac{B_{t^*}}{B_j}\cdot P^i_j\right]
\end{aligned}
$$

and use the martingale property of the reference portfolio:

$$
\begin{aligned}
BE_{t^*} &= \sum_{i\in N_2} E_{\mathcal{Q}}\left[\frac{B_{t^*}}{B_{t_0^i+T^i}}\cdot L^i_{t_0^i+T^i-1}\cdot\left(1+f^i_{t_0^i+T^i}-\frac{F_{t_0^i+T^i}}{F_{t_0^i+T^i-1}}\right)\right]\\
&\quad+E_{\mathcal{Q}}\left[L^i_{t_0^i+T^i-1}\cdot\frac{B_{t^*}\cdot\mathbf{B}^{-1}_{\mathbf{t_0^i+T^i-1}}\cdot\mathbf{F}_{\mathbf{t_0^i+T^i-1}}}{F_{t_0^i+T^i-1}}\right]-E_{\mathcal{Q}}\left[\sum_{j=t^*+1}^{t_0^i+T^i-1}\frac{B_{t^*}}{B_j}\cdot P^i_j\right]\\
&= \sum_{i\in N_2} E_{\mathcal{Q}}\left[\frac{B_{t^*}}{B_{t_0^i+T^i}}\cdot L^i_{t_0^i+T^i-1}\cdot\left(1+f^i_{t_0^i+T^i}-\frac{F_{t_0^i+T^i}}{F_{t_0^i+T^i-1}}\right)\right]\\
&\quad+E_{\mathcal{Q}}\left[\frac{B_{t^*}}{B_{t_0^i+T^i-1}}\cdot L^i_{t_0^i+T^i-1}-\sum_{j=t^*+1}^{t_0^i+T^i-1}\frac{B_{t^*}}{B_j}\cdot P^i_j\right].
\end{aligned}
$$

For $T^i+t_0^i-1$ we do the same transformations as above. We use again the alternative calculation of the policyholder account:

$$
\begin{aligned}
BE_{t^*} &= \sum_{i\in N_2} E_{\mathcal{Q}}\left[\frac{B_{t^*}}{B_{t_0^i+T^i}}\cdot L^i_{t_0^i+T^i-1}\cdot\left(1+f^i_{t_0^i+T^i}-\frac{F_{t_0^i+T^i}}{F_{t_0^i+T^i-1}}\right)\right.\\
&\quad\left.+\frac{B_{t^*}}{B_{t_0^i+T^i-1}}\cdot\left(\mathbf{L}^{\mathbf{i}}_{\mathbf{t_0^i+T^i-2}}\cdot\left(\mathbf{1}+\mathbf{f}^{\mathbf{i}}_{\mathbf{t_0^i+T^i-1}}\right)+\mathbf{P}^{\mathbf{i}}_{\mathbf{t_0^i+T^i-1}}\right)-\sum_{j=t^*+1}^{t_0^i+T^i-1}\frac{B_{t^*}}{B_j}\cdot P^i_j\right]\\
&= \sum_{i\in N_2} E_{\mathcal{Q}}\left[\frac{B_{t^*}}{B_{t_0^i+T^i}}\cdot L^i_{t_0^i+T^i-1}\cdot\left(1+f^i_{t_0^i+T^i}-\frac{F_{t_0^i+T^i}}{F_{t_0^i+T^i-1}}\right)\right.\\
&\quad\left.+\frac{B_{t^*}}{B_{t_0^i+T^i-1}}\cdot L^i_{t_0^i+T^i-2}\left(1+f^i_{t_0^i+T^i-1}-\frac{\mathbf{F}_{\mathbf{t_0^i+T^i-1}}}{\mathbf{F}_{\mathbf{t_0^i+T^i-2}}}+\frac{\mathbf{F}_{\mathbf{t_0^i+T^i-1}}}{\mathbf{F}_{\mathbf{t_0^i+T^i-2}}}\right)-\sum_{j=t^*+1}^{\mathbf{t_0^i+T^i-2}}\frac{B_{t^*}}{B_j}\cdot P^i_j\right].
\end{aligned}
$$

Tower property and the martingale property of the reference portfolio yield:

$$
\begin{aligned}
BE_{t^*} = &\sum_{i\in N_2} E_{\mathcal{Q}}\left[\sum_{j=t_0^i+T^i-2}^{t_0^i+T^i-1}\frac{B_{t^*}}{B_{j+1}}\cdot L^i_j\cdot\left(1+f^i_{j+1}-\frac{F_{j+1}}{F_j}\right)+\frac{\mathbf{B}_{\mathbf{t^*}}}{\mathbf{B}_{\mathbf{t_0^i+T^i-2}}}\cdot\mathbf{L}^{\mathbf{i}}_{\mathbf{t_0^i+T^i-2}}\right.\\
&\left.-\sum_{j=t^*+1}^{t_0^i+T^i-2}\frac{B_{t^*}}{B_j}\cdot P^i_j\right].
\end{aligned}
$$

The same transformations as above are repeated until t^*:

$$BE_{t^*} = \sum_{i \in N_2} L^i_{t^*} + E_{\mathcal{Q}} \left[\sum_{i \in N_2} \sum_{j=t^*}^{t^i_0+T^i-1} \frac{B_{t^*}}{B_{j+1}} \cdot L^i_j \cdot \left(1 + f^i_{j+1} - \frac{F_{j+1}}{F_j} \right) \right].$$

Finally we use the definition of the collective bonus (cf. equation (6)):

$$BE_{t^*} = \sum_{i \in N_2} CB^i_{t^*} + \sum_{i \in N_2} \sum_{j=t^i_0}^{t^*} P^i_j \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}}.$$

Thus, the collective bonus can also be calculated using the best estimate of liabilities and the premium payments before time t^*.

The result of *Proposition 1* is on an aggregate level (sum of all contracts of N_2). A direct consequence of *Proposition 1* is the following result for each contract $i \in N_2$:

$$BE^i_{t^*} - \sum_{j=t^i_0}^{t^*} P^i_j \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} = CB^i_{t^*}. \tag{8}$$

Proof. Analogously to proof of *Proposition 1*.

B. Proof of *Proposition 2*

Proposition 2. *Assume the model setup of Section 3.1. The market value of assets is equal to the sum of present value of future profits and best estimate of liabilities if and only if the sum of the collective boni is zero, i.e.*

$$A_{t^*} = BE_{t^*} + PVFP_{t^*} \Leftrightarrow CB_{t^*} + CB^{sh}_{t^*} = 0.$$

Proof.
We show by induction that A_{t^*} for $t^* \in \{t^1_0, \ldots, T\}$ is given as:

$$A_{t^*} = \sum_{i \in N_2} \sum_{j=t^i_0}^{t^*} P^i_j \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1} CB^i_{t^*} - \sum_{j=t^1_0+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}}.$$

First, $t^* = t^1_0$:

$$A_{t^1_0} = \sum_{i \in N_2 = \{1,\ldots,N\}} \sum_{j=t^i_0}^{t^1_0} P^i_j \cdot \prod_{k=j+1}^{t^1_0} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1 = \emptyset} CB^i_{t^*} - \sum_{j=t^1_0+1}^{t^1_0} X_j \cdot \prod_{l=j+1}^{t^1_0} \frac{F_l}{F_{l-1}} = P_{t^1_0}.$$

Next, $t^* \in \{t_0^1 + 1, \dots, T\}$. We apply the definition of the market value of assets (cf. equation (5)) and distinguish different cases for t^* in the following.

Case 1: $t^* - 1 \neq t_0^i + T^i, i \in N_1$, i.e. there is no contract matures at $t^* - 1$:

$$\begin{aligned}
A_{t^*} &= \left(A_{t^*-1} - \sum_{i \in N_1} L^i_{t_0^i + T^i} \cdot \mathbb{1}_{\{t^*-1 = t_0^i + T^i\}} \right) \cdot \frac{F_{t^*}}{F_{t^*-1}} + \sum_{i \in N_2} P^i_{t^*} - X_{t^*} \\
&= A_{t^*-1} \cdot \frac{F_{t^*}}{F_{t^*-1}} + \sum_{i \in N_2} P^i_{t^*} - X_{t^*} \\
&= \left(\sum_{i \in N_2} \sum_{j=t_0^i}^{t^*-1} P^i_j \cdot \prod_{k=j+1}^{t^*-1} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1} CB^i_{t^*-1} - \sum_{j=t_0^1+1}^{t^*-1} X_j \cdot \prod_{l=j+1}^{t^*-1} \frac{F_l}{F_{l-1}} \right) \cdot \frac{F_{t^*}}{F_{t^*-1}} + \sum_{i \in N_2} P^i_{t^*} - X_{t^*} \\
&= \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*-1} P^i_j \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1} CB^i_{t^*} - \sum_{j=t_0^1+1}^{t^*-1} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}} + \sum_{i \in N_2} P^i_{t^*} - X_{t^*} \\
&= \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*} P^i_j \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1} CB^i_{t^*} - \sum_{j=t_0^1+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}}.
\end{aligned}$$

Case 2: $t^* - 1 = t_0^i + T^i$, for at least one contract $i \in N_1$, i.e. there is at least one account i which is paid out to the policyholder at $t^* - 1$. We define

$$N_1^* = \left\{ i \in N_1 \;\middle|\; t^* - 1 = t_0^i + T^i \right\}.$$

Consequently, the insurer's portfolio consists of the contracts $i \in N_1^* \cup N_2$ at time $t^* - 1$:

$$\begin{aligned}
A_{t^*} &= \left(A_{t^*-1} - \sum_{i \in N_1} L^i_{t_0^i + T^i} \cdot \mathbb{1}_{\{t^*-1 = t_0^i + T^i\}} \right) \cdot \frac{F_{t^*}}{F_{t^*-1}} + \sum_{i \in N_2} P^i_{t^*} - X_{t^*} \\
&= \left(\sum_{i \in N_1^* \cup N_2} \sum_{j=t_0^i}^{t^*-1} P^i_j \cdot \prod_{k=j+1}^{t^*-1} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1 \setminus N_1^*} CB^i_{t^*-1} - \sum_{j=t_0^1+1}^{t^*-1} X_j \cdot \prod_{l=j+1}^{t^*-1} \frac{F_l}{F_{l-1}} - \sum_{i \in N_1^*} L^i_{t_0^i + T^i} \right) \\
&\quad \cdot \frac{F_{t^*}}{F_{t^*-1}} + \sum_{i \in N_2} P^i_{t^*} - X_{t^*} \\
&= \left(\sum_{i \in N_2} \sum_{j=t_0^i}^{t^*-1} P^i_j \cdot \prod_{k=j+1}^{t^*-1} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1 \setminus N_1^*} CB^i_{t^*-1} - \sum_{j=t_0^1+1}^{t^*-1} X_j \cdot \prod_{l=j+1}^{t^*-1} \frac{F_l}{F_{l-1}} \right. \\
&\quad \left. - \sum_{i \in N_1^*} \left(L^i_{t_0^i + T^i} - \sum_{j=t_0^i}^{t^*-1} P^i_j \cdot \prod_{k=j+1}^{t^*-1} \frac{F_k}{F_{k-1}} \right) \right) \cdot \frac{F_{t^*}}{F_{t^*-1}} + \sum_{i \in N_2} P^i_{t^*} - X_{t^*}
\end{aligned}$$

$$= \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1 \setminus N_1^*} CB_{t^*}^i - \sum_{j=t_0^1+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}}$$
$$- \sum_{i \in N_1^*} \left(L_{t_0^i+T^i}^i - \sum_{j=t_0^i}^{t^*-1} P_j^i \cdot \prod_{k=j+1}^{t^*-1} \frac{F_k}{F_{k-1}} \right) \cdot \frac{F_{t^*}}{F_{t^*-1}}.$$

Because of $P_{t^*-1}^i = 0$; $i \in N_1^*$ (no premium payment at maturity), it follows that

$$= \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1 \setminus N_1^*} CB_{t^*}^i - \sum_{j=t_0^1+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}}$$
$$- \sum_{i \in N_1^*} \left(L_{t_0^i+T^i}^i - \sum_{j=t_0^i}^{t^*-2} P_j^i \cdot \prod_{k=j+1}^{t^*-1} \frac{F_k}{F_{k-1}} \right) \cdot \frac{F_{t^*}}{F_{t^*-1}}$$
$$= \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} - \sum_{i \in N_1} CB_{t^*}^i - \sum_{j=t_0^1+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}}.$$

Now we can prove:

$$BE_{t^*} + PVFP_{t^*} - A_{t^*} \overset{\text{Proposition 1}}{=}$$
$$= \sum_{i \in N_2} CB_{t^*}^i + \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} + PVFP_{t^*} - A_{t^*}$$
$$= \sum_{i \in N_2} CB_{t^*}^i + \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} + CB_{t^*}^{sh} - \sum_{j=t_0^1+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}} - A_{t^*}$$
$$= CB_{t^*} - \sum_{i \in N_1} CB_{t^*}^i + \sum_{i \in N_2} \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}} + CB_{t^*}^{sh} - \sum_{j=t_0^1+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}} - A_{t^*}$$
$$= CB_{t^*} + CB_{t^*}^{sh}.$$

C. Average net return of the German insurance companies

Year	Average net return of the German insurance companies[4]
1990	6.78%
1991	7.44%
1992	7.39%
1993	7.59%
1994	7.15%
1995	7.37%
1996	7.37%
1997	7.46%
1998	7.57%
1999	7.58%
2000	7.51%
2001	6.12%
2002	4.68%
2003	5.05%
2004	4.90%
2005	5.18%
2006	4.82%
2007	4.65%
2008	3.54%
2009	4.18%
2010	4.27%
2011	4.13%
2012	4.59%
2013	4.68%
2014	4.63%

Glossary

This is a comparison of the notation in the present paper and the paper Hieber et al. (2019) as well as Døskeland and Nordahl (2008), we use in Section 4.

This paper	Døskeland and Nordahl (2008)
F	A
T^i	T

[4]Reference: Gesamtverband der deutschen Versicherungswirtschaft (GDV).

This paper	Hieber et al. (2019)
N	K
g^i	$g^{(i)}$
T^i	$T^{(i)}$
P^i	$P^{(i)}$
α^i	$\alpha^{(i)}$
β^i	$\beta^{(i)}$
L	P
F	A

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2 Cross Subsidizing Effects between existing and new policyholders in traditional life insurance

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Cross Subsidizing Effects between existing and new policyholders in traditional life insurance

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Abstract

For many life insurance companies, traditional life insurance with surplus participation is still a major portion of their business. As a consequence of the low interest rate environment, financial guarantees of new contracts have been decreased. Hence, new business reduces the average guaranteed rate of interest for the whole block of business. At the same time, however, new policyholders benefit from higher yields of assets in the existing asset portfolio built up with previous policyholders' contributions. Therefore, it is not obvious whether new policyholders benefit from or subsidize the existing portfolio. In this paper, we use the collective bonus introduced in Eckert et al. (2020) to measure the interaction between new policyholders and the existing insurance portfolio. Considering the situation in Germany as an example, we measure and explain cross-subsidizing effects resulting from different cohorts of policyholders being linked to the same asset portfolio. In particular, we show under which circumstances there is a benefit for or subsidization from new business.

Keywords

Life insurance; Heterogeneous portfolio; Participating contracts; Risk-neutral and real-world valuation; Interaction of contracts.

1. Introduction

As a consequence of extremely low market interest rates over a long period of time, guaranteed interest rates of traditional life insurance products have been heavily decreasing in many countries. Hence, newly issued contracts have a lower guaranteed interest rate than the already existing portfolio and might therefore subsidize old guarantees in certain scenarios. On the other hand, the new business benefits from higher returns of the existing asset portfolio that was built up with previous policyholders' premiums. These cross-subsidizing effects between different cohorts of policyholders give rise to many important questions, e.g. whether new policyholders benefit from or subsidize the existing portfolio.

Existing academic literature on the pricing, hedging and risk management of life insurance policies, e.g. Grosen and Jørgensen (2000), Bacinello (2003), Bauer et al. (2005), Graf et al. (2011) or Gatzert and Martin (2014) typically assumes a homogeneous insurance portfolio where by definition, cross-subsidizing effects do not exist. In contrast to these papers, we consider heterogeneous insurance portfolios and focus on the interdependence between different cohorts of contracts whose returns are linked to the same asset portfolio. Only few papers consider heterogeneous portfolios in their analyses, e.g. Hieber et al. (2019) who theoretically show the existence of parameter combinations which allow individual new insurance contracts to be integrated in an existing insurance portfolio without any transfer of wealth.

Even less papers explicitly analyse cross-subsidization effects between different cohorts of contracts. Døskeland and Nordahl (2008) investigate intergenerational cross-subsidization effects assuming individually fairly priced life insurance contracts with interest rate guarantees. Yi et al. (2019) consider collective pensions without an investment guarantee by the sponsor and show that reserves are then helpful to stabilize the benefits of later generations. Hieber et al. (2015) particularly analyse cross subsidizing effects on risk-return profiles of different generations. However, none of these papers use a clear measure for the ex-ante expected and ex-post realized cross subsidizing effect. Also, their model for risk sharing over time and between policyholders is rather stylized. Our model in contrast is very detailed in that respect and applies a well-defined measure (as introduced in Eckert et al. (2020)) for the ex-ante expected and ex-post realized cross subsidizing effect. In addition to Wieland (2017), who also considers interactions between new business and an existing book of business but only from the insurance company's point of view, we simultaneously look at the perspective of the shareholder, a new policyholder and an existing policyholder. Contrary to Orozco-Garcia and Schmeiser (2019), however, who show fair premiums and identical default risk cannot be simultaneously achieved for all cohorts, we neglect default risk in our analyses.

The paper is organised as follows: In Section 2, we introduce our considered asset model with stochastic stock market returns and interest rates as well as the liability model that incorporates

two cohorts of traditional participating endowment policies – one representing the existing business and one the new business written. The model also contains management rules and intertemporal risk sharing mechanisms closely modelling the current situation in the German market. In Section 3, we present Eckert et al. (2020)'s concept of the (ex-ante respectively ex-post) collective bonus which will be our measure for analysing cross subsidizing effects. In Section 4, we present the results of our analyses including several sensitivity analyses. Finally, Section 5 concludes.

2. Framework

In this section, we introduce the model of the insurance company. We use a stochastic balance sheet and cash flow projection model with a strong focus on German market practice. The model is based on Burkhart et al. (2017) who apply this model to analyse the risk sharing between policyholders and shareholders in a Solvency II setting. Besides a few minor changes in the model on the asset side, we use a different interest rate model in order to allow for negative interest rates. On the liability side, we extend the model to the case of a heterogeneous insurance portfolio.

2.1. Financial market model

We consider a frictionless and continuous financial market on a filtered probability space $(\Omega, \mathcal{F}, \mathcal{P}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0})$. There are two sources of risk, interest rates and stocks. The dynamics are considered under both, the real-world probability measure $\mathcal{P}$ and the risk-neutral measure $\mathcal{Q}$.

In order to allow for negative interest rates, we use the Hull-White model (cf. Hull and White (1990)) for the short rate process $(r_t)_{t>0}$:

$$\begin{aligned} dr_t^{\mathcal{Q}} &= \left[\theta_t - a \cdot r_t^{\mathcal{Q}}\right] \cdot dt + \sigma^r \cdot dW_t^{(1)} && \text{under } \mathcal{Q} \\ dr_t^{\mathcal{P}} &= \left[\theta_t + \lambda_r - a \cdot r_t^{\mathcal{P}}\right] \cdot dt + \sigma^r \cdot d\widetilde{W}_t^{(1)} && \text{under } \mathcal{P} \end{aligned}$$

where $W_t^{(1)}$ is a standard Wiener process adapted to $\mathcal{F}$. $\widetilde{W}_t^{(1)}$ is its counterpart under $\mathcal{P}$. The parameter a is the mean reversion rate, σ^r is the interest rate volatility and λ_r is the corresponding risk premium. The parameter θ_t can be chosen such that the short rate process matches the current term structure observed in the market (cf. e.g. Brigo and Mercurio (2006)).

The stock price $(S_t)_{t>0}$ follows a generalization of the Black-Scholes model with dynamics

$$\frac{dS_t^{\mathcal{Q}}}{S_t^{\mathcal{Q}}} = r_t^{\mathcal{Q}} \cdot dt + \rho \cdot \sigma^S \cdot dW_t^{(1)} + \sqrt{1-\rho^2} \cdot \sigma^S \cdot dW_t^{(2)}, \;\; S_0^{\mathcal{Q}} = 1 \qquad \text{under } \mathcal{Q}$$
$$\frac{dS_t^{\mathcal{P}}}{S_t^{\mathcal{P}}} = \left(r_t^{\mathcal{P}} + \lambda_S\right) \cdot dt + \rho \cdot \sigma^S \cdot d\widetilde{W}_t^{(1)} + \sqrt{1-\rho^2} \cdot \sigma^S \cdot d\widetilde{W}_t^{(2)}, \;\; S_0^{\mathcal{P}} = 1 \qquad \text{under } \mathcal{P}$$

where σ^S is the volatility, λ_S is the equity risk premium and $W_t^{(2)}$ is a standard Wiener process under $\mathcal{Q}$ and its counterpart under $\mathcal{P}$ is denoted by $\widetilde{W}_t^{(2)}$. The Wiener processes $W_t^{(1)}$ and $W_t^{(2)}$ respectively $\widetilde{W}_t^{(1)}$ and $\widetilde{W}_t^{(2)}$ are assumed to be independent. The Wiener processes driving the interest rate process and the stock prices process are correlated by the parameter ρ.

For the purpose of performing Monte Carlo simulations (cf. Section 4), the stochastic differential equations can be solved accordingly (cf. e.g. Brigo and Mercurio (2006)). It can be shown that under this model the resulting (stochastic) integrals underlying the solutions of the stochastic processes follow a joint normal distribution (cf. Zaglauer and Bauer (2008) and Bergmann (2011)). We calculate Monte Carlo paths using random realizations of this multidimensional distribution. The bank account $B_t = \exp(\int_0^t r_u du)$ serves as the discount factor in the risk-neutral framework. From the price of a zero-coupon bond $P(t,T)$ with maturity $T \geq t$ at time t as given e.g. in Brigo and Mercurio (2006), we calculate par yields that determine the coupon rates of the considered coupon bonds with annual coupon payment.

2.2. Cash flow model

In order to focus on the main (financial) effects, we neglect expenses, charges, surrender and mortality. We assume that the company's insurance portfolio consists of two different cohorts of traditional participating endowment policies. Cohort 1 consists of contracts concluded at t_0^1 with term to maturity T^1. The conclusion date of the insurance contracts of cohort 2 is t_0^2 and the term to maturity is T^2 years. Annual premium payments P_t^i $(i = 1, 2)$ are paid at the beginning of the year. The cash flow to the shareholders of period $(t-1, t)$ is denoted by X_t (cf. Section 2.4) and is paid out immediately after the balance sheet has been set up at time t (cf. Figure 1). If X_t is positive, the shareholders receive a dividend payment. If X_t is negative, the shareholders have to make a payment to cover for the losses of the insurance company. Since the cash flows are paid out after the balance sheet date, these cash flows are part of the liability side of the statutory accounting balance sheet (cf. Table 3 in Section 4).

The payment to the policyholders at maturity of the contract $\left(L^i_{t_0^i+T^i}\right)$ consists of a guaranteed maturity benefit G^i (sum insured), a bonus reserve $BR^i_{t_0^i+T^i}$ where ongoing bonus is accumulated and a terminal bonus fund $TBF^i_{t_0^i+T^i-1}$. The payment to the policyholders at the contracts' maturity is at the end of the year, immediately before the balance sheet date (cf. Figure 1).

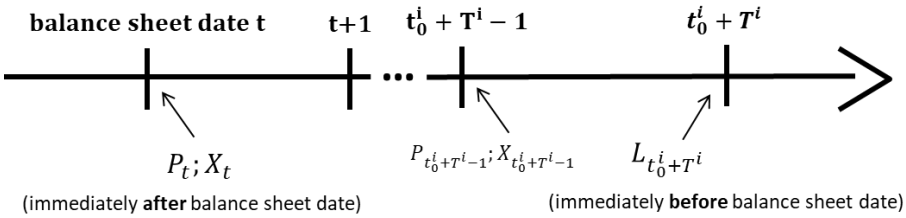


Figure 1.: Temporal overview of the different cash flows.

The asset side of the balance sheet consists of the book value of the assets. The liability side consists of the shareholders' equity $\left(Eq_t^{loc}\right)$, the payment to the shareholders (X_t), the free reserve for bonuses and rebates $(fRfB_t)$[1], the terminal bonus funds (TBF_t), the actuarial reserve (AR_t) and the bonus reserve (BR_t). The different components of the balance sheet are explained in the following sections. Table 3 in Section 4 further displays the structure of the statutory accounting balance sheet.

We denote the market value of assets immediately before (after) time t by MV_t^{A-} $\left(MV_t^{A+}\right)$ and get:

$$MV_t^{A+} = \begin{cases} MV_t^{A-} + P_t - X_t, & t \neq t_0^i + T^i, \quad i \in \{1,2\} \\ MV_t^{A-} - L_t + P_t - X_t, & t = t_0^i + T^i, \quad i \in \{1,2\} \end{cases}$$

2.3. Asset model

As in Burkhart et al. (2017), we assume that the company's asset portfolio consists of stocks and coupon bonds.[2] At the end of each year, after the coupon payments CP_{t+1} as well as the repayment of the nominal of maturing bonds $N_{t+1}^{(t+1-T_B)}$ and benefit payments L_{t+1} to policyholders, but before the balance sheet date, the asset portfolio is rebalanced to a stock ratio of q^{stock} in terms of market values. If in this process, bonds are sold, this is done proportionally to their market values. If in contrast bonds are bought, we invest in bonds with a time to maturity of T_B and annual coupon payments. Due to the rebalancing, unrealized gains or losses (UGL) stemming from book value accounting may be realized (cf. Burkhart et al. (2017)).[3]

If after the rebalancing the (relative) unrealized gains on stocks exceed a limit q_+^{UGL} (in terms of the market values), i.e. if $1 - \frac{BV_t^S}{MV_t^S} > q_+^{UGL}$ then a portion $d \cdot \left(MV_t^S - BV_t^S - q_+^{UGL} \cdot MV_t^S\right)$ of the unrealized gains on stocks is realized in order to smoothen investment returns. We further

[1]Simplified, this is an account that is collectively owned by the policyholders but not yet allocated to individual policyholders (cf. Section 3 in Alexandrova et al. (2017)).

[2]For technical reasons, we assume that $(P_t - X_t)$, i.e. the difference between premium payments and cash flows to the shareholders, is first invested in a one-year zero coupon bond before it becomes part of the "regular" asset allocation after one year.

[3]We call the ratio between the market value and the book value of the assets "asset valuation reserve" (AVR).

assume that unrealized losses on stocks exceeding a limit q_-^{UGL} (i.e. $\frac{BV_t^S}{MV_t^S} - 1 > q_-^{UGL}$) have to be realized immediately.

Overall, the investment return rate at time $t+1$ is given by

$$i_{t+1}^* = \frac{(P_t - X_t) \cdot r_t(1) + CP_{t+1} + RGL_{t+1}}{BV_t^A + P_t - X_t}$$

where RGL_{t+1} denotes the realized portion of the UGL at time $t+1$.

2.4. Liability model

2.4.1. Actuarial reserve

The relationship between the annual premium P^i and the guaranteed maturity benefit G^i is given by

$$\sum_{j=0}^{T^i-1} P^i \cdot \frac{1}{(1+g^i)^j} = G^i \cdot \frac{1}{(1+g^i)^{T^i}}$$

where g^i, $i = 1, 2$ denotes the guaranteed interest rate of the different cohorts. The actuarial reserve of contract i at outset is $AR_{t_0^i}^i = 0$ and then evolves according to

$$AR_t^i = (AR_{t-1}^i + P^i) \cdot (1+g^i) = \sum_{j=1}^{t} P^i \cdot (1+g^i)^j.$$

2.4.2. Surplus participation

This section describes how a potential surplus participation on top of the guaranteed interest is determined. First, the so-called "raw surplus" – i.e. the investment result less the guaranteed amount to be credited – is derived. Then, this surplus (if any) is distributed between policyholders and shareholders following German market practice and considering legal requirements.

Raw surplus and policyholders' share

The raw surplus Sp_{t+1} is given by the difference between the investment result R_{t+1}^* (based on the policyholders' accounts and the $fRfB$) and the guaranteed amount R_{t+1}^{gar} that has to be

credited to the policyholders' accounts (cf. Burkhart et al. (2017)):[4]

$$Sp_{t+1} = \underbrace{(AR_t + BR_t + P_t + fRfB_t + TBF_t) \cdot i^*_{t+1}}_{R^*_{t+1}} - \underbrace{\sum_{i=1}^{2}(AR^i_t + BR^i_t + P^i_t) \cdot g^i}_{R^{gar}_{t+1}}.$$

According to the regulation on minimum surplus participation (MindZV (2017)) applicable in Germany, at least 90% of the company's investment result has to be distributed to the policyholders. We use this minimum profit participation rule and, thus, the policyholders' share of surplus at time $t+1$ is given by

$$PS_{t+1} = \max\left(90\% \cdot R^*_{t+1} - R^{gar}_{t+1}; 0\right)$$

which is allocated to the $fRfB$ at first. The remaining part of raw surplus is allocated to the insurer's profit X_{t+1} (cf. equation (2)).

Negative raw surplus and German Insurance Supervision Law

§ 140 (1) of the Insurance Supervision Law (VAG) allows insurers to withdraw funds which have not yet been allocated to individual policyholders in order to avoid a pending financial imbalance of the company and to cover unforeseeable losses due to a general change of the economic environment (cf. Burkhart et al. (2017)). Thus, the $fRfB$ and the terminal bonus fund can be used to partly cover losses in case of a negative raw surplus. The share of losses that can be covered by policyholders is given by

$$q^{PH}_{t+1} = \frac{\sum_{s=1}^{T_{\S 140}} PS_{t+1-s}}{\sum_{s=1}^{T_{\S 140}} \max\left(Sp_{t+1-s}; 0\right)}$$

and depends on the relation between the policyholders' share of surplus and the (positive) raw surplus in the past $T_{\S 140}$ years. To cover the resulting amount, first, funds denoted by ${}^{\S 140, fRfB}CF_{t+1}$ are withdrawn from the $fRfB$. If they are not sufficient to cover the full policyholders' share of losses, funds denoted by ${}^{\S 140, TBF}CF_{t+1}$ from the terminal bonus funds are used. All withdrawals are limited to the respective funds available. For the sake of simplicity, we assume that the terminal bonuses from previous years which are supposed to be paid out at the end of the current period $\left(L^{term}_{t+1}\right)$ will be paid out to the policyholders as intended (cf. Burkhart et al. (2017)) and can therefore not be used to cover losses. Hence, we get

$${}^{\S 140, fRfB}CF_{t+1} = \min\left(fRfB_{t+1} + PS_{t+1}; q^{PH}_{t+1} \cdot \max\left(-Sp_{t+1}; 0\right)\right)$$

[4] Following Burkhart et al. (2017) we assume that the guaranteed interest rate also applies to the bonus reserve in subsequent years.

$$^{\S 140,TBF}CF_{t+1} = \min\left(TBF_t - L_{t+1}^{term}; q_{t+1}^{PH} \cdot \max\left(-Sp_{t+1}; 0\right) - {}^{\S 140,fRfB}CF_{t+1}\right).$$

As a consequence, the $fRfB$ evolves according to

$$fRfB_{t+1} = fRfB_t + PS_{t+1} - {}^{\S 140,fRfB}CF_{t+1} - bon_{t+1}.$$

with bon_{t+1} denoting the bonus declaration to the client described in the following section.

Bonus reserve and terminal bonus fund

At the end of each year, the bonus declaration bon_{t+1} takes place. The amount is derived from the allocations of investment surplus to the RfB in the previous T_{bon} years and is given by

$$bon_{t+1} = \frac{1}{T_{bon}} \cdot \sum_{i=0}^{T_{bon}-1} \frac{PS_{t+1-i}}{AR_{t-i} + BR_{t-i}} \cdot (AR_t + BR_t).$$

If this bonus declaration bon_{t+1} would lead to a reserve ratio outside of the corridor $[q_{min}^{fRfB}; q_{max}^{fRfB}]$, the bonus declaration is de-/increased, respectively.[5]

Next, the declared bonus is split into an ongoing and a terminal bonus $({}^{acc}bon_{t+1}, {}^{term}bon_{t+1})$ according to a fixed portion q^{TBF}. Finally, the declared bonuses are allocated to individual contracts. We follow Burkhart et al. (2017) who use a so-called natural allocation system for this allocation (cf. also Hansen and Miltersen (2002)). That means that all contracts of a certain cohort i receive the same bonus. However, contracts from different cohorts may have a different amount. Burkhart et al. (2017) describe the mechanism of bonus distribution as follows: "The investment boni are distributed such that all policyholders receive the same total yield (sum of the guaranteed interest rate g^i and the bonus rate) on their accounts value $(AR_t^i + BR_t^i)$. However, all policyholders receive at least the guaranteed interest rate. Hence, if investment boni are not sufficient for all policyholders to receive at least the highest guaranteed interest rate, the bonus rates of cohorts with a lower guaranteed interest rate are reduced accordingly".

Hence, we get

$$BR_{t+1}^i = BR_t^i \cdot (1 + g^i) + {}^{acc}bon_{t+1}^i, \quad \text{with } BR_{t_0^i}^i = 0$$

and

$$TBF_{t+1}^i = TBF_t^i + {}^{term}bon_{t+1}^i - {}^{\S 140,TBF}CF_{t+1}^i.$$

[5] The application of a "reserve corridor" is in the spirit of the much more stylized model in Kling et al. (2007). The reserve ratio is determined by the $fRfB$ and the sum of actuarial and bonus reserve.

Policyholder account

At time $t \in \{t_0^i, \ldots, t_0^i + T^i - 1\}$ the policyholder account of cohort i is given by

$$L_t^i = AR_t^i + BR_t^i + TBF_{t-1}^i \tag{1}$$

resulting in the total maturity benefit $L_{t_0^i+T^i}^i = G^i + BR_{t_0^i+T^i}^i + TBF_{t_0^i+T^i-1}^i$.

Cash flow to the shareholders

As mentioned above, one part of the shareholders' cash flow is the difference between the raw surplus and the policyholders' share of this surplus. In addition, the investment income on the shareholders' equity is paid out to the shareholders as well. Finally, we implemented a management rule keeping a constant rate of equity in the insurance company's balance sheet based on book values given by

$$q_t^{loc} = \frac{Eq_t^{loc}}{AR_t}.$$

The cash flow to the shareholders is paid out at the beginning of the next year and is given by

$$X_{t+1} = (Sp_{t+1} - PS_{t+1}) + {}^{\S 140, fRfB/TBF} CF_{t+1} + i_{t+1}^* \cdot Eq_t^{loc} + (Eq_t^{loc} - Eq_{t+1}^{loc}). \tag{2}$$

3. Valuation measures

In Eckert et al. (2020) the concept of the collective bonus (or malus) has been introduced measuring how much a contract will in expectation (ex ante) benefit or has actually benefited (ex post) from being part of the insurance collective. The collective bonus of cohort i ($i = 1, 2$) at the valuation date t^* is given by:

$$CB_{t^*}^i = CB_{t^*}^i(\text{ex ante}) + CB_{t^*}^i(\text{ex post})$$

with

$$CB_{t^*}^i(\text{ex ante}) = E_{\mathcal{Q}}\left[\sum_{j=t^*}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_{j+1}} \cdot L_j^i \cdot \left(\frac{L_{j+1}^i}{L_j^i + P_j^i} - \frac{MV_{j+1}^{A-}}{MV_j^{A+}}\right)\right]$$

and

$$CB_{t^*}^i(\text{ex post}) = L_{t^*}^i - \sum_{j=t_0^i}^{t^*-1} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{MV_k^{A-}}{MV_{k-1}^{A+}}.$$

The aggregated collective bonus of both cohorts of the insurance portfolio at time t^* is denoted by CB_{t^*}. Intuitively, the collective bonus is the difference of the contract's return and a theoretical investment of the contract's premiums in the company's asset portfolio at market value.

The ex ante collective bonus of the shareholders at time t^* is given by the so-called present value of future profits $PVFP_{t^*}$ where T is the expiration of the last contract of the insurance company:

$$PVFP_{t^*} = E_{\mathcal{Q}}\left[\sum_{j=t^*}^{T} \frac{B_{t^*}}{B_j} \cdot X_j\right].$$

The ex post collective bonus of the shareholders can be calculated assuming that all previous cash flows X_j, $j \leq t^*$, had been invested in the company's asset portfolio $\sum_{j=t_0^1+1}^{t^*-1} X_j \cdot \prod_{k=j+1}^{t^*} \frac{MV_k^{A-}}{MV_{k-1}^{A+}}$. Therefore it is given by

$$CB_{t^*}^{sh} = PVFP_{t^*} + \sum_{j=t_0^1+1}^{t^*-1} X_j \cdot \prod_{k=j+1}^{t^*} \frac{MV_k^{A-}}{MV_{k-1}^{A+}}.$$

The so-called best estimate of liabilities $BE_{t^*}^i$ of cohort i at time t^* is defined as the present value of future expected cash flows payable to the policyholders less the present value of future premium payments under the equivalent pricing measure $\mathcal{Q}$, i.e.

$$BE_{t^*}^i = E_{\mathcal{Q}}\left[\frac{B_{t^*}}{B_{t_0^i+T^i}} \cdot L_{t_0^i+T^i}^i - \sum_{j=t^*}^{t_0^i+T^i-1} \frac{B_{t^*}}{B_j} \cdot P_j^i\right].$$

The best estimate of liabilities at time t^* of all cohorts is denoted by BE_{t^*}.

Eckert et al. (2020) show that the sum of the collective bonuses of the shareholders and the policyholders is zero if and only if the market value of assets is equal to $BE_{t^*} + PVFP_{t^*}$, i.e.

$$MV_{t^*}^A = BE_{t^*} + PVFP_{t^*} \Leftrightarrow CB_{t^*} + CB_{t^*}^{sh} = 0.$$

Hence, if the market value of assets is equal to $BE_{t^*} + PVFP_{t^*}$ and one cohort of contracts has a non-zero collective bonus, the other cohort or the shareholders have to compensate for this difference. Thus, the concept of the collective bonus provides a measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio, in particular with respect to cross-subsidizing effects.

4. Analyses and results

4.1. Parameters and initial balance sheet

The valuation date $t^* = 0$ in our numerical analyses is December 31, 2017, immediately before the conclusion of cohort 2 (cf. Figure 2).

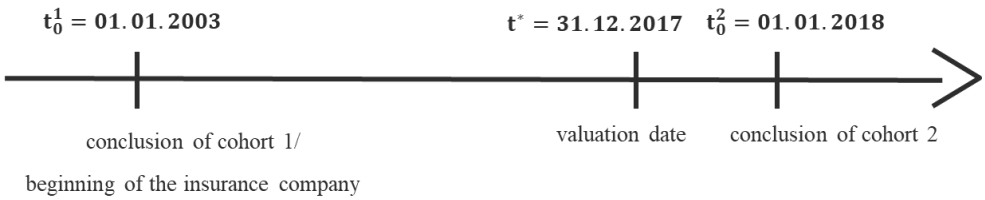


Figure 2.: Timeline of the insurance company.

The term to maturity of each cohort is $T^1 = T^2 = 20$ years, the sum insured is $G^1 = G^2 = 20,000$ € and each cohort consists of $1,000$ contracts. Cohort 1 started at t_0^1 (01/01/2003) and cohort 2 enters the insurance company at t_0^2 (01/01/2018). Thus the cohorts overlap for five years. The guaranteed interest rate of cohort 1 is chosen in accordance with the (then valid) maximum technical rate in Germany at t_0^1, i.e. $g^1 = 3.25\%$. For cohort 2 we determine the guaranteed interest rate such that the contract would be priced fairly "on an individual basis". This means that for an insurer consisting only of cohort 2 and the shareholders, the collective bonus (calculated at time t^*) of cohort 2 and of the shareholders would be zero. This results in $g^2 = 0.23\%$ assuming the parameter set explained in what follows.

For the management rules explained in Section 2, we use the parameters as summarized in Table 1:

q^{stock}	T_B	d	q_+^{UGL}	q_-^{UGL}	$T_{\S 140}$	T_{bon}	q_{min}^{fRfB}	q_{max}^{fRfB}	q^{TBF}	q^{loc}
10%	15 years	50%	15%	15%	10 years	5 years	1.5%	4.5%	$\frac{1}{3}$	0.85%

Table 1.: Parameters for management rules.

We take these parameters from Burkhart et al. (2017) who chose them consistent with regulation and market practice prevailing in the German life insurance market. We deviate from their choice only in two values: We increase the time to maturity of bonds T_B from 12 to 15 years to approximately match the asset valuation reserve rate at time $t^* = 0$ of the largest German life insurance company and use a rate of equity of $q^{loc} = 0.85\%$ to approximately match the rate of equity of Allianz (cf. Allianz (2018)). The capital market parameters for the financial model introduced in Section 2 are shown in Table 2.

r_0	a	σ^r	S_0	σ^S	ρ	λ_r	λ_A
0.0%	10%	1.30%	1.00	16.27%	10%	0.00	0.04

Table 2.: Financial market parameters.

For the parameters r_0, a, σ^r, σ^S and ρ we apply the values used in 2019 within the German standard valuation model for Solvency II (cf. DAV (2019)), developed by the German Association of Insurance Companies (GDV) – the so called "Branchensimulationsmodell" (BSM, cf. GDV (2016)). The parameter θ_t is chosen such that the zero-bond prices derived from the model match the term structure provided by EIOPA (European Insurance and Occupational Pensions Authority) for the German market[6]. For the calibration of the risk premium λ_A we follow the approach of Korn and Wagner (2018). In addition, we choose $\lambda_r = 0$. The stochastic projection is performed for $100,000$ scenarios of the financial market and has a time horizon of 20 years.

To obtain the actuarial reserve AR_0, the bonus reserve BR_0, the free reserve for bonuses and rebates $fRfB_0$, the terminal bonus fund TBF_0 and the company's equity Eq_0^{loc} at time 0, we project the first cohort in a deterministic scenario which is based on historical data from the German life insurance market (cf. Table 12 in Appendix A). At time $t^* = 0$, the book value of the asset portfolio is equal to the sum of the book values of the liabilities and the shareholders' equity. In line with Burkhart et al. (2017), we assume unrealized gains of stocks of 25%. The coupon rates of the coupon bonds at time 0 are based on historical data (cf. Table 13 in Appendix B, column "historic coupon rates"). The time to maturity of the different bonds in the bond portfolio are equally split (according to the book value of the assets) between 1 and $T_B = 15$ years. Table 3 summarizes the initial balance sheet at time $t^* = 0$:

Assets	**Liabilities**
$BV_0^S = 1,388,113$ €	$Eq_0^{loc} = 116,832$ €
$BV_0^B = 13,435,555$ €	$X_0 = 58,379$ €
	$fRfB_0 = 539,898$ €
	$TBF_0 = 116,094$ €
	$AR_0 = 13,744,974$ €
	$BR_0 = 247,492$ €

Table 3.: Balance sheet at time $t^* = 0$.

[6]European Insurance and Occupational Pensions Authority: https://www.eiopa.europa.eu/tools-and-data/risk-free-interest-rate-term-structures_en : EIOPA_RFR_20190131_Term_Structures.xlsx, accessed on August 2019.

The market value of the stocks at time 0 is $1,735,142$ € and the market value of the bonds is $15,618,090$ €. Hence, we get asset valuation reserves of the asset portfolio of 17.1 percent.

4.2. Comparison of two cohorts

4.2.1. Certainty equivalent scenario

To understand the mechanisms of the interaction between the existing business (cohort 1), new business (cohort 2) and the shareholders, we first consider the so-called certainty equivalent scenario[7]. In Figure 3, the book value performance of the reference portfolio as well as the total rate of interest allocated to cohort 1 and to cohort 2, respectively are shown:

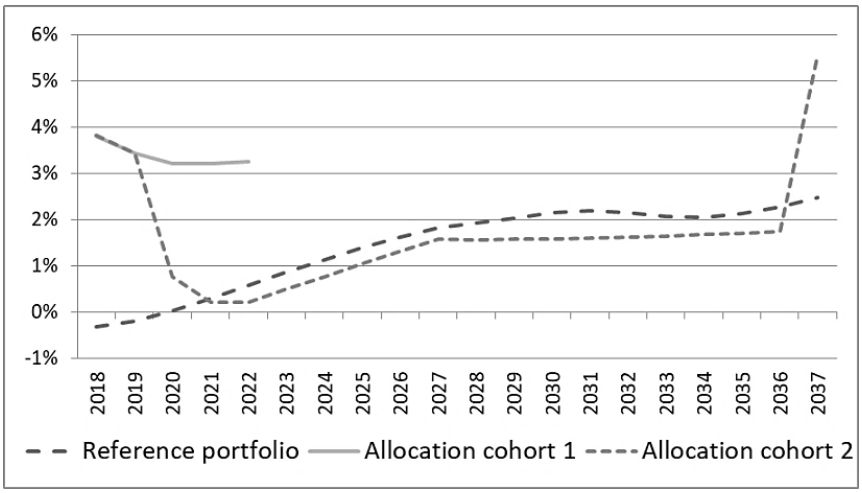


Figure 3.: Return of the reference portfolio and the resulting total rate of interest allocated to cohort 1 and cohort 2 in the certainty equivalent scenario.

Despite the poor capital market performance, the allocation to cohort 1 and cohort 2 starts at a rather high level since asset valuation reserves and $fRfB$ are used to provide high surplus participations. Starting in the year 2020, the allocation to the policyholders decreases (but for cohort 1 not below its guaranteed interest rate of 3.25%). In 2021, even funds from the terminal bonus fund have to be used to allocate the respective guaranteed rate to both contracts (cf. Section 2.4). The total rate allocated to contract 2, however, falls below the performance of the reference portfolio in this year. After 2022, the total rate of interest allocated to cohort 2 increases again above its guaranteed rate due to improved investment returns but permanently stays below the performance of the reference portfolio. At maturity of cohort 2 (year 2037), the insurance company is liquidated. The $fRfB$ as well as the asset valuation reserves are

[7]This deterministic scenario reflects the expected development of the capital market under the risk-neutral measure. The calculation is based on the assumption that all assets earn the forward rate implied by the initial yield curve (cf. Oechslin et al. (2007) and Reuß et al. (2015)).

liquidated and increase the raw surplus. Hence, the allocation to cohort 2 is very high at maturity.[8]

The present values of the realized collective bonus after $t = 0$ in the certainty equivalent scenario for each cohort and the shareholders are displayed in Table 4.

PV of realized CB of cohort 1	**PV of realized CB of cohort 2**	**PV of realized CB of shareholders**	**Sum**
2,833,334 €	-62,901 €	474,239 €	3,244,672 €

Table 4.: Present value of the realized collective bonus of cohort 1, 2 and the shareholders in the certainty equivalent scenario.

The present value of cohort 1 is highly positive due to a guaranteed interest rate which significantly exceeds the market performance. The negative value of cohort 2 implies that they subsidize the other cohort or the shareholders in this scenario. Although the shareholders have negative cash-flows in some years, their overall collective bonus is still positive. Since the sum of all collective bonuses is significantly positive, most of the subsidizing after $t = 0$ in this scenario seems to result from reserves that were generated before time $t = 0$. We will get back to this issue in the more general setting of the next subsection.

4.2.2. Ex ante collective bonus

Next, we consider the ex ante collective bonuses at time $t = 0$ (cf. Section 3) in Table 5.

ex ante CB of cohort 1	**ex ante CB of cohort 2**	**ex ante CB of shareholders**	**Sum**
3,129,461 €	341,315 €	-226,104 €	3,244,672 €

Table 5.: Ex ante collective bonus at time 0 of cohort 1, cohort 2 and the shareholders.

The positive collective bonus of both cohorts shows that, from a policyholders' perspective, the investment in the insurance contract in expectation yields higher returns than a direct investment in the reference portfolio. The reason for this is different for both cohorts: Cohort 1 has a high guaranteed interest rate which has to be granted independent of the development of the reference portfolio. The ex ante collective bonus is therefore about 3.1 Mio € (i.e. more than $3,100$ € per contract). The ex ante collective bonus of cohort 2 is also positive. The ex ante collective bonus of $341,315$ € corresponds to 1.7 percent of the sum of the premiums paid by

[8] This effect would be spread over several years in the more realistic case of an insurer that has contracts maturing every year.

cohort 2 during the term of the contract. The ex ante collective bonus of the shareholders is -0.2 Mio €. Hence in expectation, the shareholders will subsidize both cohorts. It is worth noting that the ex ante expected collective bonus for both cohorts is higher than the (deterministic) collective bonus in the certainty equivalent scenario and lower for the shareholders. This difference results primarily from the asymmetric nature of the annual guarantee: While the shareholders have to pay for 100% of losses in scenarios where the guarantee is not covered by asset returns and reserves, they only receive 10% of any existing surplus, cf. also Section 2.4.2.

Table 6 shows that the market value of assets is equal to the sum of the present value of future profits and the best estimate of liabilities. As mentioned in Section 3, this means that the sum of the collective bonuses is zero.

Market value of the assets	**Best estimate of liabilities**	**Present value of future profits**
17,353,232 €	17,579,336 €	-226,104 €

Table 6.: Market value of assets, best estimate of liabilities and present value of future profits.

Remember, the collective bonus is the sum of the ex ante part and the ex post part. The sum of the ex ante collective bonuses is about 3.24 Mio € (cf. Table 5). The ex post collective bonus of cohort 2 at time $t = 0$ is obviously zero. As a consequence, the sum of the ex post collective bonuses of cohort 1 and the shareholders at time 0 has to be -3.24 Mio €, i.e. either the policyholders from cohort 1 received lower returns than an investment directly in the reference portfolio or the shareholders invested more money than they received (or both). As we prove in Appendix C, the ex post collective bonus is the negative sum of the reserves which have not been allocated yet:

$$-CB_0(\text{ex post}) = Eq_0^{loc} + X_0 + fRfB_0 + MV_0^A - BV_0^A \tag{3}$$

Hence, the negative ex post collective bonus implies that there are reserves at time $t = 0$ which have not been allocated yet. Cohort 1, cohort 2 and the shareholders will benefit from these reserves after time $t = 0$, hence the sum of all ex ante collective bonuses is positive in this setting.

4.2.3. Ex post collective bonus

Now, we analyse the probability distribution of the ex post collective bonus under the real-world measure $\mathcal{P}$ (cf. Figure 4). The calculation of the ex post collective bonus considers the years

2018 to 2022 for cohort 1 and the years 2018 to 2037 for cohort 2 and the shareholders. Please note that we consider the ex post collective bonus directly after the maturity of the contract, i.e. the ex post collective bonus of cohort 1 is calculated in 2022 and the ex post collective bonus of cohort 2 and the shareholders is calculated in 2037.

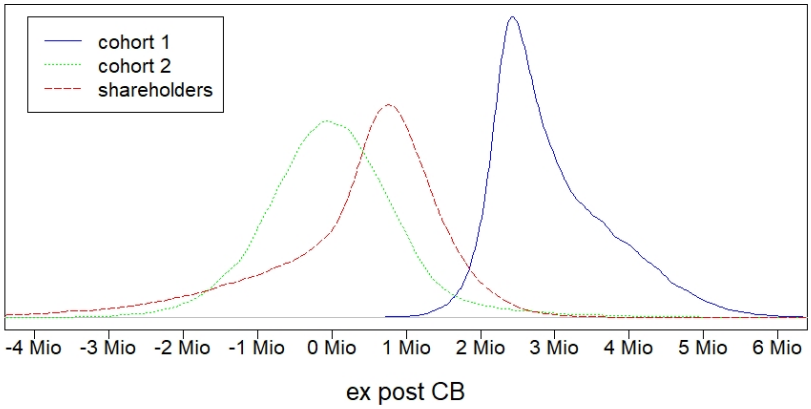


Figure 4.: Probability distribution of the ex post collective bonus of cohort 1, cohort 2 and the shareholders.

We can see that the probability distribution of cohort 2 looks rather symmetric with a median close to 0. In roughly 50 percent of the scenarios, the new business benefits from the existing portfolio (and the shareholders) and in the other 50 percent of the scenarios the new business subsidizes the existing portfolio. In scenarios with a good performance of the reference portfolio, the policyholders do not receive the entire surplus, so the total rate of interest allocated to cohort 2 is lower than a theoretical investment in the reference portfolio. The total rate of interest is downward bounded by the guaranteed rate. Hence, the ex post collective bonus is positive in scenarios with a bad performance. The probability distribution of cohort 1 on the other hand is heavily right-skewed and lies almost entirely in the positive domain. Due to the existing reserves and the high guaranteed rate the future return of the insurance contract under practically all circumstances is more attractive than a theoretical investment in the reference portfolio for cohort 1. Especially in scenarios with a bad performance of the reference portfolio compared to this cohort's guaranteed rate, the collective bonus can be very high. Consequently, the probability distribution of the shareholders is left-skewed. In scenarios with a bad performance of the reference portfolio, the shareholders' ex post collective bonus can be very low. The skewed distributions are again a consequence of the asymmetry explained in Section 4.2.2.

4.2.4. Isolated analyses of the cohorts

To better understand who benefits especially from an interaction with the other cohort and corresponding existing reserves, we consider cohort 1 and cohort 2 separately. To this end,

Table 7 displays the ex ante collective bonus for the following two situations:

Only cohort 1:
There is no new business, i.e. the insurance portfolio consists only of cohort 1.

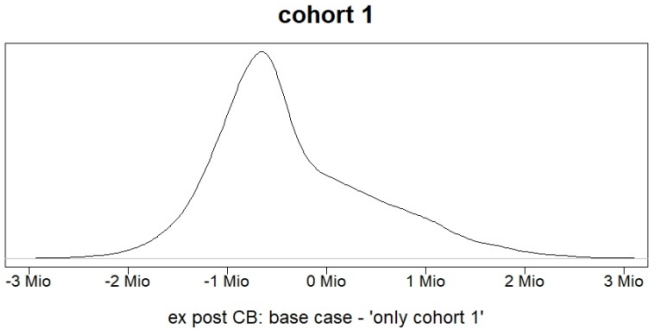
Only cohort 2:
There is no existing business, i.e. the insurance company starts with cohort 2 at time $t = 0$.

ex ante CB	cohort 1 and cohort 2	only cohort 1	only cohort 2
cohort 1	3,129,461 €	3,497,385 €	-
cohort 2	341,315 € €	-	0 €
shareholders	-226,104 €	-252,713 €	0 €

Table 7.: Comparison of the ex ante collective bonus: "cohort 1 and cohort 2", "only cohort 1" and "only cohort 2".

We can see that cohort 1 (in expectation) would be better off if the insurer did not take on cohort 2, mainly because cohort 1 would not need to share existing reserves with the new policyholders. The shareholders on the other hand benefit – albeit to a lesser extent – from the existence of cohort 2, mainly because they reduce average guarantees to be paid. This result is in line with Hieber et al. (2015). Somewhat "symmetric" to the results for cohort 1, we find that cohort 2 would be worse off if they were alone, mainly because in this case there were no "old reserves"[9] whereas the shareholders would benefit from the absence of cohort 1, mainly because in this case the high old guarantees would not exist.

In Figure 5, the probability distribution of the difference between the ex post collective bonus in the base case and the ex post collective bonus in the situation "only cohort 1" resp. "only cohort 2" are shown.



[9]Note that in the situation "only cohort 2", the ex ante collective bonus of cohort 2 (and hence also the shareholders) is zero as a consequence of the fair pricing of cohort 2.

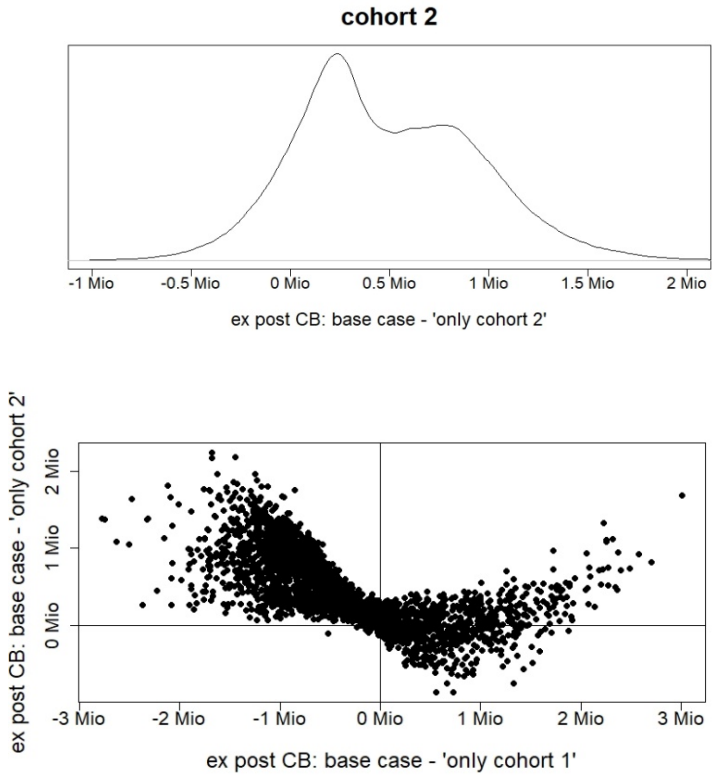


Figure 5.: Probability distribution of the difference between the ex post collective bonus in the base case and "only cohort 1", resp. "only cohort 2"; scatter plot of these differences.

Both distributions have probability mass in the positive and in the negative domain. So each cohort can either benefit or suffer from the existence of the other cohort depending primarily on the development of the reference portfolio between 2018 and 2022. The scatter plot of the ex post collective bonus differences of cohort 1 and cohort 2 (cf. Figure 5) shows the following pattern: In scenarios where the existence of existing business increases the ex post collective bonus of the new business, the existing business typically suffers from the existence of new business and vice versa. The scatter plot is – of course – not a straight line, since also collective bonus of the shareholders has an effect on the displayed quantities. The values on the top right of the scatter plot where both cohorts simultaneously profit from the existence of the respective other cohort are driven by scenarios where the shareholders have to finance guarantees for both cohorts.

4.3. Sensitivity analyses

4.3.1. New business pricing

In this section we vary the guaranteed rate of interest of the new business from -2% p.a. to 4% p.a. All other parameters remain unchanged. Figure 6, displays the resulting ex ante collective bonus of cohort 1, cohort 2 and the shareholders.

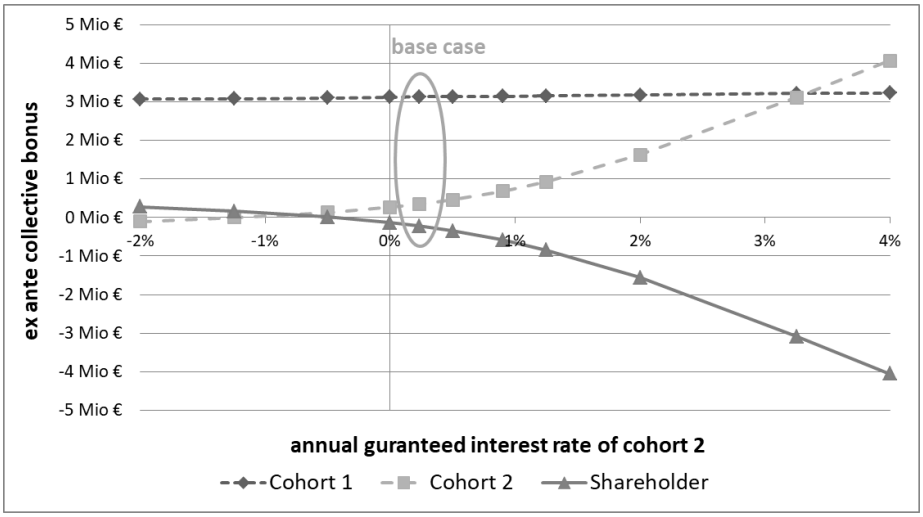


Figure 6.: Influence of the guaranteed interest rate of cohort 2 on the ex ante collective bonus.

The ex ante collective bonus of cohort 1 is only marginally affected by the change of the guarantee of the new business: It increases from $3,067,138$ € to $3,231,004$ € as the guaranteed rate of cohort 2 increases. This increase might appear counterintuitive but can be explained as follows: In our model, the $fRfB$ is proportional to the actuarial reserve. A higher guaranteed rate of interest results in a lower premium and corresponding actuarial reserve. Hence, the $fRfB$ can be reduced more to partly cover losses especially in the first few years after time 0.[10]

As expected, the ex ante collective bonus of cohort 2 increases sharply by the guaranteed rate of cohort 2 and the ex ante collective bonus of the shareholders decreases accordingly. For $g^2 = -1.25\%$, the ex ante collective bonus of cohort 2 is zero. In this case, the new business is priced "collectively fair". In Table 8, the ex ante collective bonus in this case is shown. In brackets, we show the change when compared to the individually fair pricing. For an insurance company that consists only of cohort 2, this pricing would obviously lead to subsidization from cohort 2 to the shareholders.

[10] We also varied the guaranteed rate for cohort 1. Here, as expected, the collective bonus of cohort 1 increases in the guaranteed rate whereas the collective bonuses of both, the shareholder and cohort 2 decrease.

ex ante CB	cohort 1 and cohort 2	only cohort 2
cohort 1	3,081,817 € (-47,644 €)	-
cohort 2	0 € (-341,315 €)	-323,472 € (-323,472 €)
shareholders	162,855 € (+388,959 €)	323,472 € (323,472 €)

Table 8.: Ex ante collective bonus with a "collective fair" pricing of the new business.

4.3.2. New business volume

Next, we vary the new business volume of cohort 2 (the volume of cohort 1 remains unchanged at $1,000$ contracts). As in the base case, the pricing of contract 2 is individually fair, i.e. $g^2 = 0.23\%$. In Figure 7 the ex ante collective bonus of cohort 1, cohort 2 and the shareholders for different new business volumes are plotted (left). The right plot of Figure 7 considers the ex ante collective bonus of one contract of each cohort.

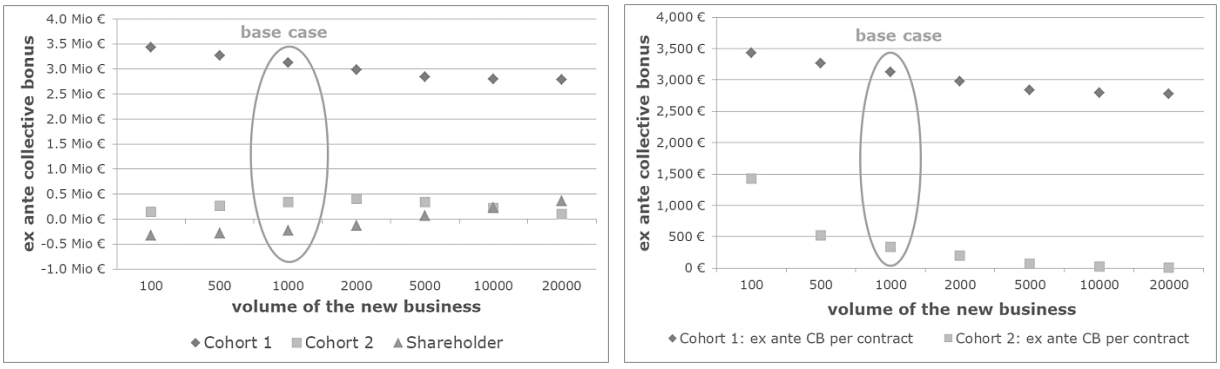


Figure 7.: Influence of the new business volume on the ex ante collective bonus: cohort 1, cohort 2 and the shareholders (left); one contract of each cohort (right).

With a higher new business volume, the ex ante collective bonus of cohort 1 decreases and the ex ante collective bonus of the shareholders increases. The ex ante collective bonus of the whole new business cohort increases to a volume of $2,000$ contracts and decreases afterwards. In contrast, the ex ante collective bonus per contract of this cohort is decreasing in the new business volume and approaches 0 – i.e. the value for cohort 2 in absence of cohort 1.

Next, we consider a different variation of new business by allowing for more than two cohorts. In each of the next twenty years, a new cohort $i+2$ is concluded at time t_0^2+i, $i \in \{1, 2, \ldots, 20\}$. Cohorts 3 through 22 are equipped with the same parameters as cohort 2. In particular, g^i is set to 0.23% for $i \in \{3, 4, \ldots, 22\}$. Each cohort consists of $1,000$ contracts. In Table 9, we compare the ex ante collective bonus per contract of cohort 1 and cohort 2 in the base case,

the case of more cohorts and the case where the new business volume of cohort 2 is increased to 20,000 contracts (cf. above).

ex ante CB	base case	more cohorts	20,000 contracts of cohort 2
per contract of cohort 1	3,129 €	2,945 €	2,781 €
per contract of cohort 2	341 €	152 €	5 €
shareholders	-226,104 €	5,805,173 €	361,299 €

Table 9.: Ex ante collective bonus with and without more new business.

The ex ante collective bonus per contract of cohort 1 decreases from 3,129 € in the base case to 2,945 € in the "more cohorts" case. Since cohort 1 matures at the conclusion date of cohort 7, it is only influenced by the 5,000 contracts of cohorts 2 to 6. Therefore, the reduction in the collective bonus is smaller than in the situation with 20,000 contracts of cohort 2. The ex ante collective bonus per contract of cohort 2 in the in the "more cohorts" case is 152 € – a reduction of more than 50 percent compared to the base case. The shareholders, however, benefit from ongoing new business: their ex ante collective bonus increases from −226,104 € to almost 6 Mio €.

In Figure 8, the ex ante collective bonus of cohort 1 to cohort 21 are plotted.

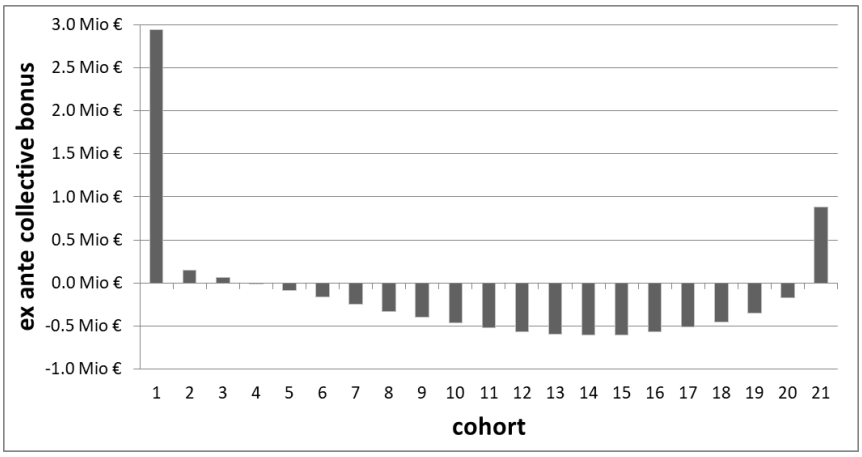


Figure 8.: Ex ante collective bonus of cohort 1 to cohort 21.

The ex ante collective bonuses of cohorts 4 through 20 are negative. The shareholders are subsidized by these cohorts, which causes the high ex ante collective bonus of the shareholders. The U-shaped structure in Figure 8 results from two effects. First, all new business cohorts are priced with the guaranteed rate that makes the cohort starting at $t = 0$ fair on an individual

basis. Since the term structure of interest rates implies increasing expected interest rates, future cohorts are not priced fairly anymore (to the disadvantage of the respective policyholders). The higher the expected interest rate, the lower would the ex ante collective bonus be ceteris paribus. Secondly, however, the $fRfB$ is liquidated to the benefit of the last few cohorts (cf. also footnote 10). Apparently, this second effect is stronger than the first effect for later cohorts explaining the increase after cohort 15.

To gain more insight on the impact of the new business volume, we compare the probability distribution of the ex post collective bonus under the real-world measure $\mathcal{P}$ in the base case and the two alternative scenarios for new business (cf. Figure 9).

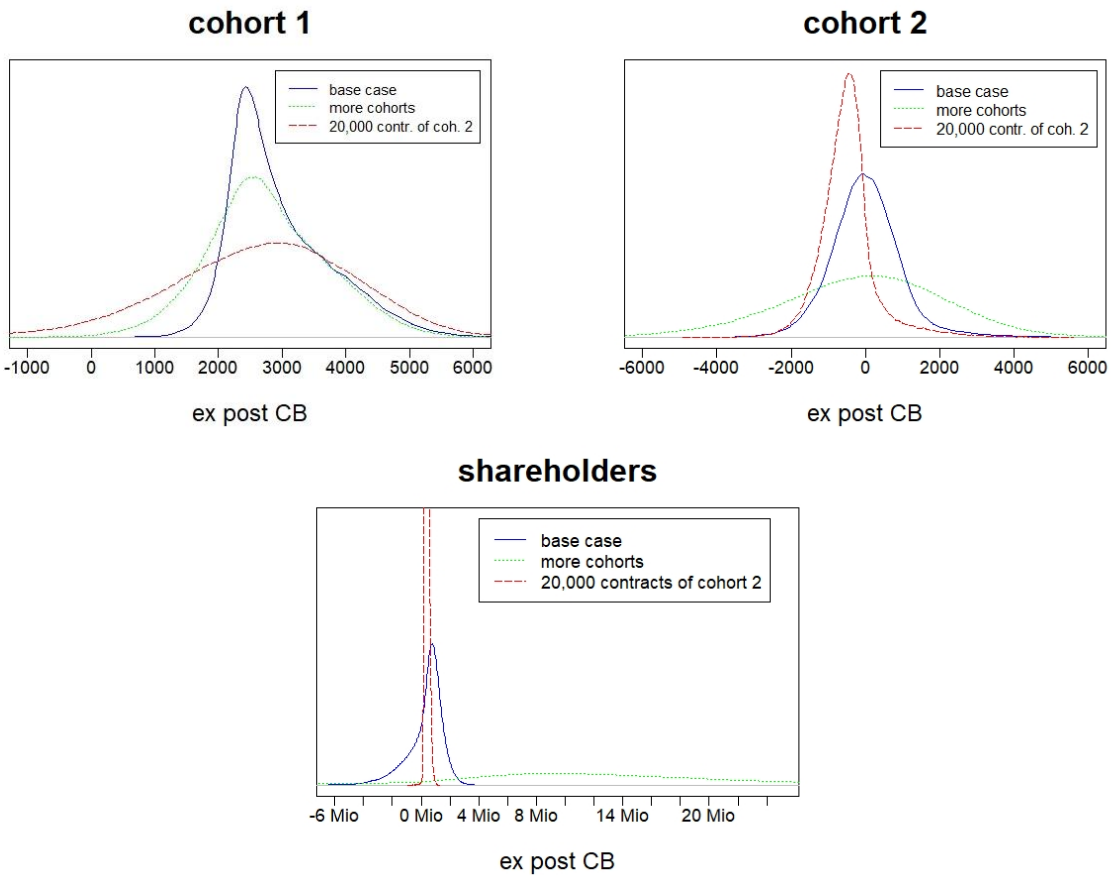


Figure 9.: Probability distribution of the ex post collective bonus: base case, "more cohorts" and "20,000 contracts of cohort 2".

As expected from the above ex ante analyses, in both alternative new business scenarios the probability distribution of the ex post collective bonus of cohort 1 and cohort 2 is shifted to the left compared to the base case. A higher number of contracts, with which cohort 1, respectively cohort 2, interacts, increases the volatility of the probability distribution. This particularly holds in the scenario "20,000 contracts of cohort 2" for cohort 1 and in the scenario "more cohorts" for cohort 2. A higher number of cohorts also increases the time period in

which the shareholders interact with policyholders increasing the volatility of the probability distribution of the shareholders' ex post collective bonus.

4.3.3. Impact of asset valuation reserves

In this section, we analyse the impact of asset valuation reserves (AVR) at time 0 on the collective bonus.[11] Recall that in our base case, we have used historic coupon rates to determine the bond portfolio (and hence the AVR) at time 0. For the sensitivity analyses, we increase the historic coupon rates by one or two percentage points (scenario "high AVR", respectively "very high AVR") and decrease them by one percentage point ("low AVR"). All other parameters remain unchanged. The coupon modifications, the resulting asset valuation reserves and the resulting ex post and ex ante collective bonus values are summarized in Table 10.

	low AVR	**base case**	**high AVR**	**very high AVR**
Coupon rates	historic minus 1%	historic	historic plus 1%	historic plus 2%
Resulting AVR	9.9%	17.1%	24.1%	31.1%
Sum ex post CB ($t = 0$)	-2,200,715 €	-3,244,672 €	-4,329,412 €	-5,373,541 €
Ex ante CB cohort 1	2,906,151 €	3,129,461 €	3,436,878 €	3,866,138 €
Ex ante CB cohort 2	122,586 €	341,315 €	623,298 €	928,349 €
Ex ante CB shareholders	-828,022 €	-226,104 €	269,236 €	579,054 €

Table 10.: Coupon rates, resulting AVR, ex post collective bonus and ex ante collective bonus for the different sensitivities of the asset valuation reserves.

As expected, the AVR are higher for higher historic coupon rates. Also, the ex post collective bonus calculated at $t = 0$ is decreasing in the AVR since higher AVR at $t = 0$ means that a larger share of asset returns has been transferred to the collective reserves rather than the cohort members' accounts (cf. equation (3)). Finally, the ex ante collective bonus of all involved parties is increasing in the AVR since higher reserves at $t = 0$ means that more money can be shared between the parties in the future.

[11] We have also varied the value of the $fRfB$ at time 0 and found structurally similar results.

4.3.4. Impact of the term structure of interest rates

In this section, we analyse the impact of the term structure of interest rates by assuming a different term structure of interest rates which is 50 basis points (bp) lower, 50 bp higher, and 100 bp higher than our base case assumption. We adjust the historic coupon rates of the coupon bonds to get the same asset valuation reserve at time 0 in all sensitivities (see Appendix B). Note that this means that we do not analyse the effect of a sudden shift in the term structure of interest rates at time 0 (which would also affect the reserve situation at $t = 0$) but rather the effect of a different interest scenario with unchanged reserves.

Table 11 shows the resulting ex ante collective bonus for all involved parties.

ex ante CB	-50 bp	base case	+50 bp	+100 bp
cohort 1	3,504,705 €	3,129,461 €	2,787,301 €	2,502,355 €
cohort 2	600,941 €	341,315 €	197,968 €	124,664 €
shareholders	-860,974 €	-226,104 €	259,403 €	617,653 €

Table 11.: Ex ante collective bonus for the different sensitivities of the term structure of interest rates.

Higher interest rates lead to a better expected future performance of the reference portfolio which primarily helps the shareholders financing the policyholders' guarantees. The policyholders' performance relative to the performance of the reference portfolio (which is measured by the collective bonus) decreases. Hence, the ex ante collective bonus of cohorts 1 and 2 is reduced by higher interest rates while the shareholder's collective bonus is increased.

5. Conclusion and Outlook

This paper deals with cross-subsidizing effects between different cohorts of policyholders and particularly analyses whether new policyholders benefit from or subsidize the existing portfolio. To answer this question, we use the recently introduced "collective bonus" (cf. Eckert et al. (2020)) as a measure and analyse two cohorts of traditional participating endowment policies in the German regulatory framework. One cohort represents the existing business with a high annual return guarantee and the other cohort represents the new business with a lower guarantee.

Both cohorts show a positive ex ante collective bonus. This means that, in expectation, both cohorts will earn higher returns than the expected return of the reference portfolio. The existing

business mainly profits from the rather high guaranteed rate that was given in the past. The new business mainly profits from existing asset valuation reserves of the company at the time they enter. The shareholders take the opposite position by carrying an ex ante collective malus.

A closer look at the probability distribution of the ex post collective malus and several scenario analyses show that cross-subsidizing effects between existing business, new business and shareholders strongly depend on the realized capital market scenario. Each stakeholders benefits from the other stakeholders in certain scenarios.

Sensitivity analyses show that a change in the guaranteed rate of interest of the new business mainly affects cross-subsidizing effects between the shareholders and the new business. Higher guarantees, c.p. lead to a higher collective bonus of cohort 2 and to a lower collective bonus of the shareholders with only a minor impact on the existing business. The volume of new business significantly influences cross-subsidizing effects between all different stakeholders: While the existing business suffers from higher new business volumes, shareholders profit from ongoing new business or higher new business volumes. Finally, as expected, higher asset valuation reserves (AVR) or higher free reserve for bonuses and rebates ($fRfB$) lead to higher future collective bonuses for all stakeholders because the larger share of past asset returns that was transferred to the collective reserves will be distributed in the future.

It is particularly noteworthy that a contract which is fairly priced on an individual basis can still have a positive ex ante collective bonus which implies a higher expected return than a direct investment in the reference portfolio. Moreover, the ex ante collective bonus is an expected value and therefore does not consider volatility (both, with respect to value fluctuations in time and with respect to the variance of the terminal benefit). Hence, even if a new contract comes with a negative ex ante collective bonus it might still be attractive for a new policyholder due to so-called return smoothing elements of surplus participation mechanisms. Building and resolving hidden reserves as well as the use of collective buffer accounts like the RfB or the terminal bonus fund reduce volatility (in particular with respect to interim value fluctuations) when compared to a direct investment in the reference portfolio. Hence, even if the expected return is below the expected return of the reference portfolio, an investment in the contract might still be rational (or at least subjectively attractive) depending on the investor's risk aversion.[12]

The focus of this paper was on the interaction of different "generations" of traditional participating endowment policies. In recent years, we have observed a trend in many countries towards products "between" pure unit-linked products and traditional participating products which use an investment in the traditional product's assets (including the return smoothing elements) as

[12] Cf. e.g. Albrecht and Weinmann (2015) for an analysis of the effect of return smoothing mechanisms in German life insurance contracts, or Ruß and Schelling (2020) for subjective (behavioral) aspects of the attractiveness of collective investments.

a "building block". Such products include participating life insurance products with alternative guarantees (cf. Reuß et al. (2015) and Reuß et al. (2016)), equity-linked life insurance based on traditional products where surplus is leveraged using suitable options (Alexandrova et al. (2017)), (dynamic) hybrid products with a (path dependently changing) combination of traditional and unit-linked assets (Kochanski and Karnarski (2011)) which all share the same asset pool and the same bonus reserves. This gives rise to an abundant range of questions with respect to fairness (cross-subsidizing effects between generations and product types) and risk management (new risks resulting e.g. from path dependent in- and outflow to the traditional cover fund) that are subject to further research. In particular, the question of fairness between different product categories (as opposed to different generations of policyholders in the same product category) has become increasingly relevant over the last years and has recently explicitly been addressed by the German regulator, cf. BaFin (2020). The regulator explicitly demands that systematic cross-subsidizing effects between different product categories have to be observed, quantified and compensated for.

Appendix

A. Net investment return[13] for deterministic scenario

Time	2003	2004	2005	2006	2007	2008	2009	2010
Net investment return	5.05%	4.90%	5.18%	4.82%	4.65%	3.54%	4.18%	4.27%

Time	2011	2012	2013	2014	2015	2016	2017
Net investment return	4.13%	4.59%	4.68%	4.63%	4.52%	4.36%	4.49%

Table 12.: Net investment return of the assets of the German life insurance companies[14].

[13]The net investment return for the deterministic scenario corresponds to the investment return rate defined in Section 2.3.

[14]GDV: 2018_0697_RS_LV_Anlage_Kapitalanlagen_2017.

B. Historic coupon rates of the coupon bonds of the portfolio at time t^*

Time	Historic coupon rates	Rates in scenario "-50bp"	Rates in scenario "+50bp"	Rates in scenario "+100bp"
2003-12	4.87%	4.34%	5.40%	5.93%
2004-12	4.39%	3.86%	4.92%	5.45%
2005-12	3.94%	3.41%	4.47%	5.00%
2006-12	4.17%	3.64%	4.70%	5.23%
2007-12	4.94%	4.41%	5.47%	6.00%
2008-12	4.47%	3.94%	5.00%	5.53%
2009-12	4.45%	3.92%	4.98%	5.51%
2010-12	4.21%	3.68%	4.74%	5.27%
2011-12	3.86%	3.33%	4.39%	4.92%
2012-12	2.91%	2.38%	3.44%	3.97%
2013-12	3.16%	2.63%	3.69%	4.22%
2014-12	1.37%	0.84%	1.90%	2.43%
2015-12	1.48%	0.95%	2.01%	2.54%
2016-12	1.16%	0.63%	1.69%	2.22%
2017-12	1.34%	0.81%	1.87%	2.40%

Table 13.: Historic data for the coupon rates of the coupon bonds[15] and adjusted coupon rates for the sensitivity: "Influence of the term structure of interest rates".

C. Proof of relation between the balance sheet and the ex post collective bonus

The ex post part of the collective bonus is the negative sum of the reserves in the balance which are not allocated yet

$$-CB_0(\text{ex post}) = Eq_0^{loc} + X_0 + fRfB_0 + MV_0^A - BV_0^A$$

Proof.
From the statutory accounting balance sheet (cf. Table 3) we know:

$$BV_0^A = Eq_0^{loc} + X_0 + fRfB_0 + TBF_0 + AR_0 + BR_0.$$

[15]Deutsche Bundesbank Eurosystem: https://www.bundesbank.de/dynamic/action/de/statistiken/zeitreihen-datenbanken/zeitreihen-datenbank/759778/759778?listId=www_skms_it04b, accessed on 02 January 2019.

From equation (1) we get:

$$L_0^1 = TBF_0 + AR_0 + BR_0.$$

There are no payments to the policyholders before time 0. Hence (cf. Section 2.2)

$$MV_i^A = MV_i^{A-}, \;\; i \in \{t_0^1, \ldots, 0\}.$$

Furthermore the market value of assets at time 0 is equal to (cf. Section 2.2)

$$\begin{aligned} MV_0^A = MV_0^{A-} = MV_0^{A-} \cdot \frac{MV_{-1}^{A+}}{MV_{-1}^{A+}} &= \frac{MV_0^{A-}}{MV_{-1}^{A+}} \cdot \left(MV_{-1}^{A-} + P_{-1}^1 - X_{-1}\right) = \\ &= \frac{MV_0^{A-}}{MV_{-1}^{A+}} \cdot \left(P_{-1}^1 - X_{-1}\right) + \frac{MV_0^{A-}}{MV_{-1}^{A+}} \cdot \frac{MV_{-1}^{A-}}{MV_{-2}^{A+}} \cdot \left(MV_{-2}^{A-} + P_{-2}^1 - X_{-2}\right) = \cdots = \\ &= \sum_{j=t_0^1}^{-1} \left(P_j^1 - X_j\right) \cdot \prod_{k=j+1}^{0} \frac{MV_k^{A-}}{MV_{k-1}^{A+}}. \end{aligned}$$

Hence, we get

$$\begin{aligned} Eq_0^{loc} + X_0 + fRfB_0 + MV_0^A - BV_0^A &= MV_0^A - TBF_0 - AR_0 - BR_0 \\ &= MV_0^A - L_0^1 \\ &= \sum_{j=t_0^1}^{-1} \left(P_j^1 - X_j\right) \cdot \prod_{k=j+1}^{0} \frac{MV_k^{A-}}{MV_{k-1}^{A+}} - L_0^1 \\ &= -\sum_{j=t_0^1}^{-1} X_j \cdot \prod_{k=j+1}^{0} \frac{MV_k^{A-}}{MV_{k-1}^{A+}} - \left(L_0^1 - \sum_{j=t_0^1}^{-1} P_j^1 \cdot \prod_{k=j+1}^{0} \frac{MV_k^{A-}}{MV_{k-1}^{A+}}\right) \\ &= -CB_0^{sh}(\text{ex post}) - CB_0^1(\text{ex post}) \\ &= -CB_0(\text{ex post}). \end{aligned}$$

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3 Werden bei einer Differenzierung der Überschussbeteiligung in der Lebensversicherung Neukunden zu Lasten des Bestands bevorzugt?

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Werden bei einer Differenzierung der Überschussbeteiligung in der Lebensversicherung Neukunden zu Lasten des Bestands bevorzugt?

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Abstract

Klassische Lebensversicherungsverträge mit Überschussbeteiligung bilden weiterhin einen großen Anteil des Geschäftes von Lebensversicherungsunternehmen. Aufgrund der anhaltenden Niedrigzinsphase werden die Garantien von neuen Versicherungsprodukten zunehmend reduziert. Als Ausgleich für die reduzierten Garantien wird Neukunden bei manchen Anbietern eine erhöhte Beteiligung an den erwirtschafteten Überschüssen in Aussicht gestellt. Es wird somit bei der Überschussdeklaration zwischen Bestandskunden und Neukunden differenziert. In dieser Arbeit sollen die Auswirkungen einer Differenzierung der Überschussbeteiligung auf Bestandskunde und Neukunde sowie auf die Profitabilität des Aktionärs untersucht werden. Zur Ermittlung der Höhe der Differenzierung werden die im Ergebnisbericht der Deutschen Aktuarvereinigung e.V. (DAV (2017)) beschriebenen Methoden verwendet. Es wird im Speziellen gezeigt, dass in dem verwendeten Basisszenario die erwartete Rentabilität der Bestandskunden bei einer Neugeschäftsstrategie mit abgesenkter Garantie und erhöhter Überschussbeteiligung leicht geringer ausfällt als bei einer Neugeschäftsstrategie ohne Reduktion der Garantie und ohne Differenzierung der Überschussbeteiligung. Außerdem wird darauf eingegangen, wie die Abnahme der erwarteten Rentabilität der Bestandskunden kompensiert werden kann. Das verwendete Modell legt einen starken Fokus auf die Überschussbeteiligung im deutschen Versicherungsmarkt.

Schlüsselwörter
Lebensversicherung; Versicherungsverträge mit Überschussbeteiligung; Heterogenes Versicherungsportfolio; Interaktion von Versicherungsverträgen; Differenzierung der Überschussbeteiligung.

1. Einleitung

Am europäischen Lebensversicherungsmarkt werden immer häufiger neben klassischen Versicherungsprodukten auch Produkte mit reduzierten Garantien und erhöhter Überschussbeteiligung angeboten. Bei einem klassischen Produkt wird das gesamte Vertragsguthaben jährlich mindestens mit einem über die gesamte Vertragsdauer konstanten Garantiezins, welcher in der Regel dem zum Zeitpunkt des Vertragsschlusses gültigen Höchstrechnungszins für die Ermittlung der Deckungsrückstellung entspricht, verzinst. Dieser Garantiezins wird bei neuen Produkten nun häufig reduziert. Beide Produktausgestaltungen, das klassische Produkt und das Produkt mit reduziertem Garantiezins, investieren in das allgemeine Sicherungsvermögen des Lebensversicherungsunternehmens. Der Kunde mit reduzierter Garantie wird zum Ausgleich stärker an den erwirtschafteten Überschüssen beteiligt. Es erfolgt eine Differenzierung der Überschussbeteiligung zwischen Kunden mit einem klassischen Produkt und Kunden mit einem Produkt mit reduzierter Garantie. In dieser Arbeit wird angenommen, dass die Bestandskunden ein klassisches Produkt abgeschlossen haben. Anschließend werden verschiedene Strategien, unter anderem ein reduzierter Garantiezins und eine erhöhte Überschussbeteiligung, für das Pricing des Neugeschäfts betrachtet und deren Auswirkung auf die Bestandskunden und den Aktionär analysiert.

Während die Bundesanstalt für Finanzdienstleistungsaufsicht (im Folgenden abgekürzt als BaFin) in einer Verlautbarung aus dem Jahr 2004 (siehe VerBaFin (2004)) die Differenzierung der Überschussbeteiligung bei einheitlicher Kapitalanlage aufgrund des Gleichbehandlungsgrundsatzes nach §138 Absatz 2 Versicherungsaufsichtsgesetz (VAG)[1] untersagt hatte, wird in einem Ergebnisbericht der Deutschen Aktuarvereinigung e.V. (DAV) aus dem Jahr 2017 (siehe DAV (2017)) argumentiert, dass die Verlautbarung der BaFin sich aufgrund von deutlich unterschiedlichen Rahmenbedingungen nicht ohne Weiteres auf die heutige Situation übertragen lässt und somit eine Überschussdifferenzierung gerechtfertigt ist. Unterschiedliche Rahmenbedingungen bedeuten für die DAV die fehlende Wahlmöglichkeit im Jahr 2004 zwischen Produkten mit unterschiedlich werthaltigen Garantien und die annähernde Gleichheit des Höchstrechnungszinses in Deutschland bei allen damaligen Vertragsgenerationen. Deshalb liegen für die DAV dann gleiche Voraussetzungen im Sinne von §138 Absatz 2 VAG vor, wenn „Art und Struktur“ (DAV (2017)) der Produkte, ausgenommen reduzierte Garantien, miteinander vergleichbar sind. Weiter wird argumentiert, dass Gleichbehandlung nicht bedeuten kann, dass bei gleicher Prämie in jedem erdenklichen Zukunftsszenario die Leistung gleich ist. Vielmehr müssen Verträge „wertgleich“ sein. Diese Wertgleichheit könne beispielsweise durch die Differenzierung der Überschussbeteiligung hergestellt werden (DAV (2017)).

[1] §138 Absatz 2 VAG: „Bei gleichen Voraussetzungen dürfen Prämien und Leistungen nur nach gleichen Grundsätzen bemessen werden.“

In dieser Arbeit werden die Auswirkungen einer Differenzierung der Überschussbeteiligung zwischen Bestandskunde und Neugeschäft analysiert und im Speziellen auf die Frage einer Benachteiligung des Bestandes aufgrund einer Überschussdifferenzierung eingegangen. Dabei wird im Gegensatz zu Grosen und Jørgensen (2000), Bacinello (2003), Bauer et al. (2005), Graf et al. (2011), Gatzert und Martin (2014) und vielen weiteren Autoren, welche von einem homogenen Versicherungsportfolio für das Pricing, Hedging und Risikomanagement ausgehen, ein heterogenes Versicherungsportfolio betrachtet. Konkret wird die Interaktion zwischen 20 verschiedenen Vertragsgenerationen von klassischen Lebensversicherungen mit Überschussbeteiligung analysiert. Nur wenige Arbeiten betrachten die Quersubventionierung zwischen verschiedenen Vertragsgenerationen. Døskeland und Nordahl (2008) untersuchen beispielsweise die Quersubventionierung zwischen mehreren Generationen von individuell fair bewerteten klassischen Versicherungen, und Hieber et al. (2015) analysieren den Effekt von Quersubventionierung auf das Risikoprofil des Ertrages verschiedener Vertragsgenerationen. Dabei wird jedoch bei der Überschussbeteiligung nicht differenziert. Dies geschieht beispielsweise in Reuß et al. (2016), die das Zusammenspiel aus Asset Allokation und Höhe der Überschussbeteiligung bei unterschiedlichen Produktausgestaltungen untersuchen. In dieser Arbeit wird das Konzept des Kollektivbonus (collective bonus (CB)) angewendet, um zu messen, wie sehr ein Kollektiv von Verträgen von einem anderen Kollektiv profitiert, bzw. wie sehr ein Kollektiv ein anderes Kollektiv subventioniert. Dieses Konzept wird in Eckert et al. (2020a) eingeführt und ermöglicht es, sowohl erwartete (ex ante) als auch bereits realisierte (ex post) Quersubventionierungen zu bestimmen.

Diese Arbeit ist folgendermaßen aufgebaut: In Abschnitt 2 wird das stochastische Modell, welches unserer Analyse zugrunde liegt, beschrieben. In Abschnitt 3 werden unterschiedliche Neugeschäftsstrategien vorgestellt und anschließend die Auswirkung dieser Strategien auf das Neugeschäft, die Bestandskunden und den Aktionär analysiert. Nachdem als Abschluss von Abschnitt 3 verschiedene Sensitivitäten betrachtet werden, schließt Abschnitt 4 mit einem Fazit und einem Ausblick für zukünftige Forschungsfragen.

2. Modell

In diesem Abschnitt wird das Modell für das Versicherungsunternehmen, welches unseren Analysen zugrunde liegt, und im Speziellen das Asset Liability Management beschrieben. Das Modell wurde anhand der im deutschen Versicherungsmarkt üblichen Praxis entwickelt und basiert auf Burkhart et al. (2017) sowie Eckert et al. (2020b), wobei die Möglichkeit einer Differenzierung der Überschussbeteiligung zwischen Neugeschäft und Bestand ergänzt wurde.

2.1. Finanzmarktmodell

Analog zu Eckert et al. (2020b) wird von einem vollständigen, stetigen Finanzmarkt auf einem filtrierten Wahrscheinlichkeitsraum $(\Omega, \mathcal{F}, \mathcal{P}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0})$ ausgegangen. Auf diesem werden Zinsen und Aktien modelliert. Diese werden sowohl unter dem Wahrscheinlichkeitsmaß $\mathcal{P}$ sowie unter dem zu $\mathcal{P}$ eindeutigen äquivalenten risikoneutralen Maß $\mathcal{Q}$ betrachtet.

Für die Modellierung des Short Rate Prozesses $(r_t)_{t>0}$ wird das Hull-White Modell (cf. Hull und White (1990)) verwendet,

$$\begin{aligned} dr_t^{\mathcal{Q}} &= \left[\theta_t - a \cdot r_t^{\mathcal{Q}}\right] \cdot dt + \sigma^r \cdot dW_t^{(1)} && \text{unter } \mathcal{Q} \\ dr_t^{\mathcal{P}} &= \left[\theta_t + \lambda_r - a \cdot r_t^{\mathcal{P}}\right] \cdot dt + \sigma^r \cdot d\widetilde{W}_t^{(1)} && \text{unter } \mathcal{P} \end{aligned}$$

und der Aktienpreis $(S_t)_{t>0}$ wird mithilfe eines verallgemeinerten Black-Scholes-Modells (cf. Black und Scholes (1973)) dargestellt:

$$\begin{aligned} \frac{dS_t^{\mathcal{Q}}}{S_t^{\mathcal{Q}}} &= r_t^{\mathcal{Q}} \cdot dt + \rho \cdot \sigma^S \cdot dW_t^{(1)} + \sqrt{1-\rho^2} \cdot \sigma^S \cdot dW_t^{(2)}, \;\; S_0^{\mathcal{Q}} = 1 && \text{unter } \mathcal{Q} \\ \frac{dS_t^{\mathcal{P}}}{S_t^{\mathcal{P}}} &= \left(r_t^{\mathcal{P}} + \lambda_S\right) \cdot dt + \rho \cdot \sigma^S \cdot d\widetilde{W}_t^{(1)} + \sqrt{1-\rho^2} \cdot \sigma^S \cdot d\widetilde{W}_t^{(2)}, \;\; S_0^{\mathcal{P}} = 1 && \text{unter } \mathcal{P}. \end{aligned}$$

Die risikolose Anlage $B_t = \exp(\int_0^t r_u du)$ dient als Diskontfaktor bei der risikolosen Bewertung. Auf die Bezeichnung und Funktionsweise der einzelnen Parameter, die Vereinfachung der oben genannten Gleichungen für die Monte-Carlo Simulation und die Herleitung der Berechnung der Kuponzinsen wird an dieser Stelle verzichtet und auf Eckert et al. (2020b) verwiesen.

2.2. Bilanz des Versicherungsunternehmens und Zahlungszeitpunkte

Der Fokus dieser Arbeit liegt auf den Auswirkungen der Differenzierung der Überschussbeteiligung. Daher werden Kosten, Storno und Sterblichkeit nicht berücksichtigt. Das Versicherungsportfolio besteht aus 20 verschiedenen Generationen von je $1,000$ ([2]) klassischen Erlebensfallversicherungen mit Überschussbeteiligung und jährlicher vorschüssiger Prämienzahlung P_t^i $(i = 1, \ldots, 20)$. Die Laufzeit der Verträge beträgt 20 Jahre $(T^i = 20\ , i = 1, \ldots, 20)$, die garantierte Erlebensfallleistung ist $G^i = 20,000$ $(i = 1, \ldots, 20)$. In Abbildung 1 ist die Laufzeit der einzelnen Verträge abgebildet. Die Verträge beginnen jeweils zum Jahresanfang (t_0^i, $i = 1, \ldots, 20$), die erste Generation im Jahr 2000 ($t_0^1 = 01.01.2000$). Jedes Jahr kommt eine weitere Generation hinzu. Zum ersten Januar 2019 beginnt Generation 20, welche das Neugeschäft repräsentiert.

[2]Es wird die englische Notation für Dezimalzahlen verwendet.

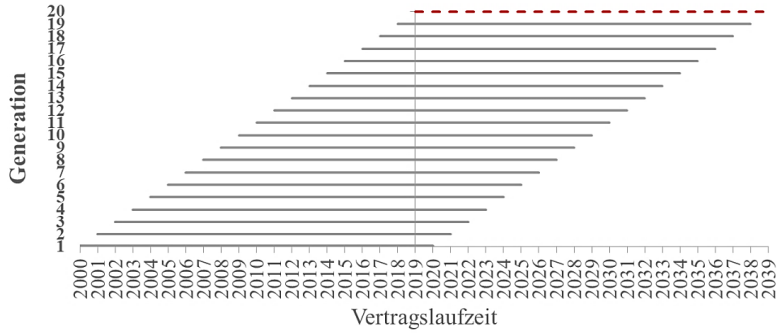


Abbildung 1.: Laufzeit der einzelnen Vertragsgenerationen.

Die Dividendenzahlung des Zeitraums $(t-1,t)$ an den Aktionär wird mit X_t bezeichnet. Ist X_t positiv, erhält der Aktionär eine Dividendenzahlung. Ist X_t negativ, muss der Aktionär eine Ausgleichszahlung für die Verluste des Versicherungsunternehmens tätigen. Die Zahlung der Dividende für das vergangene Jahr findet zum Beginn des Folgejahres statt (siehe Abbildung 2), das heißt X_t wird zu Beginn der Periode $(t,t+1)$ entrichtet und ist somit noch Teil der Passivseite der Bilanz zum Zeitpunkt t (siehe Tabelle 4). Die Leistung $L^i_{t_0^i+T^i}$, die an die Kunden bei Ablauf des Vertrages ausbezahlt wird, setzt sich aus garantierter Leistung G^i, laufendem Überschuss $(BR^i_{t_0^i+T^i})$ und Schlussüberschuss $(TBF^i_{t_0^i+T^i-1})$ zusammen. Der Kunde erhält die endfällige Leistung am Jahresende, unmittelbar vor dem Bilanzstichtag (siehe Abbildung 2).

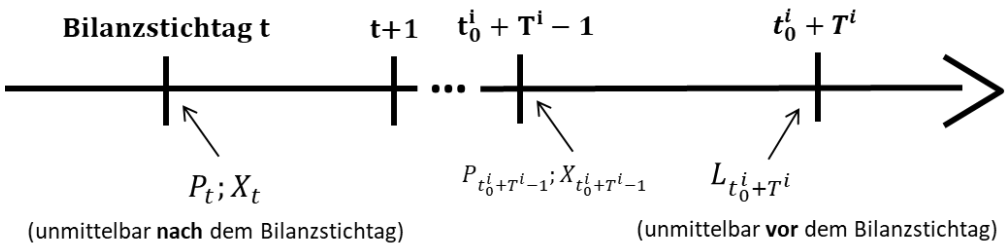


Abbildung 2.: Überblick der Zahlungszeitpunkte.

Die Aktiva des Versicherungsunternehmens bestehen aus den Buchwerten der Anlagen (BV_t^A), zusammengesetzt aus den Buchwerten der Aktien (BV_t^S) und der Bonds (BV_t^B). Die Passiva sind Eigenkapital des Aktionärs $(Eq_t^{loc}$ [3]$)$, die noch ausstehende Dividendenzahlung (X_t), die freie Rückstellung für Beitragsrückerstattung $fRfB_t$, die Schlussüberschussbeteiligung (TBF_t), das Deckungskapital (AR_t) und die laufende Überschussbeteiligung (BR_t). Tabelle 4 gibt einen Überblick über die Bilanz des Versicherungsunternehmens. Der Marktwert der Assets unmittelbar vor (nach) dem Zeitpunkt t wird definiert mit MV_t^{A-} (MV_t^{A+}):

$$MV_t^{A+} = \begin{cases} MV_t^{A-} + P_t - X_t, & t \neq t_0^i + T^i, \quad i \in \{1,\dots,20\} \\ MV_t^{A-} - L_t + P_t - X_t, & t = t_0^i + T^i, \quad i \in \{1,\dots,20\}. \end{cases}$$

[3] loc = local accounting rule.

2.3. Modellierung der Aktivseite

Es wird dieselbe Modellierung der Aktivseite verwendet wie in Burkhart et al. (2017). Im Asset Portfolio des Versicherungsunternehmens sind Aktien und Kuponbonds enthalten. Für die Differenz aus Dividendenzahlung X_t und Prämienzahlung P_t wird (aus modelltechnischen Gründen) angenommen, dass diese zunächst in einen einjährigen Zerokuponbond investiert wird. Am Jahresende finden die Kuponzahlungen CP_{t+1}, die Nennwertrückzahlung $N_{t+1}^{(t+1-T_B)}$ bei Ablauf von Bonds und die Ausschüttung der Leistung an die Kunden statt. Im Anschluss (unmittelbar vor dem Bilanzstichtag) wird das Asset Portfolio anhand der Aktienquote q^{stock}, bezogen auf die Marktwerte der Assets, neu ausgerichtet. Dafür werden entsprechend Aktien ge- oder verkauft und falls notwendig, Kuponbonds proportional zu ihren Marktwerten verkauft oder Kuponbonds mit Laufzeit T_B und jährlichen Kuponzahlungen gekauft. Beim Anpassen des Asset Portfolios kann es zur Realisierung von stillen Reserven oder stillen Lasten kommen. Außerdem wird ein Teil d der stillen Reserven auf Aktien oberhalb einer Grenze q_+^{UGL} [4] realisiert, um die Buchwertrendite zu stabilisieren, das heißt, wenn $1 - \frac{BV_t^S}{MV_t^S} > q_+^{UGL}$ ist, wird $d \cdot (MV_t^S - BV_t^S - q_+^{UGL} \cdot MV_t^S)$ realisiert. Nicht realisierte Verluste unterhalb einer Grenze q_-^{UGL} werden direkt realisiert ($\frac{BV_t^S}{MV_t^S} - 1 > q_-^{UGL}$).

Somit ergibt sich die Buchwertrendite zum Zeitpunkt $t+1$ als

$$i_{t+1}^* = \frac{(P_t - X_t) \cdot r_t(1) + CP_{t+1} + RGL_{t+1}}{BV_t^A + P_t - X_t},$$

wobei RGL_{t+1}[4] die realisierten stillen Reserven oder Lasten zum Zeitpunkt $t+1$ bezeichnen.

2.4. Modellierung der Passivseite

Deckungskapital

Mithilfe des Äquivalenzprinzips ergibt sich die Prämie P^i je Generation i

$$\sum_{j=0}^{T^i-1} P^i \cdot \frac{1}{(1+g^i)^j} = G^i \cdot \frac{1}{(1+g^i)^{T^i}},$$

wobei mit g^i, $i = 1, \ldots, 20$ der jeweilige Rechnungszins bezeichnet wird. Das Deckungskapital $AR_{t_0^i}^i$ von Generation i ist zu Beginn null. Anschließend ist das Deckungskapital AR_t^i am Ende

[4] UGL = unrealized gains or losses; RGL = realized gains or losses.

des Jahres t gegeben durch

$$AR_t^i = (AR_{t-1}^i + P^i) \cdot (1 + g^i) = \sum_{j=1}^{t} P^i \cdot (1 + g^i)^j.$$

Überschussbeteiligung

Die Bestimmung der erwirtschafteten Überschüsse läuft analog zu Eckert et al. (2020b) ab. Bei der Zuteilung der Überschüsse auf die einzelnen Vertragsgenerationen wird jedoch abgewichen, da eine Differenzierung der Überschüsse zwischen Bestandskunden und Neugeschäft vorgenommen wird. Die Bestimmung und Verteilung der Überschüsse findet immer am Jahresende nach dem Anpassen des Asset Portfolios statt:

1. Der Rohüberschuss Sp_{t+1} wird als Differenz aus Investmenterträgen R_{t+1}^* (basierend auf den Vertragsguthaben und der *fRfB*) und vertraglich garantierten Erträgen R_{t+1}^{gar}, die den Vertragsguthaben gutgeschrieben werden, ermittelt[5]

$$Sp_{t+1} = \underbrace{(AR_t + BR_t + P_t + fRfB_t + TBF_t) \cdot i_{t+1}^*}_{R_{t+1}^*} - \underbrace{\sum_{i=1}^{20} (AR_t^i + BR_t^i + P_t^i) \cdot g^i}_{R_{t+1}^{gar}}.$$

2. Ist der Rohüberschuss positiv, so wird dieser teilweise als Überschussbeteiligung den Guthaben der Kunden zugeteilt und teilweise als Dividendenzahlung an den Aktionär ausgeschüttet. Die Aufteilung erfolgt nach der Mindestzuführungsverordnung (siehe MindZV (2017)). Der Teil, der den Kunden gutgeschrieben wird (PS_{t+1}[6]), ergibt sich als

$$PS_{t+1} = \max\left(90\% \cdot R_{t+1}^* - R_{t+1}^{gar}; 0\right). \tag{1}$$

   Zunächst wird PS_{t+1} der *fRfB* zugeteilt. Der restliche Teil des Rohüberschusses ist Teil der Dividendenzahlung X_{t+1} (siehe Gleichung (2)).

   Ist der Rohüberschuss negativ, wird §140 Absatz 1 VAG[7] angewendet, welcher es ermöglicht, noch nicht zugeteilte Geldmittel zu verwenden, um unvorhersehbare Verluste

[5] Wie in Burkhart et al. (2017) wird angenommen, dass die laufende Überschussbeteiligung mit dem jeweiligen Garantiezins verzinst wird.

[6] PS = policyholders' share; PH = policyholder.

[7] Versicherungsaufsichtsgesetz §140 Absatz 1: [...] In Ausnahmefällen kann die Rückstellung für Beitragsrückerstattung, soweit sie nicht auf bereits festgelegte Überschussanteile entfällt, mit Zustimmung der Aufsichtsbehörde im Interesse der Versicherten herangezogen werden, um einen drohenden Notstand abzuwenden, unvorhersehbare Verluste aus den überschussberechtigten Versicherungsverträgen auszugleichen [...].

auszugleichen. Die Schlussüberschussbeteiligung und die $fRfB$ können somit genutzt werden, um einen negativen Rohüberschuss zu minimieren. Hierbei repräsentiert

$$q_{t+1}^{PH^6} = \frac{\sum_{s=1}^{T_{\S 140}} PS_{t+1-s}}{\sum_{s=1}^{T_{\S 140}} \max\left(Sp_{t+1-s}; 0\right)}$$

den Anteil der Verluste, der durch die Kunden getragen wird. Dieser Anteil ermittelt sich aus dem Verhältnis der Zuteilung des Rohüberschusses an die Kunden und dem gesamten Rohüberschuss (sofern positiv). Betrachtet werden die letzten $T_{\S 140}$ Jahre. Zunächst wird aus der $fRfB$ ($^{\S 140, fRfB}CF_{t+1}$) entnommen

$$^{\S 140, fRfB}CF_{t+1} = \min\left(fRfB_{t+1} + PS_{t+1}; q_{t+1}^{PH} \cdot \max\left(-Sp_{t+1}; 0\right)\right).$$

Reicht dies nicht, um die Verluste auszugleichen, wird zusätzlich von den noch nicht zugeteilten Schlussüberschussbeteiligungen ($^{\S 140, TBF}CF_{t+1}$) entnommen

$$^{\S 140, TBF}CF_{t+1} = \min\left(TBF_t - L_{t+1}^{term}; q_{t+1}^{PH} \cdot \max\left(-Sp_{t+1}; 0\right) - {}^{\S 140, fRfB}CF_{t+1}\right).$$

Dabei wird angenommen, dass der Schlussüberschuss (L_{t+1}^{term}), der bereits ein Jahr vor Ablauf dem auslaufenden Vertrag zugeteilt wird, im TBF bis zur Auszahlung verbleibt, aber nicht zum Ausgleich von Verlusten verwendet werden darf.

3. Die Dividendenzahlung an den Aktionär setzt sich aus dem Aktionärsanteil des Rohüberschusses und der Rendite des Eigenkapitals zusammen. Außerdem ist im Modell eine Eigenkapitalquote implementiert

$$q_t^{loc} = \frac{Eq_t^{loc}}{AR_t}.$$

Jährlich wird das Eigenkapital entsprechend angepasst. Diese Anpassung fließt in die Berechnung der Dividendenzahlung mit ein

$$X_{t+1} = (Sp_{t+1} - PS_{t+1}) + {}^{\S 140, fRfB/TBF}CF_{t+1} + i_{t+1}^{*} \cdot Eq_t^{loc} + (Eq_t^{loc} - Eq_{t+1}^{loc}). \qquad (2)$$

4. Am Jahresende findet die Überschussdeklaration bon_{t+1} statt. Die Höhe der Deklaration wird aus der Zuteilung der Überschüsse zur *RfB* der vergangenen T_{bon} Jahre ermittelt

$$bon_{t+1} = \frac{1}{T_{bon}} \cdot \sum_{i=0}^{T_{bon}-1} \frac{PS_{t+1-i}}{AR_{t-i} + BR_{t-i}} \cdot (AR_t + BR_t).$$

Falls die Reduzierung der *RfB* durch die Entnahme der Überschussdeklaration zu einer Reservequote außerhalb des Korridors $\left[q_{min}^{fRfB}; q_{max}^{fRfB}\right]$ führt, wird die Überschussbeteili-

gung entsprechend angepasst.[8] Die Aufteilung des Überschusses in laufenden und Schlussüberschuss ($^{acc}bon_{t+1}$, $^{term}bon_{t+1}$) erfolgt anhand eines festgelegten Faktors q^{TBF}. Zuletzt findet die Zuteilung auf die einzelnen Verträge statt. Wie in Burkhart et al. (2017) wird ein sogenanntes natürliches Beteiligungssystem (siehe Hansen und Miltersen (2002)) verwendet. Das bedeutet, dass alle Verträge einer bestimmten Generation i denselben Überschuss erhalten. Die Höhe der Überschüsse von Verträgen unterschiedlicher Generationen kann sich unterscheiden, insbesondere wird zwischen Neugeschäft und Bestandskunde differenziert. Der Aufteilungsmechanismus ist wie folgt: Die Überschüsse werden so aufgeteilt, dass alle Bestandskunden – sofern möglich – dieselbe Gesamtverzinsung erhalten. Für das Neugeschäft wird bei ausreichendem erwirtschafteten Überschuss eine höhere Gesamtverzinsung (Gesamtverzinsung der Bestandskunden plus Spread s) deklariert. Bei nicht ausreichendem Überschuss wird der deklarierte Überschuss bei allen Verträgen gleichermaßen gekürzt, wobei jeder Vertrag mindestens den Garantiezins erhält.

Somit ergibt sich für die Ansammlung der laufenden Überschüsse von Generation i

$$BR^i_{t+1} = BR^i_t \cdot (1 + g^i) + {}^{acc}bon^i_{t+1}$$

mit $BR^i_{t^i_0} = 0$. Die Ansammlung der Schlussüberschüsse von Generation i zum Zeitpunkt $t + 1$ ist

$$TBF^i_{t+1} = TBF^i_t + {}^{term}bon^i_{t+1} - {}^{\S 140, TBF}CF^i_{t+1}.$$

Zu den Zeitpunkten $t \in \{t^i_0, \ldots, t^i_0 + T^i - 1\}$ ergibt sich das Vertragsguthaben als

$$L^i_t = AR^i_t + BR^i_t + TBF^i_{t-1},$$

was zu folgender Ablaufleistung von Generation i führt:

$$L^i_{t^i_0+T^i} = G^i + BR^i_{t^i_0+T^i} + TBF^i_{t^i_0+T^i-1}.$$

5. Zuletzt wird die *fRfB* um den Anteil der Rohüberschüsse, die an die Kunden fließen, erhöht und um die Geldmittel, die im Zuge der Anwendung von §140 VAG benutzt werden, sowie den deklarierten Überschüssen reduziert:

$$fRfB_{t+1} = fRfB_t + PS_{t+1} - {}^{\S 140, fRfB}CF_{t+1} - bon_{t+1}.$$

[8] Die Verwendung eines Korridors für die Reservequote beruht auf Kling et al. (2007). Die Reservequote ist proportional zur Summe aus Deckungskapital und laufender Überschussbeteiligung.

3. Ergebnisse

3.1. Parameter und Ausgangsbilanz

Der Bewertungszeitpunkt der Analysen t^* (= 0) ist der 31. Dezember 2018, unmittelbar vor dem Vertragsbeginn von Generation 20 (siehe Abbildung 1). Es werden die gleichen Parameter wie in Eckert et al. (2020b) verwendet, welche im Einklang mit der aktuellen Regulatorik und der marktüblichen Praxis auf dem deutschen Lebensversicherungsmarkt sind (siehe Tabelle 1).

q^{stock}	T_B	d	q_+^{UGL}	q_-^{UGL}	$T_{\S 140}$	T_{bon}	q_{min}^{fRfB}	q_{max}^{fRfB}	q^{TBF}	q^{loc}
10%	15 Jahre	50%	15%	15%	10 Jahre	5 Jahre	1.5%	4.5%	$\frac{1}{3}$	0.85%

Tabelle 1.: Überblick über die Parameter für die Modellierung der Aktiv- und Passivseite.

Bei der Kalkulation der Verträge der Generationen 1 bis 19 wird als Rechnungszins und Garantiezins jeweils der zu Vertragsabschluss gültige Höchstrechnungszins der Lebensversicherung in Deutschland verwendet (siehe Tabelle 2). Für das Neugeschäft (Generationen 20) werden in Abschnitt 3.2 verschiedene Pricing-Strategien verglichen.

Generation i	1	2 bis 4	5 bis 7	8 bis 12	13 bis 15	16, 17	18, 19
Rechnungszins g^i	4.00%	3.25%	2.75%	2.25%	1.75%	1.25%	0.90%

Tabelle 2.: Rechnungszins der einzelnen Vertragsgenerationen.

Die Parameter für das Finanzmarktmodell (siehe Abschnitt 2.1) sind passend zum Bewertungszeitpunkt gewählt (siehe Tabelle 3). Für die Wahl der Parameter r_0, a, σ^r, σ^S und ρ werden die Werte des Branchensimulationsmodells (siehe GDV (2016) und DAV (2018)) verwendet. Für die Risikoprämie λ_A wird dem Ansatz von Korn und Wagner (2018) gefolgt und λ_r wird gleich Null gesetzt. Zur Kalibrierung des zeitabhängigen Parameters θ_t wird die Zinsstrukturkurve vom 31. Dezember 2018, die von der EIOPA (European Insurance and Occupational Pensions Authority) für den deutschen Markt zur Verfügung gestellt wird[9], verwendet. Wie in Eckert et al. (2020b) werden 100,000 Szenarien des Finanzmarktes mit einer Zeitspanne von 20 Jahren stochastisch simuliert.

Zur Ermittlung der Ausgangssituation zum Zeitpunkt 0, also insbesondere für das Eigenkapital Eq_0^{loc}, die freie Rückstellung für die Beitragsrückerstattung $fRfB_0$, die Schlussüberschussbeteiligung TBF_0, das Deckungskapital AR_0 und die laufende Überschussbeteiligung BR_0, wird

[9] European Insurance and Occupational Pensions Authority: https://www.eiopa.europa.eu/tools-and-data/risk-free-interest-rate-term-structures_en: EIOPA_RFR_20181231_Term_Structures.xlsx, Zugriff im März 2020.

r_0	a	σ^r	S_0	σ^S	ρ	λ_r	λ_A
0.0%	10%	1.48%	1.00	16.95%	20%	0.00	0.04

Tabelle 3.: Übersicht über die Parameter des Finanzmarktmodells.

ein deterministisches Szenario für die Jahre von 2000 bis 2018 verwendet, welches auf historischen Werten deutscher Lebensversicherer beruht (siehe Tabelle 8, Anhang A). Zum Zeitpunkt 0 sind die Buchwerte der Aktiva gleich den Buchwerten der Passiva. Wie in Burkhart et al. (2017) wird 25% als Wert für die stillen Reserven der Aktien angenommen. Die Kuponzinsen der Kuponbonds zum Zeitpunkt 0 basieren auf historischen Daten (siehe Tabelle 9, Anhang B). Die Buchwerte der Kuponbonds werden gleichmäßig auf Bonds mit Laufzeit zwischen 1 und $T_B = 15$ Jahren aufgeteilt. In Tabelle 4 ist die resultierende Bilanz zum Zeitpunkt $t^* = 0$ abgebildet.

Aktiva	**Passiva**
$BV_0^S = 17,321,127$ €	$Eq_0^{loc} = 1,487,793$ €
$BV_0^B = 172,716,826$ €	$X_0 = 536,642$ €
	$fRfB_0 = 6,966,448$ €
	$TBF_0 = 1,919,041$ €
	$AR_0 = 175,034,521$ €
	$BR_0 = 4,093,507$ €

Tabelle 4.: Ausgangsbilanz zum Bewertungszeitpunkt 31. Dezember 2018.

Der Marktwert der Aktien zum Zeitpunkt 0 beträgt 21,651,408 € und der Marktwert der Bonds zum Zeitpunkt 0 ist 194,862,674 €.

3.2. Strategien für die Kalkulation des Neugeschäfts

In diesem Abschnitt werden vier verschiedene Strategien für das Pricing des Neugeschäfts gegenübergestellt und analysiert. Beim Neugeschäft handelt es sich weiterhin um klassische Erlebensfallversicherungen mit Überschussbeteiligung, bei denen nur die Höhe des Garantiezinses und des Spreads bei der Überschussdifferenzierung angepasst werden. Alle anderen Parameter, insbesondere auch die in Abschnitt 2.4 beschriebene Berechnung des Deckungskapitals, des laufenden Überschusses und des Schlussüberschusses bleiben unverändert. Wie im Ergebnisbericht der DAV (siehe DAV (2017)) beschrieben, wird von einem konstanten Spread über die gesamte Vertragslaufzeit ausgegangen, der einmalig zu Vertragsbeginn ermittelt wird.

Strategie A (Keine Reduktion der Garantie)

Das Neugeschäft hat als Rechnungszins den bei Vertragsabschluss im Jahr 2019 gültigen Höchstrechnungszins von 0.90%. Es wird keine Differenzierung der Überschussbeteiligung zwischen Bestand und Neugeschäft vorgenommen, d. h. der Spread s^A ist 0%.

Strategie B (Reduktion der Garantie ohne Spread)

Das Neugeschäft hat einen Rechnungszins von 0%. Es wird keine Differenzierung der Überschussbeteiligung vorgenommen, d. h. der Spread s^B ist 0%.

Strategie C (Reduktion der Garantie mit individuell kalkuliertem Spread)

Der Rechnungszins des Neugeschäfts wird auf 0% reduziert. Im Gegenzug erhalten die Verträge von Generation 20 eine höhere Überschussbeteiligung. Zur Ermittlung des Spreads s^C bei der Überschussbeteiligung wird Generation 20 *individuell*, also ohne die anderen Vertragsgenerationen/ohne die Bestandskunden bewertet. Es gibt somit nur eine Interaktion zwischen dem Neugeschäft und dem Aktionär. Zunächst wird Generation 20 *individuell* nach Strategie A bewertet. Anschließend wird in einer zweiten Bewertung der Rechnungszins von Generation 20 auf 0% reduziert und die Überschussbeteiligung solange erhöht, bis eine Wertgleichheit im Sinne von DAV (2017)[10] bei beiden voneinander unabhängigen Bewertungen vorliegt:

$$\begin{aligned} &E_{\mathcal{Q}}\left[L_{20}^{20}\left(g^{20}=0.90\%; s^A=0\%\right)\cdot B_{20}^{-1}-\sum_{j=0}^{19}P_j^{20}\left(g^{20}=0.90\%\right)\cdot B_j^{-1}\right] \\ &\quad = E_{\mathcal{Q}}\left[L_{20}^{20}\left(g^{20}=0.00\%; s^C\right)\cdot B_{20}^{-1}-\sum_{j=0}^{19}P_j^{20}\left(g^{20}=0.00\%\right)\cdot B_j^{-1}\right]. \end{aligned}$$

In Eckert et al. (2020a) wird gezeigt, dass für Generation 20, das Neugeschäft, gilt

$$CB_0^{20}\,(\text{ex ante}) = E_{\mathcal{Q}}\left[L_{20}^{20}\cdot B_{20}^{-1}-\sum_{j=0}^{19}P_j^{20}\cdot B_j^{-1}\right].$$

[10] Nach dem Ergebnisbericht der DAV (siehe DAV (2017)) ist eine „Wertgleichheit“ gegeben, wenn bei einer stochastischen Simulation der Erwartungswert der diskontierten Leistungen bereinigt um die Prämienhöhe gleich ist. Dabei wird der Erwartungswert unter dem risikoneutralen Maß $\mathcal{Q}$ gebildet, da beim Vergleich des Erwartungswertes unter $\mathcal{P}$ ein risikoaverser Kunde stets das Produkt mit der höheren Garantie wählen würde.

Somit entspricht der ex ante Kollektivbonus[11] der zu erwartenden diskontierten Leistung bereinigt um die Prämienhöhe unter $\mathcal{Q}$ und ist zur Bestimmung des Spreads geeignet. Bei einem Spread $s^C = 0.36\%$ sind Strategie A und C bei *individueller* Betrachtung der Verträge wertgleich: Das Neugeschäft hat jeweils einen ex ante Kollektivbonus von 361,798 €[12]. In Anhang C ist die Auswirkung der Überschussdifferenzierung auf die Entwicklung des Vertragsguthabens dargestellt.

Strategie D (Reduktion der Garantie mit kollektiv kalkuliertem Spread)

Auch bei dieser Strategie findet eine Reduktion des Garantiezinses auf 0% und eine Erhöhung der Überschussbeteiligung statt. Der Spread wird nun unter der Annahme ermittelt, dass die Generationen 1 bis 19 bereits vorhanden sind. Im Folgenden wird diese Bewertung *kollektiv* genannt. Betrachtet man den ex ante Kollektivbonus (*kollektiv*) nach Strategie A, so ergibt sich 777,113 € und für Strategie C 336,901 €. *Individuell* betrachtet sind Strategie A und C wertgleich, *kollektiv* betrachtet jedoch nicht. Für Strategie D wird der Spread bei der Überschussbeteiligung nun so ermittelt, dass für Generation 20 eine Wertgleichheit nach Strategie A und Strategie D bei *kollektiver* Betrachtung vorliegt (Vorgehen analog zu Strategie C mit dem Unterschied, dass nun die Interaktion mit den Bestandskunden bei den Bewertungen mit einbezogen wird). Dies ergibt einen Spread s^D von 0.97% (siehe dazu Anhang C).

Strategie E (Kein Neugeschäft)

Es gibt kein Neugeschäft. Die stochastische Simulation endet somit nach 19 Jahren mit dem Ablauf von Generation 19.

3.3. Auswirkungen der unterschiedlichen Strategien auf den Bestand und den Aktionär

Um die Auswirkungen der unterschiedlichen Neugeschäftsstrategien messbar zu machen, wird erneut das Konzept des Kollektivbonus (siehe Eckert et al. (2020a)) verwendet. Der ex ante

[11] Konzept des Kollektivbonus: Der ex ante Kollektivbonus eines Vertrages gibt an, wie sehr dieser Vertrag in Erwartung von anderen Verträgen profitiert, bzw. wie sehr er diese in Erwartung subventioniert. Der ex post Kollektivbonus misst, wie sehr dieser Vertrag in der Vergangenheit von anderen Verträgen profitiert hat, bzw. wie sehr er diese subventioniert hat.

[12] Es wird von einer fairen Bewertung des Neugeschäfts gesprochen, wenn der *individuell* ermittelte Kollektivbonus des Neugeschäfts 0 ist. Dies entspricht einer Situation, in der weder das Neugeschäft in Erwartung den Aktionär subventioniert noch umgekehrt (siehe Eckert et al. (2020a)). Das Neugeschäft nach Strategie A ist unter den getroffenen Annahmen nicht fair bewertet. In Abschnitt 3.5 wird ein fair bewertetes Neugeschäft untersucht.

Kollektivbonus von Vertragsgeneration i ($i = 1, \ldots, 19$) zum Bewertungszeitpunkt $t^* = 0$ ergibt sich als

$$CB_0^i\,(\text{ex ante}) = E_{\mathcal{Q}}\left[\sum_{j=0}^{t_0^i+T^i-1} L_j^i \cdot B_{j+1}^{-1} \cdot \left(\frac{L_{j+1}^i}{L_j^i + P_j^i} - \frac{MV_{j+1}^{A-}}{MV_j^{A+}}\right)\right].$$

Der Kollektivbonus ermittelt sich als Differenz aus der Entwicklung des Vertragsguthabens und einem theoretischen Investment der Prämien in das Asset Portfolio des Versicherungsunternehmens. Für den ex ante Kollektivbonus (CB_0^{sh} (ex ante)) des Aktionärs wird der sogenannte „Present Value of Future Profits“ (PVFP) verwendet (siehe Eckert et al. (2020a)), welcher sich aus den erwarteten zukünftigen Zahlungen aus Sicht des Aktionärs berechnet

$$CB_0^{sh}\,(\text{ex ante}) = PVFP_0 = E_{\mathcal{Q}}\left[\sum_{j=0}^{T} X_j \cdot B_j^{-1}\right].$$

Zum Zeitpunkt T endet der letzte Vertrag des Versicherungsunternehmens.

	Strategie A	**Strategie B**	**Strategie C**	**Strategie D**	**Strategie E**
Garantie| Spread	0.90%| 0.00%	0.00%| 0.00%	0.00%| 0.36%	0.00%| 0.97%	Kein Neugeschäft
Gen. 1	1,260,944 €	1,260,504 €	1,260,502 €	1,260,499 €	1,265,367 €
Gen. 2	1,765,483 €	1,762,394 €	1,761,247 €	1,759,291 €	1,772,681 €
Gen. 3	2,237,687 €	2,234,836 €	2,233,698 €	2,231,775 €	2,243,344 €
Gen. 4	2,634,976 €	2,632,640 €	2,631,562 €	2,630,693 €	2,638,921 €
Gen. 5	2,761,322 €	2,757,916 €	2,756,008 €	2,752,971 €	2,768,926 €
Gen. 6	2,913,951 €	2,911,467 €	2,909,628 €	2,906,713 €	2,919,764 €
Gen. 7	3,001,130 €	2,999,786 €	2,998,027 €	2,995,283 €	3,004,811 €
Gen. 8	2,691,579 €	2,689,171 €	2,685,126 €	2,678,591 €	2,703,404 €
Gen. 9	2,614,367 €	2,613,392 €	2,609,251 €	2,602,617 €	2,622,605 €
Gen. 10	2,511,040 €	2,511,576 €	2,507,360 €	2,500,679 €	2,515,261 €
Gen. 11	2,399,441 €	2,401,155 €	2,396,849 €	2,390,120 €	2,409,816 €
Gen. 12	2,273,994 €	2,277,304 €	2,272,850 €	2,265,932 €	2,285,552 €
Gen. 13	1,603,080 €	1,605,246 €	1,594,783 €	1,578,124 €	1,611,735 €
Gen. 14	1,454,106 €	1,458,983 €	1,447,498 €	1,429,102 €	1,461,400 €
Gen. 15	1,343,851 €	1,352,038 €	1,338,684 €	1,317,036 €	1,361,102 €
Gen. 16	728,834 €	733,276 €	706,759 €	663,926 €	773,013 €
Gen. 17	690,307 €	699,479 €	667,572 €	615,494 €	761,835 €
Gen. 18	355,200 €	356,034 €	305,450 €	223,562 €	511,412 €
Gen. 19	426,809 €	436,098 €	369,580 €	262,137 €	865,003 €
Summe 1-19 (Bestand)	35,668,104 €	35,693,296 €	35,452,437 €	35,064,544 €	36,495,949 €
Gen. 20	777,113 €	67,806 €	336,901 €	777,113 €	-
Aktionär	6,189,427 €	6,873,542 €	6,845,306 €	6,792,987 €	6,138,695 €

Tabelle 5.: Übersicht über die ex ante Kollektivboni für Strategie A bis E.

In Tabelle 5 sind die ex ante Kollektivboni für die Generationen 1 bis 20 und dem Aktionär für die unterschiedlichen Strategien abgebildet. Im Folgenden werden einzelne Aspekte von Tabelle 5 genauer analysiert.

Situation ohne Neugeschäft (Strategie E)

Da die vom Bestand aufgebauten Reserven (Bewertungsreserven, *fRfB*) in der Situation ohne Neugeschäft zwischen einer geringeren Anzahl an Vertragsgenerationen aufgeteilt werden, sind in der Situation ohne Neugeschäft unter der Voraussetzung, dass alle Managementregeln (siehe Abschnitt 2 und Abschnitt 3.1) unverändert bleiben, die ex ante Kollektivboni und somit die erwartete Rentabilität des Bestandes am größten. Im Gegensatz dazu sinkt der erwartete Profit des Aktionärs im Vergleich zu allen betrachteten Situationen mit Neugeschäft. Der Aktionär wird in Erwartung vom Neugeschäft subventioniert (siehe dazu Eckert et al. (2020b)).

Ex ante Kollektivbonus von Generation 20 und des Aktionärs

Durch eine Senkung des Garantiezinses reduziert sich der ex ante Kollektivbonus des Neugeschäfts von 777,113 € (Strategie A) auf 67,806 € (Strategie B) (siehe Abbildung 3 (links)). Da der Aktionär bei schlechter Kapitalmarktperformance des Asset Portfolios weniger Garantie finanzieren muss, profitiert er in Erwartung von einem niedrigeren Garantiezins des Neugeschäfts (siehe Abbildung 3 (rechts)), und sein ex ante Kollektivbonus steigt bei Strategie B um 684,115 € im Vergleich zu Strategie A.

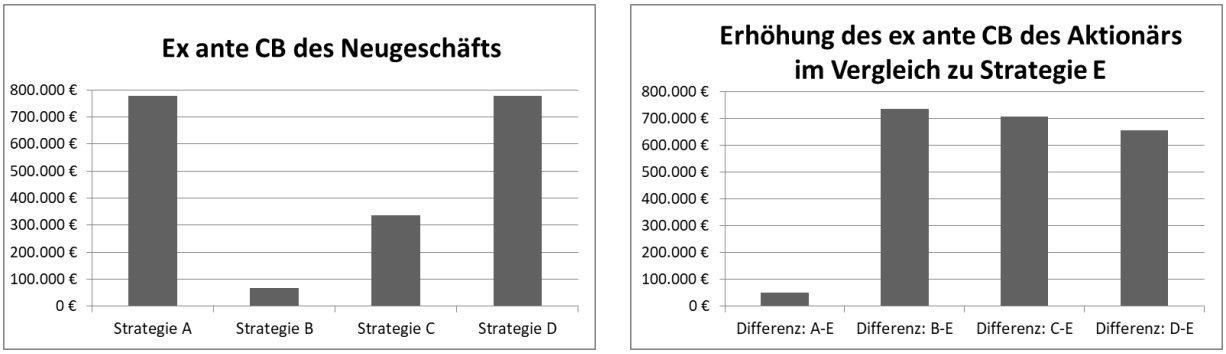


Abbildung 3.: Ex ante Kollektivbonus von Gen. 20 in A bis D (links) und Erhöhung des ex ante Kollektivbonus des Aktionärs in A bis D im Vergleich zu Strategie E (rechts).

Eine höhere Überschussbeteiligung für das Neugeschäft reduziert den erwarteten Profit des Aktionärs und erhöht die erwartete Rentabilität des Neugeschäfts, wobei der Anstieg der erwarteten Rentabilität bei Strategie C und D (im Vergleich zu B) größer als die Reduktion des erwarteten Profits des Aktionärs ist. Die Überschüsse fließen größtenteils in die mit dem

Garantiezins verzinste laufende Überschussbeteiligung, welche bei höherer Überschussbeteiligung entsprechend größer ist. Dies reduziert den Rohüberschuss und somit die Dividende des Aktionärs.

Vergleich ohne (Strategie A) und mit Überschussdifferenzierung (Strategie C und D)

In Abbildung 4 ist die Differenz der ex ante Kollektivboni der Generationen 1 bis 19 bei Strategie A und C, bzw. Strategie A und D, dargestellt. Die Differenzen sind positiv: Durch das Absenken der Garantie des Neugeschäfts und der Einführung einer Differenzierung der Überschussbeteiligung reduzieren sich die ex ante Kollektivboni des Bestandes.

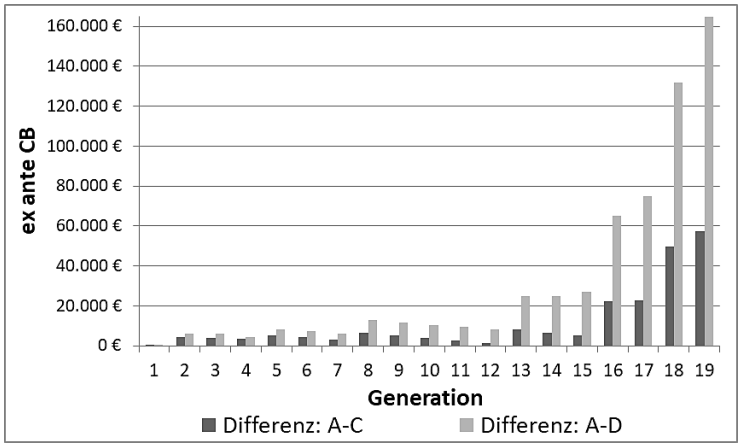


Abbildung 4.: Vergleich ex ante Kollektivbonus bei Strategie A und C bzw. A und D.

Der höhere Spread in Strategie D im Vergleich zu Strategie C führt zu einer stärkeren Abnahme der erwarteten Rentabilität des Bestandes. Das Neugeschäft hat in Strategie D einen höheren ex ante Kollektivbonus als in Strategie C (siehe Abbildung 3). Speziell die jüngeren Vertragsgenerationen, welche die längste Zeit mit Generation 20 interagieren, subventionieren das Neugeschäft. Bei guter Performance des Asset Portfolios ist die Überschussbeteiligung der Bestandskunden in Strategie C, D leicht geringer als in A, da der Neukunde eine höhere Beteiligung erhält. Bei schlechter Performance des Asset Portfolios bekommen die Bestandskunden sowohl in Strategie A als auch in C und D ihre garantierte Zuteilung. Dies führt insgesamt zu einer Abnahme der erwarteten Rentabilität des Bestandes.

Ex post Betrachtung: Vergleich von Strategie A und Strategie C

In diesem Abschnitt wird der ex post Kollektivbonus[13] des Neugeschäfts, des Bestandes (exemplarisch Generation 10) und des Aktionärs für Strategie A und Strategie C betrachtet:

$$CB_t^i\,(\text{ex post}) = L_t^i - \sum_{j=t_0^i}^{t-1} P_j^i \cdot \prod_{k=j+1}^{t} \frac{MV_k^{A-}}{MV_{k-1}^{A+}}.$$

Nach Eckert et al. (2020a) ergibt sich der ex post Kollektivbonus des Aktionärs als

$$CB_t^{sh}\,(\text{ex post}) = \sum_{j=t_0^1+1}^{t-1} X_j \cdot \prod_{k=j+1}^{t} \frac{MV_k^{A-}}{MV_{k-1}^{A+}}.$$

Der ex post Kollektivbonus wird jeweils zum Ablaufzeitpunkt der einzelnen Vertragsgenerationen betrachtet. Das bedeutet für Generation 10 wird der ex post Kollektivbonus zum Ende des Jahres 2028 (unmittelbar nach der Auszahlung der Leistung) und für Generation 20 sowie den Aktionär zum Ende des Jahres 2038 ermittelt. In Abbildung 5 sind die Wahrscheinlichkeitsverteilungen des ex post Kollektivbonus in Strategie A und C dargestellt.

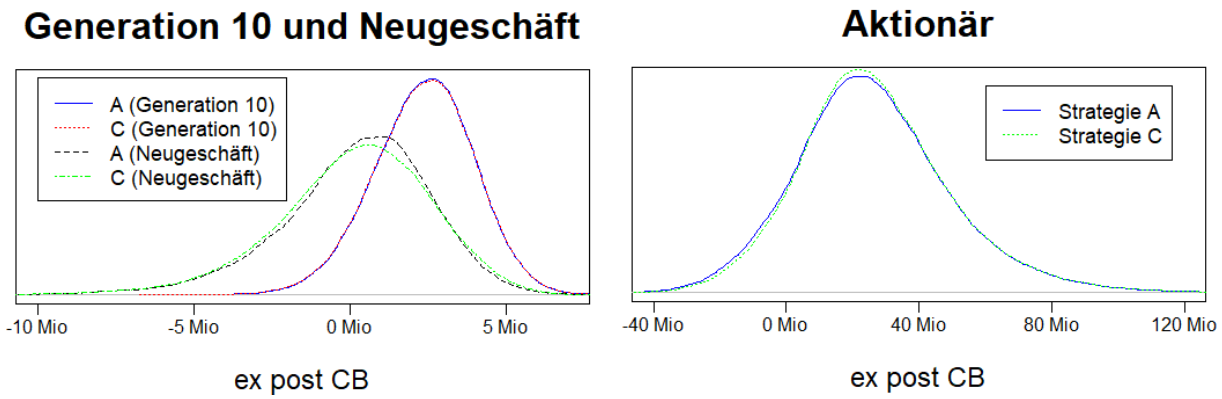


Abbildung 5.: Wahrscheinlichkeitsverteilungen des ex post Kollektivbonus in A und C.

Durch die Reduktion des Rechnungszinses und die Erhöhung der Überschussbeteiligung ergibt sich eine höhere Volatilität beim ex post Kollektivbonus des Neugeschäfts. Die Differenzierung der Überschussbeteiligung führt zu einer Reduktion des ex post Kollektivbonus von Generation 10 (minimale Verschiebung nach links). Beim Aktionär bewirkt die Reduktion der garantierten Verzinsung des Neugeschäfts besonders in Kapitalmarktszenarien, in denen der Aktionär einen negativen ex post Kollektivbonus hat, er somit Garantien auffüllen muss, eine Erhöhung.

Als Nächstes wird die Wahrscheinlichkeitsverteilung der Differenz aus dem ex post Kollektivbonus in Strategie A und dem ex post Kollektivbonus in Strategie C (siehe Abbildung 6)

[13] Der ex post Kollektivbonus für Generation i zum Zeitpunkt t ergibt sich als Differenz aus der Entwicklung des Vertragsguthabens bis zum Zeitpunkt t und einem theoretischen Investment der Prämien in das Asset Portfolio des Versicherungsunternehmens vor dem Zeitpunkt t (siehe Eckert et al. (2020a)).

betrachtet.

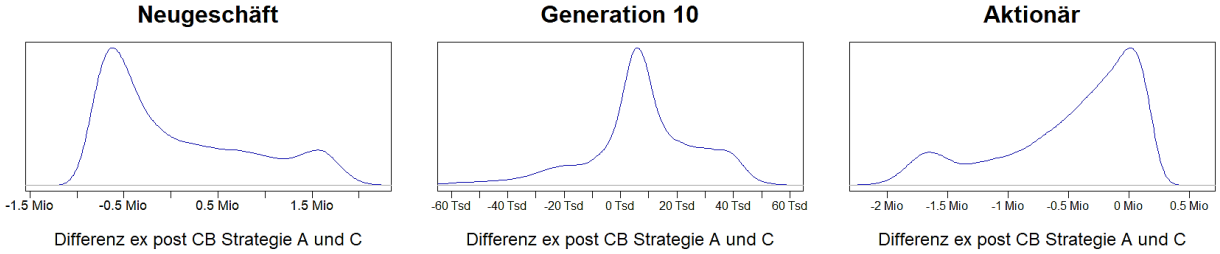


Abbildung 6.: Wahrscheinlichkeitsverteilungen der Differenz aus ex post Kollektivbonus in Strategie A und in Strategie C für das Neugeschäft, Generation 10 und den Aktionär.

Aufgrund der erhöhten Überschussbeteiligung ist in der Mehrheit der Szenarien der ex post Kollektivbonus des Neugeschäfts in Strategie C größer als in Strategie A, wobei dieser Unterschied nur gering ist. Kommt in einem Szenario vermehrt der Garantiewert des Vertrages zum Tragen, so ist Strategie A für das Neugeschäft deutlich rentabler. Für Generation 10 hat die Verteilung eine Gewichtung nach rechts: Strategie A führt für den Bestandskunden in der Mehrheit der Szenarien zu einem höheren ex post Kollektivbonus. Die Wahrscheinlichkeitsverteilung des Aktionärs hat eine starke Gewichtung nach links. Er interagiert über einen langen Zeitraum mit dem Neugeschäft. Muss der Aktionär Garantien auffüllen, so ist für ihn Strategie C profitabler als Strategie A. Werden jedoch Überschüsse erwirtschaftet, kehrt sich dies um.

3.4. Möglichkeiten, die Abnahme der Rentabilität des Bestandes zu kompensieren

Abschnitt 3.3 hat ergeben, dass die erwartete Rentabilität der Bestandskunden bei einer Neugeschäftsstrategie mit abgesenkter Garantie und erhöhter Überschussbeteiligung leicht geringer ausfällt als bei einer Neugeschäftsstrategie ohne Reduktion der Garantie und ohne Differenzierung der Überschussbeteiligung.[14] In diesem Abschnitt werden zwei Möglichkeiten vorgestellt, wie die Abnahme der erwarteten Rentabilität der Bestandskunden kompensiert werden kann und gleichzeitig für das Neugeschäft eine Wertgleichheit zu Strategie A vorhanden ist.

Chancenreichere Kapitalanlage

Die niedrigere Garantie des Neugeschäfts bei Strategie C ermöglicht dem Versicherungsunternehmen eine chancenreichere Kapitalanlage. Bisher wurde eine Aktienquote von 10% der Marktwerte aller Assets angenommen. In diesem Abschnitt werden zwei Alternativen zu Strategie C

[14] Bei einer Neugeschäftsstrategie mit Rechnungszins von 0% und einem Spread von 0.04% ist die Summe der ex ante Kollektivboni der Bestandskunden genauso groß wie bei Strategie A. Allerdings gilt für diese Neugeschäftsstrategie keine Wertgleichheit nach DAV (2017).

betrachtet: Zum einen wird die Aktienquote auf 11% (Strategie C (11%)) und zum anderen auf 12.5% (Strategie C (12.5%)) erhöht. Dabei werden bei der Berechnung des Ausgangszustands zum Zeitpunkt 0 die stillen Reserven der Aktien reduziert, sodass das Versicherungsunternehmen dieselben Bewertungsreserven wie zuvor hat (die Bewertungsreserven der Bonds bleiben unverändert). Wie in Abschnitt 3.2 unter Strategie C beschrieben, wird der Spread mit den erhöhten Aktienquoten neu ermittelt. Man erhält für Strategie C (11%) einen Spread von 0.35% und für Strategie C (12.5%) ist der Spread 0.33%. Eine höhere Aktienquote führt zu einem niedrigeren Spread.

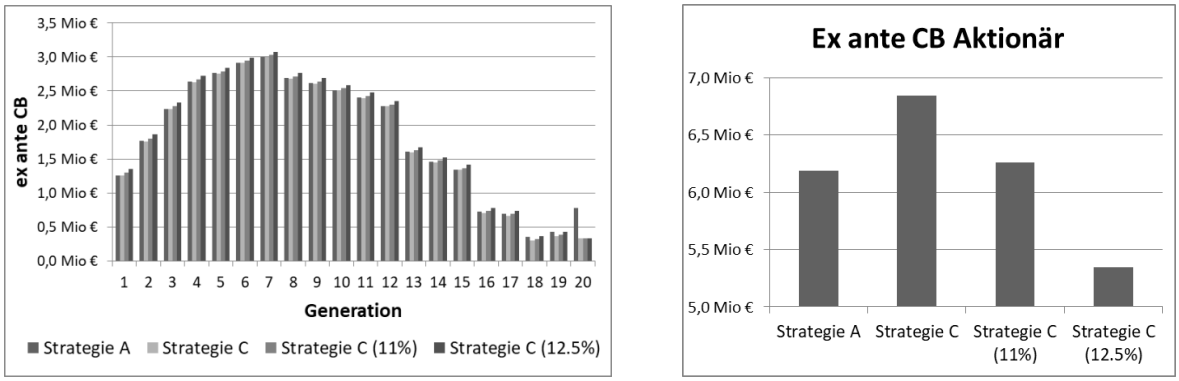


Abbildung 7.: Übersicht der ex ante Kollektivboni des Aktionärs und der Vertragsgenerationen bei verschiedenen Aktienquoten.[15]

Das Neugeschäft hat trotz Überschussdifferenzierung und erhöhter Aktienquote bei Strategie C, Strategie C (11%) und Strategie C (12.5%) eine geringere erwartete Rentabilität, verglichen mit Strategie A. Der ex ante Kollektivbonus des Neugeschäfts ist in allen Fällen jedoch positiv. In Erwartung profitiert das Neugeschäft von den Bestandskunden bzw. vom Aktionär. Die höhere Aktienquote führt zu einer Reduktion des erwarteten Profits des Aktionärs und einer Erhöhung der erwarteten Rentabilität der Vertragskunden (siehe Abbildung 7). Bei einer Aktienquote von 11% steigert sich trotz der Einführung einer Überschussdifferenzierung und einer Reduktion der Garantie des Neugeschäfts die erwartete Rentabilität des größten Teils der Bestandskunden im Vergleich zu Strategie A. Auch der erwartete Profit des Aktionärs steigert sich. Bei einer Aktienquote von 12.5% erhöht sich die erwartete Rentabilität aller Bestandskunden. Der Aktionär hat jedoch einen geringeren erwarteten Profit als in A.

Höhere RfB-Zuführung

Eine zweite Möglichkeit, die Abnahme der erwarteten Rentabilität der Bestandskunden auszugleichen, bietet die Erhöhung der *RfB*-Zuführung. Bisher erfolgt die Aufteilung des Rohüberschusses anhand der Mindestzuführungsverordnung, d. h. es wird die Differenz aus 90%

[15]Die genauen Werte der ex ante Kollektivboni sind in Tabelle 10 in Anhang D zu finden.

der Investmenterträge und den garantierten Zinserträgen der *RfB* zugeführt (siehe Gleichung (1)). Strategie C wird angepasst, indem zum einen der Anteil der Investmenterträge auf 92% (Strategie C (92%)) bzw. 95% (Strategie C (95%)) erhöht wird und zum anderen der Spread mit diesen erhöhten *RfB*-Zuführungen neu ermittelt wird. Der Spread bei Strategie C (92%) ist 0.30% und nimmt durch die höhere *RfB*-Zuführung bei Strategie C (95%) auf 0.21% ab.

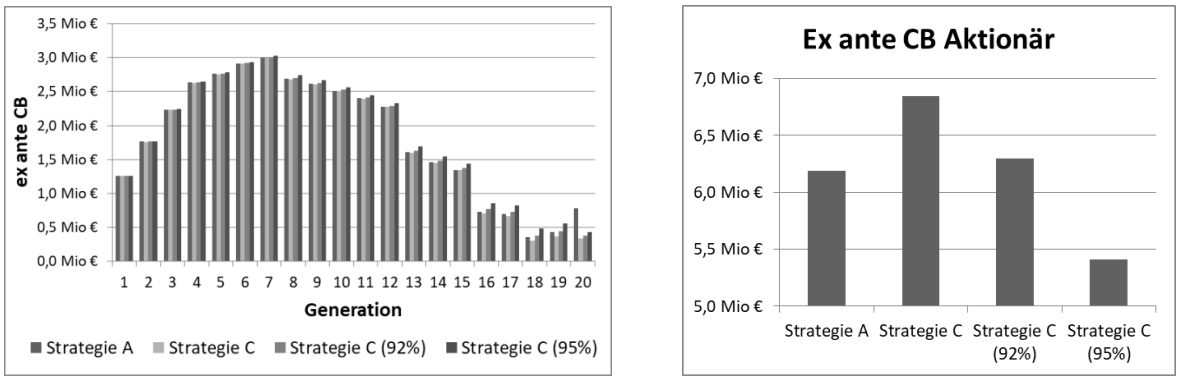


Abbildung 8.: Übersicht der ex ante Kollektivboni des Aktionärs und der Vertragsgenerationen bei verschiedenen *RfB*-Zuführungen.[16]

Das Neugeschäft hat trotz Überschussdifferenzierung und erhöhter *RfB*-Zuführung bei Strategie C und den Alternativen zu C im Vergleich zu Strategie A eine geringere erwartete Rentabilität, profitiert jedoch erneut in allen betrachteten Fällen von den Bestandskunden bzw. vom Aktionär (positiver ex ante Kollektivbonus) (siehe Abbildung 8 (links)). Für den Aktionär führt eine höhere *RfB*-Zuführung zu einem geringeren Anteil am Rohüberschuss und somit zu einer geringeren Dividendenzahlung. Der ex ante Kollektivbonus reduziert sich bei einer höheren *RfB*-Zuführung, wobei er in Strategie C (92%) noch größer als bei Strategie A ist (siehe Abbildung 8 (rechts)). Im Gegensatz dazu erhöht sich der ex ante Kollektivbonus der Vertragskunden bei einer höheren *RfB*-Zuführung (siehe Abbildung 8 (links)). Bei einer *RfB*-Zuführung von 92% profitieren in Erwartung die Bestandskunden und der Aktionär von einer Neugeschäftsstrategie mit Überschussdifferenzierung und reduzierter Garantie verglichen mit einer Neugeschäftsstrategie ohne Differenzierung und ohne Reduktion der Garantie.

3.5. Sensitivitäten

Auswirkung des Neugeschäftsvolumens auf die Höhe des Spreads

In diesem Abschnitt wird das Volumen des Neugeschäfts modifiziert und jeweils der Spread sowohl *individuell* als auch *kollektiv* ermittelt.

[16]Die genauen Werte der ex ante Kollektivboni sind in Tabelle 11 in Anhang E zu finden.

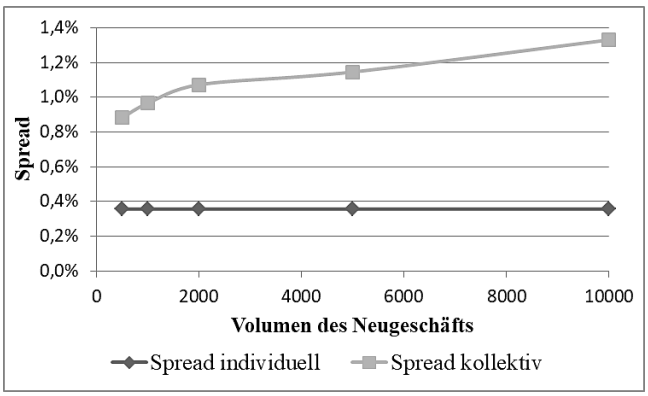


Abbildung 9.: Spread bei *individueller* und *kollektiver* Betrachtung für eine unterschiedliche Anzahl von Neugeschäftsverträgen.

Unabhängig vom Volumen des Neugeschäfts beträgt der Spread bei der *individuellen* Betrachtung stets 0.36% (siehe Abbildung 9). Es gibt keine Interaktion mit dem Bestand, nur eine zwischen Neugeschäft und Aktionär. Daher hat das Volumen des Neugeschäfts keinen Einfluss auf die Höhe des Spreads. Bei der *kollektiven* Spread-Ermittlung wird mit zunehmendem Neugeschäftsvolumen ein höherer Spread benötigt, um dieselbe erwartete Rentabilität für das Neugeschäft bei reduzierter (Rechnungszins: 0%) und nicht reduzierter Garantie (Rechnungszins: 0.90%) zu erhalten.

Faire Bewertung des Neugeschäfts

Wie in Abschnitt 3.2 erwähnt, ist das Neugeschäft nach Strategie A mit einem ex ante Kollektivbonus von 361,798 € nicht fair bewertet. Eine faire Bewertung führt zu einem Garantiezins

	Garantie g^{20}	**Spread s**	**ex ante Kollektivbonus (*individuell*)**	**ex ante Kollektivbonus (*kollektiv*)**
Strategie A (fair)	0.24%	0.00%	0 €	260,032 €
Strategie C (fair)	0.00%	0.07%	0 €	124,378 €
Strategie D (fair)	0.00%	0.27%	248,864 €	260,032 €

Tabelle 6.: Ex ante Kollektivbonus und Spread nach Strategie A (fair), C (fair) und D(fair).

von 0.24% (alle anderen Parameter sind unverändert wie in Strategie A). Ermittelt man nun den Spread nach Strategie C, so beträgt dieser 0.07%. Nach Strategie D ergibt sich ein Spread von 0.27% (siehe Tabelle 6). Aufgrund des geringeren Garantiezinses von Strategie A (fair) sind die Spreads in Strategie C (fair) und Strategie D (fair) entsprechend kleiner. In Tabelle 7 sind die ex ante Kollektivboni der Generationen 1 bis 20 und des Aktionärs gegenübergestellt. Die Strategien A, C und D wurden entsprechend der fairen Bewertung angepasst. Strategie B und E sind unverändert.

	Strategie A (fair)	**Strategie B**	**Strategie C (fair)**	**Strategie D (fair)**	**Strategie E**
Garantie| Spread	0.24%| 0.00%	0.00%| 0.00%	0.00%| 0.07%	0.00%| 0.27%	Kein Neugeschäft
Gen. 1	1,260,625 €	1,260,504 €	1,260,504 €	1,260,503 €	1,265,367 €
Gen. 2	1,763,267 €	1,762,394 €	1,762,155 €	1,761,577 €	1,772,681 €
Gen. 3	2,235,643 €	2,234,836 €	2,234,598 €	2,234,024 €	2,243,344 €
Gen. 4	2,633,303 €	2,632,640 €	2,632,414 €	2,631,871 €	2,638,921 €
Gen. 5	2,758,878 €	2,757,916 €	2,757,511 €	2,756,548 €	2,768,926 €
Gen. 6	2,912,172 €	2,911,467 €	2,911,074 €	2,910,146 €	2,919,764 €
Gen. 7	3,000,174 €	2,999,786 €	2,999,409 €	2,998,520 €	3,004,811 €
Gen. 8	2,689,856 €	2,689,171 €	2,688,313 €	2,686,270 €	2,703,404 €
Gen. 9	2,613,677 €	2,613,392 €	2,612,511 €	2,610,420 €	2,622,605 €
Gen. 10	2,511,440 €	2,511,576 €	2,510,676 €	2,508,547 €	2,515,261 €
Gen. 11	2,400,684 €	2,401,155 €	2,400,232 €	2,398,055 €	2,409,816 €
Gen. 12	2,276,387 €	2,277,304 €	2,276,348 €	2,274,099 €	2,285,552 €
Gen. 13	1,604,584 €	1,605,246 €	1,603,019 €	1,597,738 €	1,611,735 €
Gen. 14	1,457,549 €	1,458,983 €	1,456,538 €	1,450,744 €	1,461,400 €
Gen. 15	1,349,679 €	1,352,038 €	1,349,208 €	1,342,473 €	1,361,102 €
Gen. 16	731,692 €	733,276 €	727,655 €	714,275 €	773,013 €
Gen. 17	696,504 €	699,479 €	692,728 €	676,638 €	761,835 €
Gen. 18	354,683 €	356,034 €	345,274 €	319,768 €	511,412 €
Gen. 19	432,076 €	436,098 €	422,069 €	388,543 €	865,003 €
Summe 1-19 (Bestand)	35,682,875 €	35,693,296 €	35,642,235 €	35,520,759 €	36,495,949 €
Gen. 20	260,032 €	67,806 €	124,378 €	260,032 €	-
Aktionär	6,691,737 €	6,873,542 €	6,868,031 €	6,853,853 €	6,138,695 €

Tabelle 7.: Übersicht über die ex ante Kollektivboni für A (fair), B, C (fair), D (fair) und E.

Auch bei der fairen Bewertung des Neugeschäfts reduziert sich bei einer Differenzierung der Überschussbeteiligung zwischen Neugeschäft und Bestand sowie einer Reduktion des Garantiezinses des Neugeschäfts der ex ante Kollektivbonus des Bestandes, und es erhöht sich der ex ante Kollektivbonus des Aktionärs im Vergleich zu einer Neugeschäftsstrategie ohne Differenzierung und ohne Reduktion des Garantiezinses. Beide Effekte sind aufgrund der geringeren Spreads nicht so stark ausgeprägt.

4. Fazit und Ausblick

In diesem Paper wird die Frage „Werden bei einer Differenzierung der Überschussbeteiligung in der Lebensversicherung Neukunden zu Lasten des Bestands bevorzugt?“ analysiert. Dazu werden 20 verschiedene Generationen von klassischen Lebensversicherungsverträgen mit Überschussbeteiligung betrachtet. Die Generationen 1 bis 19 repräsentieren die Bestandskunden und Generation 20 das Neugeschäft. Es werden verschiedene Alternativen für das Pricing des Neuge-

schäfts vorgestellt, wobei nur die beiden Parameter „Garantiezins des Versicherungsvertrages“ und „Höhe der Überschussbeteiligung“ verändert werden. Dabei werden zwei Möglichkeiten zur Ermittlung des Spreads, welcher die Differenzierung der Überschussbeteiligung zwischen Bestandskunden und Neugeschäft angibt, basierend auf DAV (2017) betrachtet: eine *kollektive*, d. h. mit Berücksichtigung des vorhandenen Bestandes, und eine *individuelle* Ermittlung. Anschließend werden die Auswirkungen der Differenzierung der Überschussbeteiligung analysiert.

Der erwartete Profit des Aktionärs ist bei Neugeschäft mit niedrigerem Garantiezins und höherer Überschussbeteiligung größer als in der Situation mit höherem Garantiezins und niedrigerer Überschussbeteiligung. Ein niedrigerer Garantiezins hat zur Folge, dass der Aktionär in schlechten Kapitalmarktszenarien weniger Kapital zur Verfügung stellen muss. Des Weiteren wird gezeigt, dass sich die erwartete Rentabilität der Bestandskunden bei einer Neugeschäftsstrategie mit abgesenkter Garantie und erhöhter Überschussbeteiligung im Vergleich zu einer Neugeschäftsstrategie ohne Reduktion der Garantie und ohne Differenzierung der Überschussbeteiligung reduziert.

Anschließend werden zwei Möglichkeiten vorgestellt, wie diese Abnahme der erwarteten Rentabilität des Bestandes aufgrund der Überschussdifferenzierung ausgeglichen werden kann: Die Reduktion der Garantie des Neugeschäfts ermöglicht eine chancenreichere Kapitalanlage, von der auch der Bestandskunde profitiert. Eine weitere Möglichkeit bietet die Erhöhung der *RfB*-Zuführung. Durch diese Maßnahmen ist es möglich, dass sich die erwartete Rentabilität der Bestandskunden und der erwartete Profit des Aktionärs durch eine Neugeschäftsstrategie mit Überschussdifferenzierung und reduzierter Garantie, verglichen mit einer Neugeschäftsstrategie ohne Differenzierung und ohne Reduktion der Garantie, erhöhen.

In dieser Arbeit werden die Auswirkungen einer Differenzierung der Überschussbeteiligung auf den Bestandskunden, das Neugeschäft und den Aktionär analysiert. Dabei werden Kosten, Storno und Sterblichkeit nicht betrachtet. Ein weiteres Forschungsfeld bietet die Interaktion zwischen verschiedenen Verträgen aufgrund von Sterblichkeit, Stornowahrscheinlichkeit und Kundenverhalten.

Anhang

A. Buchwertrendite der Kapitalanlage deutscher Lebensversicherer[17]

Zeitpunkt	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Buchwertrendite	7.51%	6.12%	4.68%	5.05%	4.90%	5.18%	4.82%	4.65%	3.54%	4.18%

Zeitpunkt	2010	2011	2012	2013	2014	2015	2016	2017	2018
Buchwertrendite	4.27%	4.13%	4.59%	4.68%	4.63%	4.52%	4.36%	4.49%	3.60%

Tabelle 8.: Überblick der Buchwertrendite der Kapitalanlage deutscher Lebensversicherer.

B. Historische Kuponzinsen[18] der Kuponbonds zum Zeitpunkt 0

Zeitpunkt	2004/12	2005/12	2006/12	2007/12	2008/12	2009/12	2010/12	2011/12
hist. Kuponzinsen	4.39%	3.94%	4.17%	4.94%	4.47%	4.45%	4.21%	3.86%

Zeitpunkt	2012/12	2013/12	2014/12	2015/12	2016/12	2017/12	2018/12
hist. Kuponzinsen	2.91%	3.16%	1.37%	1.48%	1.16%	1.34%	1.34%

Tabelle 9.: Überblick der historischen Kuponzinsen der Kuponbonds zum Zeitpunkt $t^* = 0$.

C. Auswirkung der Überschussdifferenzierung auf die Entwicklung des Guthabens

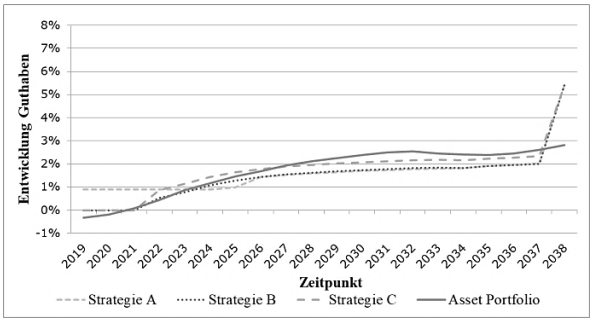
In Abbildung 10 wird die Veränderung des Vertragsguthabens nach Strategie A und Strategie C bei *individueller* Betrachtung (links) sowie Strategie A und Strategie D bei *kollektiver* Betrachtung (rechts) im „certainty equivalent (CE)“ Szenario[19] dargestellt. Um die Auswirkung der Erhöhung der Überschussbeteiligung zu zeigen, ist zusätzlich jeweils die Veränderung des Vertragsguthabens nach Strategie B abgebildet.

Bei der *individuellen* Betrachtung erhalten die Verträge in Strategie B und C in den ersten Jahren ihren Garantiewert von 0% und in Strategie A von 0.90%. Mit steigender Performance des

[17] Quelle: Gesamtverband der Deutschen Versicherungswirtschaft (GDV).

[18] Deutsche Bundesbank Eurosystem: https://www.bundesbank.de/dynamic/action/de/statistiken/zeitreihen-datenbanken/zeitreihen-datenbank/759778/759778?listId=www_skms_it04b, Stand: 02.12.2019.

[19] Das deterministische Szenario entspricht der erwartenden Entwicklung des Kapitalmarkts unter dem risikoneutralen Maß (cf. Oechslin et al. (2007) und Reuß et al. (2015)).



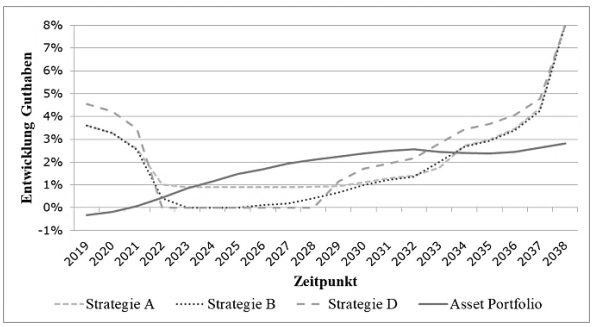


Abbildung 10.: Veränderung des Vertragsguthabens nach Strategie A, B, C und D im CE-Szenario; links *individuelle* Betrachtung und rechts *kollektive* Betrachtung.

Asset Portfolios bekommen die Verträge nach C eine höhere Überschussbeteiligung. Das Vertragsguthaben entwickelt sich mit einem höheren Prozentsatz als in Strategie A und B. Bei der *kollektiven* Betrachtung ist die Entwicklung des Vertragsguthabens in den ersten Jahren trotz niedriger Performance des Asset Portfolios sehr hoch. Dies liegt an den bestehenden Reserven, die von den Generationen 1 bis 19 und dem Aktionär aufgebaut wurden und nun zur Stabilisierung der Überschussbeteiligung verwendet werden (siehe Eckert et al. (2020b)). Anschließend reduziert sich die Entwicklung des Vertragsguthabens bis auf das jeweilige Garantieniveau. Mit steigender Entwicklung des Asset Portfolios erhöht sich auch die Entwicklung der Vertragsguthaben. Hierbei wird wiederum der Einfluss der Überschussdifferenzierung deutlich, welche zu einer höheren Entwicklung des Vertragsguthabens bei Strategie D als bei Strategie A führt. Im Jahr 2038 laufen die Verträge aus, und somit endet auch das Versicherungsunternehmen. Die *fRfB* und noch vorhandene Bewertungsreserven werden aufgelöst. Dies führt zu einem starken Anstieg des Vertragsguthabens.

D. Chancenreichere Kapitalanlage

	Strategie A	**Strategie C**	**Strategie C (11%)**	**Strategie C (12.5%)**
Garantie| Spread	0.90%| 0.00%	0.00%| 0.36%	0.00%| 0.35%	0.00%| 0.33%
Aktienquote	10%	10%	11%	12.5%
Gen. 1	1,260,944 €	1,260,502 €	1,295,395 €	1,348,660 €
Gen. 2	1,765,483 €	1,761,247 €	1,802,203 €	1,863,866 €
Gen. 3	2,237,687 €	2,233,698 €	2,273,286 €	2,332,713 €
Gen. 4	2,634,976 €	2,631,562 €	2,668,907 €	2,724,910 €
Gen. 5	2,761,322 €	2,756,008 €	2,790,787 €	2,842,900 €
Gen. 6	2,913,951 €	2,909,628 €	2,942,320 €	2,991,836 €
Gen. 7	3,001,130 €	2,998,027 €	3,029,705 €	3,078,165 €
Gen. 8	2,691,579 €	2,685,126 €	2,714,536 €	2,760,278 €
Gen. 9	2,614,367 €	2,609,251 €	2,639,064 €	2,685,672 €
Gen. 10	2,511,040 €	2,507,360 €	2,537,723 €	2,585,173 €
Gen. 11	2,399,441 €	2,396,849 €	2,427,298 €	2,474,825 €
Gen. 12	2,273,994 €	2,272,850 €	2,303,065 €	2,350,218 €
Gen. 13	1,603,080 €	1,594,783 €	1,624,973 €	1,672,390 €
Gen. 14	1,454,106 €	1,447,498 €	1,477,323 €	1,524,174 €
Gen. 15	1,343,851 €	1,338,684 €	1,367,541 €	1,412,826 €
Gen. 16	728,834 €	706,759 €	734,434 €	778,102 €
Gen. 17	690,307 €	667,572 €	692,883 €	733,272 €
Gen. 18	355,200 €	305,450 €	327,154 €	362,952 €
Gen. 19	426,809 €	369,580 €	388,743 €	428,217 €
Summe 1-19 (Bestand)	35,668,104 €	35,452,437 €	36,037,340 €	36,951,148 €
Gen. 20	777,113 €	336,901 €	337,669 €	339,237 €
Aktionär	6,189,427 €	6,845,306 €	6,259,634 €	5,344,259 €

Tabelle 10.: Übersicht über die ex ante Kollektivboni bei verschiedenen Aktienquoten.

E. Höhere *RfB*-Zuführung

	Strategie A	**Strategie C**	**Strategie C (92%)**	**Strategie C (95%)**
Garantie| Spread	0.90%| 0.00%	0.00%| 0.36%	0.00%| 0.30%	0.00%| 0.21%
RfB-Zuführung	90%	90%	92%	95%
Gen. 1	1,260,944 €	1,260,502 €	1,260,624 €	1,260,876 €
Gen. 2	1,765,483 €	1,761,247 €	1,764,746 €	1,770,705 €
Gen. 3	2,237,687 €	2,233,698 €	2,238,024 €	2,245,598 €
Gen. 4	2,634,976 €	2,631,562 €	2,635,978 €	2,643,849 €
Gen. 5	2,761,322 €	2,756,008 €	2,766,263 €	2,784,242 €
Gen. 6	2,913,951 €	2,909,628 €	2,919,793 €	2,937,821 €
Gen. 7	3,001,130 €	2,998,027 €	3,007,935 €	3,025,652 €
Gen. 8	2,691,579 €	2,685,126 €	2,706,021 €	2,741,916 €
Gen. 9	2,614,367 €	2,609,251 €	2,629,968 €	2,665,678 €
Gen. 10	2,511,040 €	2,507,360 €	2,527,771 €	2,563,020 €
Gen. 11	2,399,441 €	2,396,849 €	2,416,612 €	2,450,632 €
Gen. 12	2,273,994 €	2,272,850 €	2,292,465 €	2,326,219 €
Gen. 13	1,603,080 €	1,594,783 €	1,630,427 €	1,688,761 €
Gen. 14	1,454,106 €	1,447,498 €	1,484,236 €	1,544,079 €
Gen. 15	1,343,851 €	1,338,684 €	1,377,021 €	1,439,094 €
Gen. 16	728,834 €	706,759 €	763,992 €	853,414 €
Gen. 17	690,307 €	667,572 €	726,222 €	817,703 €
Gen. 18	355,200 €	305,450 €	376,310 €	484,950 €
Gen. 19	426,809 €	369,580 €	442,638 €	554,790 €
Summe 1-19 (Bestand)	35,668,104 €	35,452,437 €	35,967,046 €	36,798,998 €
Gen. 20	777,113 €	336,901 €	373,876 €	426,016 €
Aktionär	6,189,427 €	6,845,306 €	6,293,721 €	5,409,630 €

Tabelle 11.: Übersicht über die ex ante Kollektivboni bei verschiedenen *RfB*-Zuführungen.

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Curriculum Vitae

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Higher Education

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Positions and Practical Experience

Since 02/2016	**Institut für Finanz- und Aktuarwissenschaften**, Ulm Actuarial Consultant
10/2011 - 09/2015	**Ulm University**, Ulm Student assistant
08/2014 - 10/2014	**Allianz Private Krankenversicherungs-AG**, Unterföhring Internship (actuarial office)
08/2012 - 10/2012	**TNG Technology Consulting GmbH**, Unterföhring Internship (software development)

Scientific Publications and Working Papers

2020	**A measure to analyse the interaction of contracts in a heterogeneous life insurance portfolio** (with Stefan Graf and Alexander Kling) *European Actuarial Journal: DOI: 10.1007/s13385-020-00225-2.*
2020	**Cross Subsidizing Effects between existing and new policyholders in traditional life insurance** (with Stefan Graf, Alexander Kling and Jochen Ruß) *Working Paper.*
2020	**Werden bei einer Differenzierung der Überschussbeteiligung in der Lebensversicherung Neukunden zu Lasten des Bestands bevorzugt?** *Working Paper.*

Further Publications

2015	**The Cycle Spectrum of Claw-free Hamiltonian Graphs** (with Felix Joos and Dieter Rautenbach) *Graphs and Combinatorics 32, 93–101: https://doi.org/10.1007/s00373-015-1530-9.*

Research Conferences (with Contributed Talks)

06/2018	31st International Congress of Actuaries (ICA) 2018, Berlin
10/2018	Ulm Actuarial Day, Ulm
11/2018	2nd Frankfurt Insurance Research Workshop 2018, Frankfurt
11/2019	WiMa-Kongress 2019, Ulm
10/2020	Ulm Actuarial Day, Ulm

Ulm, May 2021

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