Research Group for Cosmology and Quantum Gravitation
Institute of Theoretical Physics
Faculty of Natural Sciences

Induced Gravity
with
Higgs Potential
Elementary Interactions and Quantum Processes

Doctoral Thesis

for obtaining the academical degree of Doctor of natural sciences
(Doctor rerum naturalium)
of the Faculty of Natural Sciences of the University of Ulm

presented by
Nils Manuel Bezares Roder
from Mexico City
Ulm, 2010

Referee
Prof. Dr. Frank Steiner
Prof. Dr. Werner Balser

Arbeitsgruppe für Kosmologie und Quantengravitation
Institut für Theoretische Physik
Fakultät für Naturwissenschaften

Induzierte Gravitation mit Higgspotential

Elementare Wechselwirkungen und Quantenprozesse

Dissertation

zur Erlangung des Doktorgrades Dr. rer. nat.
(Doctor rerum naturalium)
der Fakultät für Naturwissenschaften der Universität Ulm

vorgelegt von
Nils Manuel Bezares Roder
aus Mexiko-Stadt
Ulm, 2010

Erstgutachter: Prof. Dr. Frank Steiner, Institut für Theoretische Physik
Zweitgutachter: Prof. Dr. Werner Balser, Institut für Angewandte Analysis
Amtierender Dekan: Prof. Dr. Axel Groß, Institut für Theoretische Chemie
Tag der Promotion: 29. Juni 2010

Summary

This work is intended to first serve as introduction in fundamental subjects of physics in order to be then able to review the mechanism of symmetry breakdown and its essential character in physics. It is discussed how this mechanism is indeed fundamental for a better understanding of physics in all its fields, especially in relation to elementary-particle and condensed-matter physics, including superconductivity in its usual as well as dual form which is investigated within gluodynamics. This work then introduces the concept of scalar–tensor theories of gravity based on Bergmann–Wagoner models with a Higgs potential. The main physical context aimed is the problem of Dark Matter and Dark Energy which are addressed in this work in an attempt to better understand those phenomenological subjects of astrophysics and cosmology.
On the one hand, there is gravitation. It is especially relevant for astrophysical phenomena and for analyses of the structure of the Universe as a whole. Within this context, we have Dark Matter as an especially relevant concept. Dark Matter is the name given to most of the matter in the Universe, and it is necessary to reproduce measured astrophysical data within standard dynamics. The latter assumes (electromagnetically uncoupled) dynamical matter which may still (and should) be produced in experiments in order to clarify its nature. Dark Matter comprises *ca.* 90 percent of the whole matter density, whereas matter density only comprises about one third of the total energy density there is. Almost all other density (hence around 2/3 of all energy density) is called Dark Energy. This energy acts gravitationally repulsive and leads to the measured accelerated expansion of the current Universe.
On the other hand, there is elementary-particle physics where mass is explained as a consequence of rupture of symmetry by means of an interaction between the massless matter states and some scalar fields. A scalar field of this kind is coupled here to gravitation in order to obtain new gravitational dynamics besides the usual ones. As a consequence, antigravitation and antiscreening of matter will be expected as phenomenological Dark Matter.
This work entails the following main contributions:

- General features of Einstein's theory are introduced together with generalities of the different elementary interactions of physics from which the concepts of dark sectors and Higgs Mechanism are derived (Chapters 1 and 2).

- The concept of symmetry breaking and especially the Higgs Mechanism of mass generation are discussed in their relevance for the most different subjects of physics, especially in relation to the Standard Model of elementary particle physics with elementary Higgs fields (Chapters 3).

- The mechanism of symmetry breakdown with Higgs scalar fields, as essence for the phenomenology of superconductivity in condensed matter physics, is shown within the problem of confinement of quarks in hadrons, *i.e.* of the constituents within nuclear particles. This Chapter shows the relevance and universal properties of mechanisms of symmetry breakdown in physics. It was carried out within

a joint work with Dr. Hemwati Nandan of the Centre for Theoretical Physics in New Delhi. Parting from Wick-transformed propagators for dual-symmetric systems, it continues earlier works of his and reenforces the concept of symmetries in nature and the assumption that Higgs fields may lead to thin flux-tube formation for color-electric charges constricting analogously to magnetic fields in superconductors, whereas Cooper pairs in BCS models act as effective Higgs bosons (Chapter 4 and especially 4.3).

- Scalar–Tensor Theories are introduced historically (Chapter 5) in order to build in them the process of Higgs Mechanism. This is then fulfilled with a theory of induced gravity with a Higgs potential (Chapter 6) which seems renormalizable according to deWitt's power counting criterion, and with mass-generating Higgs fields which only couple gravitationally as well as with Higgs fields which act analogously to cosmon fields.

- Higgs fields in general interact gravitationally so that they are coupled here to act within gravitation indeed. Further, the energy density of the gravitational field is derived for the specific model of induced gravity from an analogy to electrodynamics (Chapters 6.4 and 7.3). It is shown that a non-vanishing value of pressure related to the scalar field is necessary in order to reproduce standard linear solar-relativistic dynamics. Within astrophysical considerations for flat rotation curves of galaxies, a possible dark-matter behavior is concluded within spherical symmetry (Chapters 7.7 and 7.8). The scalar field and the dark-matter profile of total energy density are derived. An analogous relation between density and pressure in galactodynamics to that of solar-relativistic behavior appears for the dominance of phenomenological Dark Matter in galaxies.

- Within spherical symmetry (Chapter 7), gravitationally repulsive issues of induced gravity are concluded. These may lead to weakening of horizons of Black Holes ("grey stars"; Chapter 7.4) as well as to Reissner–Nordström-like behavior in galaxies and Black Holes (Chapter 7.5). This may account to potentially relevant astrophysical consequences on weak-field solutions such as geodesic motion (Chapter 7.7) and solar-relativistic effects (Chapter 7.6).

- Fundamental relations of cosmology within induced gravity with Higgs potential are derived for a Friedmann–Robertson–Walker symmetry (Chapter 8). Cosmic acceleration and dark-matter phenomenology are analyzed in virtue of the generalized Friedmann equations, the equations of state, cosmic deceleration and density parameters.

- Indications of a possible finite initial state of the Universe are achieved for a Friedmann cosmology (Chapters 8.7 and 8.8) together with accelerating behavior in such a state as well as in the current Universe. Absence of matter leads to anti-stiff, quintessential (antigravitational) behavior in the Universe (Chapter 8.2).

Übersicht

Diese Arbeit beabsichtigt als erstes, eine kurze Einleitung in grundlegende Gebiete der Physik zu sein, um so einen Überblick des Mechanismus der Symmetriebrechung und seiner wesentlichen Merkmale in der Physik wieder zugeben. Es wird besprochen, wie dieser Mechanismus tatsächlich grundlegend für ein besseres Verständnis der Physik in vielen Gebieten ist, ganz besonders in Verbindung mit der Elementarteilchenphysik und der Physik kondensierter Materie, einschließlich der Supraleitung in ihrer gewöhnlichen, wie auch in ihrer dualen Form, welche innerhalb der Gluodynamik untersucht wird.
Diese Arbeit führt dann in das Konzept der Skalar-Tensortheorien der Gravitation ein, die auf Bergmann-Wagoner-Modellen mit einem Higgspotential basieren. Der wesentliche physikalische Kontext, den diese Arbeit bezweckt, sind die Probleme der Dunklen Materie und der Dunklen Energie. Diese werden untersucht, um solche phänomenologische Fachgebiete der Astrophysik und der Kosmologie besser zu verstehen. Auf der einen Seite liegt die Gravitation vor. Sie befasst sich im wesentlichen mit den astrophysikalischen Phänomenen und mit der Struktur des Universums an sich. In diesem Kontext ist die Dunkle Materie von besonderer Relevanz. Der größte Anteil an Materie im Universum wird als Dunkle Materie bezeichnet. Sie ist notwendig, um die innerhalb der Standarddynamik gemessenen astrophysikalischen Daten wiederzugeben. Standarddynamik benötigt, (nicht an den Elektromagnetismus gekoppelte) dynamische Materie, welche aber noch experimentell nachgewiesen werden muss. Dunkle Materie umfasst ca. 90 Prozent der gesamten Materiedichte, wobei die Materiedichte wiederum nur rund ein Drittel der gesamten Energiedichte darstellt. Fas alle restliche Dichte, also knapp zwei Drittel der gesamten Energiedichte, wird als Dunkle-Energie-Dichte bezeichnet. Diese Energie agiert gravitativ abstoßend und wird zur Erklärung der gemessenen beschleunigten Expansion unseres Universums herangezogen.
Auf der anderen Seite liegt die Elementarteilchenphysik vor, in der die Masse als Folge gewisser Symmetriebrechung erklärt wird und zwar mittels Wechselwirkungen zwischen den masselosen Materiezuständen und einer bestimmten Art skalarer Felder. Ein skalares Feld dieser Art wird hier an die Gravitation gekoppelt, um somit neue gravitative Dynamik zu erlangen, die zu den gewöhnlichen hinzuzufügen ist. Als Folge werden Antigravitation und Gegenabschirmung der Materie als phänomenologische Dunkle Materie erwartet.
Diese Arbeit beinhaltet folgende Hauptbeiträge:

- Allgemeine Bestandteile der einsteinschen Theorie werden zusammen mit allgemeingültigen Aspekten der elementaren Wechselwirkungen der Physik eingeleitet. Aus diesen stammen die Konzepte der dunklen Sektoren und des Higgsmechanismus ab (Kapitel 1 und 2).

- Das Konzept der Symmetriebrechung und insbesondere des Higgsmechanismus der Massenerzeugung werden im Sinne ihrer Bedeutung für die unterschiedlichen Gebiete der Physik besprochen, vornehmlich in Bezug auf das Standardmodell der Elementarteilchenphysik mit elementaren Higgsfeldern (Kapitel 3).

- Der Mechanismus der Symmetiebrechung mit Higgsfeldern wird vorgeführ. Dieser ist Kern der phänomenologischen Supraleitung innerhalb der Physik kondensierter Materie und wird hier im Sinne des Problems der Einsperrung (*Confinement*) der Quarks in Hadronen, also der Konstituenten innerhalb nuklearer Teilchen, betrachtet. Dieses Kapitel zeigt die Bedeutung und universellen Eigenschaften des Mechanismus der Symmetriebrechung in der Physik. Dies wurde unter Mitwirkung von Dr. Hemwati Nandan des *Centre for Theoretical Physics* in Neu-Delhi erarbeitet. Basierend auf Wick-transformierten Propagatoren für duale Systeme setzt es frühere Arbeiten von ihm fort und verstärkt das Konzept der Symmetrien in der Natur, zusammen mit der Vermutung, dass Higgsfelder zur Bildung dünner Flussröhren (*flux tubes*) farbelektrischer Ladungen führen, welche sich analog zu Magnetfeldern in Supraleitern verengen, wobei Cooperpaare in BCS-Modellen als Higgsbosonen auftreten (Kapitel 4 und hauptsächlich 4.3).

- Skalar-Tensortheorien werden historisch eingeführt (Kapitel 5), um in diese den Higgsmechanismus einzubauen. Dies wird mit einer Theorie induzierter Gravitation mit einem Higgspotential erreicht (Kapitel 6), welche gemäß des Abzählbarkeitskriteriums von deWitt renormalisierbar zu sein scheint. Sowohl massenerzeugende Higgsfelder (die nur gravitativ koppeln) als auch Higgsfelder (die analog zu Kosmonfeldern agieren) werden als skalare Felder gewählt.

- Higgsfelder im Allgemeinen wechselwirken gravitativ. Somit koppeln sie hier derart, dass sie innerhalb der Gravitation agieren. Des weiteren wird die Energiedichte des Gravitationsfeldes für das spezifische Modell der induzierten Gravitation aus einer Analogie mit der Elektrodynamik abgeleitet (Kapitel 6.4 und 7.3). Es wird gezeigt, dass ein nichtverschwindender Wert des mit dem Skalarfeld verbundenes Druckes notwendig ist, um die standardsolarrelativistische Dynamik wiederzugeben. Innerhalb astrophysikalischer Abwägungen für flache Rotationskurven wird mögliches Dunkle-Materie-Verhalten bei sphärischer Symmetrie schlussgefolgert (Kapitel 7.7 und 7.8). Das skalare Feld und das Profil Dunkler Materie der gesamten Energiedichte werden abgeleitet. Eine ähnliche Beziehung zwischen Dichte und Druck der galaktischen Dynamik zu der solarrelativistischen Verhaltens tritt bei der Dominanz phänomenologischer Dunkler Materie in Galaxien auf.

- Bei zentraler Symmetrie (Kapitel 7) werden Indizien gravitativ abstoßender Wirkungen schlussgefolgert. Solche können zu einer Abschwächung des Horizonts Schwarzer Löcher (“graue Sterne”; Kapitel 7.4) führen, sowie zu Reißner-Nordström Verhalten in Galaxien und Schwarzen Löchern (Kapitel 7.5). Dies kann potenziell relevante astrophysikalische Folgen haben, z.B. bei Schwachfeldlösungen sowie bei der geodätischen Bewegung (Kapitel 7.7) und solarrelativistischen Effekten (Kapitel 7.6).

- Es werden fundamentale Beziehungen der Kosmologie innerhalb der induzierten Gravitation mit Higgspotential für die Friedmann-Robertson-Walker-Symmetrie abgeleitet (Kapitel 8). Die kosmische Beschleunigung und die Phänomenologie Dunkler Materie werden auf der Grundlage verallgemeinerter Friedmanngleichungen untersucht, aber auch unter Betracht der Zustandsgleichung, der kosmischen Dezelerations- und der Dichteparameter.

- Indizien auf einen möglichen endlichen Anfangszustand des Universums werden für eine Friedmann-Kosmologie erhalten (Kapiteln 8.7 und 8.8). Darüber hinaus wird ein Beschleunigungsverhalten solcher Zustände und des jetzigen Zustandes des Universums hergeleitet. Dabei würde die Abwesenheit von Materie zu einem antisteifen, quintessenziellen (antigravitativen) Verhalten im Universum führen (Kapitel 8.2).

Contents

Abbreviations, acronyms and basic symbols

The exact meaning of the symbols may be gotten by means of the context. Furthermore, hats, primes, subscripts and further indices are used for differentiation within the text. Such are not shown in the list above. For further information, the reader may use the **extended list of mathematical symbols** and the **index** at the end of this work. Second (s), meter (m), kilogram (kg), Newton (N), Ampère (A) and subdivisions and multiples of them (and SI prefixes in general) are not shown.

Abbreviation	Symbol / Basic usage	Abbreviation	Symbol / Basic usage
α	Reissner–Nordström-like charge term / Strength / He-nuclei in radiation	β	(anti-)electrons in ew radiation
Γ	Christoffel symbol (connection)	γ	Dirac matrix / polytropic index Charge–coupling ratio / Photons in radiation
Δ	Difference / Density ratio	δ	Kronecker delta / Delta distribution
ϵ	Energy density / Levi–Civita	ε	Permittivity / Geodesic parameter
κ	Ginzburg–Landau parameter / Gravitational coupling	Λ	Cosmological term (function, constant) / Lorentz transformation
ΛCDM	Cold Dark Matter Model with cosmological constant	λ	Gauge / Higgs parameter / Metric component
μ	Higgs parameter / Metric component / Muon / Permeability	ν	Metric component / Neutrino
ξ	Scalar-field excitation	Π	Polarization tensor
ϱ	Density	σ	Magnetic charge density / Cross section / Pauli matrix
τ	Tauon / Eigentime / Generator	Φ	Potential / Scalar field
χ	Covariant distance	Ψ	Potential
ψ	Potential / Scalar field / Wave function	φ	Scalar-field excitation / Angle
Ω	Density parameter / Unit sphere	ω	Jordan–Brans–Dicke coupling

Abbreviation	Symbol / Basic usage	Abbreviation	Symbol / Basic usage
A	Spinor index	AHM	Abelian Higgs Model
ATLAS	A Tiroidal LHC Apparatus	AU	Astronomical Unit
a	Scale parameter / Isospin index / Halo scale / “Outside” subscript	ad. loc.	*Ad locum*: In the place
ad. val.	*Ad valorem*: According to the value		
B	Baryon / Spinor index	B	Newtonian field amplitude
BCS	Bardeen–Cooper–Schrieffer	BD	Brans–Dicke
BH	Black Hole	BW	Bergmann–Wagoner
C	Color	CCC	Cosmic Censorship Conjecture
CDM	Cold Dark Matter	CERN	*Conseil Européen pour la Recherche Nucléaire*
CMB	Cosmic Microwave Background	CP	Conjugation–Parity
c	Lightspeed	ca.	*Circa*: about
cf.	*Confer*: compare, consult	cos	Cosine
cosh	hyperbolic cosine	$const.$	constant
D	Dyon	DE	Dark Energy
DESY	*Deutsches Elektronensynchrotron*	DM	Dark Matter
DME	Dual Meissner Effect	dom	Dominance
dyn	Dynamical		
E	Energy	Ei	Exponential integral
EOS	Equation of state	EP	Equivalence principle
EPR	Electron Paramagnetic Resonance	ESR	Electron Spin Resonance
EV	Expectation value	e	Electric charge
ed.	Editor, edition	eff	Effective
e.g.	*Exempli gratia*: for instance	em	Electromagnetic
eq.	equation	et al.	*Et alii*: and others
etc.	*Et cetera*: and the rest, and so on	eV	Electronvolt
ew	electroweak	exp.	Experimental
F	Field-strength / Force	FLAG	Finite Length-Scale Anti-Gravity
FLRW	Friedmann–Lemaître–Robertson–Walker	FRW	Friedmann–Robertson–Walker
f	Family index (isospin); Flavor	f	*Folium*: and following page,
ff	*Foliis*: and following pages, from pages	fig.	Figure
G	Gravitational coupling constant	GL	Ginzburg–Landau / General Linear
GR	General Relativity	GRB	Gamma-ray burst
GSW	Glashow–Salam–Weinberg	GUT	Grand Unified Theory
g	Coupling constant / Metric		

Abbreviation	Symbol / Basic usage	Abbreviation	Symbol / Basic usage
H	Higgs / Hubble	HDM	Hot Dark Matter
HE	Hilbert–Einstein	h	Metric correction / Planck's constant
$\hbar$	Reduced Planck's constant	h.c.	Hermite conjugate
h.t.	*Hoc titulo*: under/in this title		
ISCO	Innermost stable circular orbit	i	"Inside" subscript
i.a.	*Inter alia*: among other things	i.e.	*Id est*: that is
ibid.	*Ibidem*: in the same place		
J	Jordan	JBD	Jordan–Brans–Dicke
j	Current		
K	Curvature / Force / Mass parameter	KK	Kaluza–Klein
k	Boltzmann constant / Force density		
L	"Left-handed" subscript	L	Lagrange function / Angular momentum
$\mathcal{L}$	Lagrangian (Lagrange density)	LEP	Large Electron–Positron Collider
LHC	Large Hadron Collider	LIGO	Laser Interferometer Gravitational Wave Observatory
LISA	Laser Interferometer Space Antenna	LNT	Linear No Threshold
l	Compton wave length (length scale) / lepton index	loc. cit.	*Loco citato* (l.c.): In the place cited
log	Logarithm	ly	Lightyear
M	Matter	M	Mass (Higgs) / Mass parameter
ME	Meissner Effect	MOG	Modified Gravity
MOND	Modified Newtonian Dynamics	MRI	Magnetic Resonance Imaging
m	mass / Fermionic index	min	Minimum
N	Newton	N	Gauge fixing tensor / Isobar quantity
N.B.	*Nota bene*: Note well	NFW	Navarro–Frenk–White
NMR	Nuclear Magnetic Resonance		
O(N)	Orthogonal group of degree N	op.cit.	*Opus citatum*: cited work
P	Perihelion	p	Momentum / Pressure
par	parameter	pc	Parsec
pg.	Page	pro tem.	*Pro tempore*: For the time, temporarily
Q	Charge	QAD	Quantum Asthenodynamics
QCD	Quantum Chromodynamics	QED	Quantum Electrodynamics
QM	Quantum Mechanics	q	Deceleration parameter / Scalar-field excitation / Quark

Abbreviation	Symbol / Basic usage	Abbreviation	Symbol / Basic usage
R	Radiation / Radius / Curvature (scalar) / "Right-handed" subscript	RMS	Root mean square
RN	Reissner–Nordström	RW	Robertson–Walker
r	Distance / Orbit	rad	Radiation
S	Action / Spin	SB	Symmetry Breaking
SEP	Strong equivalence principle	SF, sf	Scalar field
SI	*Système International d'unités*	SM	Standard Model / Stiff Matter
SNe	Super Novae	SR	Special Relativity
SSB	Spontaneous Symmetry Breaking	STT	Scalar–Tensor Theory
STVG	Scalar–Tensor–Vector Gravity	SO(N)	Special Orthogonal Group of degree N
SU(N)	N-Dimensional Special Unitary Group	SUSY	Supersymmetry
Shi	Sinushyperbolicus Integral	SNe	Super Novae
s	Energy–momentum	sc.	*Scilicet*: one may know
sin	Sine	sinh	Hyperbolic sine
s.p.	*Sine prole*: without issue	sr	steradian
T	Total	T	Energy–stress / Kinetic energy / Isospin / Temperature
$\mathcal{T}$	Kineric energy density	TeVeS	Tensor–Vector–Scalar
t	Time	tan	Tangent
tanh	Hyperbolic tangent	tot	Total
U	BW cosmological term / Unitary transformation	U(N)	N-Dimensional Unitary Group
u	Potential-term mixture / Reciprocal orbit / energy density	UMi	Ursa Minor-is
$V, \mathcal{V}$	Potential (density)	VE	Vacuum expectation
VEV	Vacuum expectation value	v	Ground-state value / velocity
viz	*Videlicet, videre licet*: namely, to wit	v.i.	*Vide infra*: see below
vs.	*Versus*: against	v.s.	*Vide supra*: see above
W	Charged weakon (weak gauge boson)	WEP	Weak equivalence principle
WIMP	Weakly Interacting Massive Particle	WMAP	Wilkinson Microwave Anisotropy Probe
w	EOS parameter / potential mixture		
X	RN correction	x	Coordinate / Yukawa matrix
Y	Hypercharge	YM	Yang–Mills
Z	Neutral weakon / Partition function	Zw	Zwanziger
z	Redshift		

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Introduction

Maxwell's theory of electrodynamics shows an inextricable symmetry which locks together electricity and magnetism. The four Maxwell's equations describe the fate of the electric and magnetic fields at a spacetime point, and from these it is possible to derive that each of the fields obeys a wave equation representing that light propagates as electromagnetic waves. Historically, they significantly contributed to the development of Special Relativity. Furthermore, the broken dual symmetric structure of electromagnetism leads to the theoretically fascinating aspect of the possibility of monopoles. Their existence would mean a further unification in nature.

Electrodynamics (usual as well as comprising monopoles) can be given on the grounds of gauge theories so that electromagnetic forces may be interpreted as consequences of local gauge transformation with gauge potentials. Furthermore, local gauge transformation of Special Relativity leads to General Relativity with external transformations of spacetime as consequence of gauge, and with Christoffel symbols as generalized potentials of what is linearly perceived as the gravitational force at relatively small velocities compared to light speed.

Indeed, all elementary interactions of nature – the fundamental forces between elementary particles – can be given by means of gauge theories, and the Standard Model of elementary particle physics provides a concise and accurate description of all fundamental interactions except gravitation.

Modern quantum theories of elementary interactions ground on Maxwell's theory by means of Yang–Mills theories which generalize the structure of electrodynamics for more complex, non-abelian gauge groups for theories of quantum dynamics. Furthermore, the answer of the fundamental problem about which mechanism allows the elementary particles to become heavy is now addressed in terms of the Higgs boson in the Standard Model. The Higgs Mechanism is, therefore, a powerful tool of modern particle physics which makes the models mathematically consistent and able to explain the nature of fundamental interactions in a manifest way. The bosons and fermions are believed to gain mass through a phase transition via Higgs Mechanism. In this way, the particles can be coupled with experiments, and a theoretical explanation may be given of how the mass generation takes place.

The Higgs particles, belonging to the Higgs field, are still not experimental reality and need to be observed to make any model complete. The search for Higgs particles is a very important task in physics, and it is believed that their mass will be achievable with the future generation of high energy experiments as the LHC in Geneva, specifically at energies less than 250 GeV and higher than 130 GeV. Yet, this may be model dependent, whereas the exact properties of the Higgs field, their couplings and their source are of special relevance. Furthermore, Higgs particles in general appear effectively in all branches of physics as they are, for instance, basic in the understanding of superconductivity where they appear as composites within the concept of the Cooper pairs. Then they lead to an effective mass of photons, which itself leads to the Meissner effect. Furthermore, within strong interactions they may lead to a dual Meissner effect and hence to a possible explanation of the confinement of quarks and color charges in hadrons.

The nature of Higgs fields is still not completely understood. Universally, they interact in a gravitative and Yukawa form in every model. However, their exact properties may depend on the specific model used. Actually, given their gravitative nature, if they are coupled non-minimally to gravitation, unlike in the Standard Model, Higgs fields may decouple completely from the fermionic sector or couple only very weakly and further even possess an (almost) vanishing mass in addition to a finite ground-state value. This is a main issue of this work, which in this way intends to contribute to unification issues of nature.
Within astrophysics, there exist the problems of Dark Matter and Dark Energy. Within the standard theory far more mass is necessary than mass from luminous matter can be measured. Furthermore, cosmic counter-gravitative interactions are measured. The nature of these issues of cosmology is unclear and also if they may be related to still unknown mechanisms of further generalizations of the theories. This work relates them to the concept of scalar–tensor theories and the Higgs Mechanism of Spontaneous Symmetry Breaking. Hence, a cosmon-like theory of induced gravity is presented which may contribute to the phenomenon of the dark sectors of cosmology as well as to the dynamics of the primeval Universe. There, it may account to the subject of the cosmological Inflation, *i.e.* the era of very high acceleration after the Big Bang. Further, it may also account to an understanding of the Big Bang itself as new gravitational dynamics coming from the scalar field may dominate at early stages of the Universe and act as further matter with negative pressure or density. Dark Energy and Dark Matter may be a remanence of such dynamics.
What composes our Universe? Which are the dynamics of dark sectors of energy density? Which is the relation between scalar fields and astrophysical and microscopic phenomena, if any? May the Higgs Mechanism further show an even more universal character with a relation to geometrized gravity? What consequences would that have in our picture and interpretation of astrophysical phenomena? What consequences would that have for the early stages of the Universe as what we now can perceive of it? All these are subjects of this work, which intends to hold on to their physical context, especially within elementary-particle physics and astrophysics. In an attempt to answer part of these questions, some review of nuclear and elementary-particle physics is necessary, together with some grounding of fundamental physics towards Higgs Mechanism of mass generation, superconductivity and Abelian Higgs Models for confinement of quarks in baryons as Dual Meissner Effect, Jordan and Bergmann–Wagoner models for induced gravity, and central as well as Friedmann–Robertson–Walker symmetry for galactic dynamics and cosmology.

Part I

Elementary particles and Gravitation

Chapter 1

On the geometrical basics of gravitation

– General features of Einstein's theory are introduced. Homogeneous Maxwell-like systems are derived for a geometrical field-strength tensor related to the Ricci scalar. They may be partly found published in [23]. –

1.1 Transformations and the metrical tensor

Both quantum physics and gravitational physics comprise altogether all known elementary interactions of physics. Nuclear forces, *i.e.* the relevant forces between nuclear constituents (purely of quantum-mechanical nature), act effectively only at short distances. Electrodynamics (Lorentz forces and general consequences of electromagnetism from and on charged particles) and gravitation (as a consequence of mass and energy), on the other hand, are long-ranged. However, electromagnetism cancels out because of negative and positive charges so that for long scales only gravitation, the weakest of all elementary interactions, dominates. Further elementary interactions, namely electro-weak and strong interactions (which effectively lead to nuclear and electromagnetic forces) can be understood in terms of quantum phenomena while gravitation cannot yet be fully understood on those grounds. Still, all may be explained in terms of gauge theories (*cf.* [56] on Quantum Cosmology) and ground on the covariant formalism with a 4-dimensional (lorentzian) manifold of spacetime. Elementary-particle theories (with quantum electromagnetism), however, ground on **inner transformations** as are the ones of spin and isospin, while gravitation grounds on **external transformations** as are the ones of spacetime itself.
Historically, in nuclear physics the **isospin**, originally called *isotopic* or *isotonic* spin, is a defined property of particles which originally differentiates between nucleonic particles or nucleons (neutrons and protons). Without concerning the isospin, both nucleons are interpretable as the same particle within nuclear forces (the "nucleon"), given that forces between nucleons are (nearly) independent of the particle's charges [4] (let us say, they are ***isotopic*** to each other). Within nuclear forces, both isotopic particles are indistinguishable between each other. Hence, since the isospin makes a differentiation between protons and nucleons, the isospin is the one quantum property which leads to the existence of different types of atoms (nuclides) of the same chemical element, each of them having a different atomic mass (isotopes).
The quantum state of nucleons can be given by a two-vector in isospin space whereas each component possesses an isospin. Nuclides with different amount of nucleons but the same chemical properties (same amount of protons) are isotopes of each other. Nuclides with the same amount of neutrons but different amount of protons are isotones. Isotopes and isotones differ in their isospin. Therefore the name isotopic and isotonic spin.

If in an atom the amount of each nucleon is the same and the configuration of isospins differs, we speak about isobars [84]. This is the case in mirror nuclei ($_1H^3 - {_2He^3}$, $_6C^{13} - {_7N^{13}}$ *etc.*). Neglecting Coulomb forces, they show that protons and neutrons have approximately the same bounding contributions (*e.g.* [91]). Nuclear forces (as stated) do basically not differ between isospin and are independent of the electric charge. With the advent of elementary-particle physics, the concept of nucleonic isospin has been generalized. Hence, an isospin vector or isovector possess N different components in isospin space [155]. The particles related to each isospin depend on the specific group within which they are indistinguishable between each other (isotopic). Isospin is defined as an intrinsic property of quantum mechanical states, and transformations within isospin space have implications on particles themselves. These transformations can be followed up to the consequences on a nuclear and chemical level.

Another intrinsic property of particles is the **spin**. It gives the statistics which a particle follows. The spin is a quantum number which categorizes between general types of particles. Transformations in spin space imply, among others, transformations such as between fermions and bosons[1] which not only follow different quantum mechanical statistics but have an antagonical relation towards elementary interactions. On an elementary-particle level, fermions "feel" interactions and bosons transmit them [158]. Such is, however, a matter of Yang–Mills theories, especially in the context of the Standard Model of elementary particle physics based on gauge to introduce bosons acting on fermionic multiplets (*cf.* Chapter 3.3). Gravitation as an elementary interaction of physics, however, does not ground on transformations of matter itself. Transformations within gravitation are the ones of spacetime which, in analogy to spin and isospin transformations (internal) are called external ones. These transformations may be defined by the locally gauged homogeneous or inhomogeneous (Poincaré) **Lorentz** group.[2] Homogeneous Lorentz transformations are the generalization of rotations in the 3 dimensional euclidean space onto the Minkowski space ($x^\mu \to x^{\mu'} = \Lambda^\mu{}_\nu x^\nu$). The group of proper (det$\Lambda = +1$), orthochronous ($\Lambda^0{}_0 \geq 1$) transformations is isomorph to the group SO$^+$(3,1) (restricted Lorentz group) of special pseudo-orthogonal transformations in 4 dimensions. It is spanned by usual 3-dim rotations and the special Lorentz transformations (*boost* transformations) [99]. Further components of the Lorentz group are gotten from parity, time and parity–time transformations (representatives of a coset class related to the factor (or quotient) group O(3)/SO(3) $\simeq \mathbb{Z}_2 = \{1, -1\}$ and the Klein four-group $\mathbb{Z}_2 \times \mathbb{Z}_2$). Poincaré transformations ($x^\mu \to x^{\mu'} = \Lambda^\mu{}_\nu x^\nu + a^\mu$) is the symmetry group of the Minkowski space. Space-time transformations are related to the properties of the metrical tensor $g_{\mu\nu}$, often simply known as metric (see Appendix A).

If the metric is spacetime dependent, the derivative along tangent vectors of a manifold is to introduce a connection on the manifold by means of a differential operator which entail the properties of the spacetime metric which we represent by $g_{\mu\nu}$. This operator is the **covariant derivative**. It is related to the local gauge of the transformation (Lorentz) group (see Appendix A.1). The connection components within GR are the Christoffel symbols $\Gamma^\mu_{\nu\lambda}$ [218] as objects which are analogous to Yang–Mills' gauge potentials (fields) A^μ (related to gauge bosons) of the different isospin and spin components. As a matter of fact, local gauge transformations of the Lorentz group leads to (the geometrical part of) gravitation in form of a **geometrization of gravity** (see Appendix A.2).

[1] Named after *Enrico Fermi* (1901-1954) and Fermi–Dirac statistics. and after *Satyedra Nath Bose* (1894-1974) and Bose–Einstein statistics.

[2] After Jules Henri Poincaré (1854-1912) and Hendrik Antoon Lorentz (1853-1928).

1.2 Maxwell equations of gravitation

4-vectors a^μ in $\mathbb{R}^4$ are classified according to their scalar product as timelike, spacelike or lightlike (null). With the signature used in this work, there is

(i) $a^\mu a_\mu > 0$ for a timelike vector a^ν.

(ii) $a^\mu a_\mu < 0$ for a spacelike vector a^ν.

(iii) $a^\mu a_\mu = 0$ for a lightlike vector a^ν.

In the case of $a^\mu = dx^\mu$, the scalar product is the (square of the) line element, ds^2. In the case (i), cds is called eigentime. In (ii), $\sqrt{-ds^2}$ is called eigenlength, and in (iii) the worldline runs through a light cone. As generally known, causally linked events lie within a light cone (*cf.* [218]).

With a timelike vector field $u_\mu = \frac{dx_\mu}{ds}$ with

$$u_\mu u^\mu = \frac{dx_\mu}{ds}\frac{dx^\mu}{ds} = \frac{ds^2}{ds^2} = 1$$

and

$$u_{\mu;\sigma} u^\mu = 0 \, ,$$

the field equations for a given observer may be written as

$$u_{\mu;\lambda} u^\lambda = K_\mu \tag{1.2.1}$$

with K_μ as the nongravitational part of a (to mass normalized) force from a mass carried by the observer. On the other hand, there is an equilibrium between nongravitational "forces" K_μ and inertial ones E_μ which maintain the mass in a geodisical trajectory such that

$$E_\mu + K_\mu = 0 \, . \tag{1.2.2}$$

The force E_μ may be written as

$$E_\mu = -u_{\mu;\sigma} u^\sigma = (u_{\sigma;\mu} - u_{\mu;\sigma}) u^\sigma = (u_{\sigma,\mu} - u_{\mu,\sigma}) u^\sigma = \tilde{F}_{\mu\sigma} u^\sigma \, . \tag{1.2.3}$$

The latter defines a field-strength tensor $\tilde{F}_{\mu\nu}$ of the same structure as within electrodynamics (abelian), as a rotation of a 4-vector,

$$\tilde{F}_{\mu\nu} = u_{\nu;\mu} - u_{\mu;\nu}, \tag{1.2.4}$$

with gauge variables u^μ (*cf.* [64]). For the (gravitational) field-strength (1.2.4), **Maxwell-like equations** are obviously valid [23, 64],

$$\tilde{F}_{(\lambda\mu,\nu)} \equiv \tilde{F}_{\mu\nu,\lambda} + \tilde{F}_{\lambda\mu,\nu} + \tilde{F}_{\nu\lambda,\mu} = 0 \, . \tag{1.2.5}$$

In that sense, the gravitational or inertial force E_μ which appears for the observer has the form of the electric part of the Lorentz force. It is related to the Ricci tensor as it can be written using the divergence of $\tilde{F}_{\mu\nu}$. The Ricci tensor from equation (A.3.24) may be rewritten as follows,

$$-R^\lambda{}_\mu u_\lambda = (u^\lambda{}_{;\mu} - u^\alpha{}_{;\alpha} \delta_\mu{}^\lambda)_{;\sigma} = H^\lambda{}_{\mu;\lambda} \, . \tag{1.2.6}$$

Be this the definition of a tensor $H_{\mu\nu}$ which is symmetric. This tensor give the (skew symmetric) field-strength tensor $\tilde{F}_{\mu\nu}$ by

$$\tilde{F}_{\mu\nu} = H_{\nu\mu} - H_{\mu\nu} = u_{\nu;\mu} - u_{\mu;\nu} \tag{1.2.7}$$

with $\tilde{F}_{\mu\nu}$ as an antisymmetric tensor and u^{μ} as vector potential of the gravitational field strength. Equivalently, be

$$\begin{aligned} H_{\nu\mu} + H_{\mu\nu} &= u_{\mu;\nu} + u_{\nu;\mu} - 2u^{\alpha}{}_{;\alpha} g_{\mu\nu} \\ &\equiv Q_{\mu\nu} \,. \end{aligned} \tag{1.2.8}$$

This defines a tensor $Q_{\mu\nu}$. With equation (1.2.8), the Ricci tensor may be written in terms of the field strength as follows,

$$-R^{\lambda}{}_{\mu} = \frac{1}{2}\tilde{F}_{\mu}{}^{\lambda}{}_{;\lambda} + \frac{1}{2}Q_{\mu}{}^{\lambda}{}_{;\lambda} \,. \tag{1.2.9}$$

The divergence of $Q_{\mu}{}^{\lambda}$ is as below,

$$\begin{aligned} Q_{\mu}{}^{\lambda}{}_{;\lambda} &= u_{\mu}{}^{;\lambda}{}_{;\lambda} + u^{\lambda}{}_{;\mu;\lambda} - 2u^{\alpha}{}_{;\alpha;\mu}\delta_{\lambda}{}^{\mu} \\ &= 2u^{\lambda}{}_{;\mu;\lambda} - 2u^{\lambda}{}_{;\lambda;\mu} \,. \end{aligned} \tag{1.2.10}$$

Consequently, there is the following equality,

$$\begin{aligned} Q_{\mu}{}^{\lambda}{}_{;\lambda}u^{\mu} &= u_{\mu}{}^{;\lambda}{}_{;\lambda}u^{\mu} + u^{\lambda}{}_{;\mu;\lambda}u^{\mu} - 2^{\lambda}{}_{;\lambda;\mu}u^{\mu} \\ &= u_{\mu}{}^{;\lambda}{}_{;\lambda}u^{\mu} + u^{\lambda}{}_{;\mu;\lambda}u^{\mu} - 2u^{\lambda}{}_{;\lambda;\mu}u^{\mu} \\ &= 4\left(u^{\lambda}{}_{;\mu}u^{\mu}\right)_{;\lambda} - 4u^{\lambda}{}_{;\mu}u^{\mu}{}_{;\lambda} - 2u^{\lambda}{}_{;\mu;\lambda}u^{\mu} - 2u^{\lambda}{}_{;\lambda;\mu}u^{\mu}. \end{aligned} \tag{1.2.11}$$

With $g^{\mu\lambda}g_{\lambda\mu} = 1$, there is equivalently

$$Q_{\mu}{}^{\lambda}{}_{;\lambda}u^{\mu} = 4\left(u^{\lambda}{}_{;\mu}u^{\mu}\right)_{;\lambda} - 4u_{\mu}{}^{;\lambda}u^{\mu}{}_{;\lambda} - 2u^{\lambda}{}_{;\mu;\lambda}u^{\mu} - 2u^{\lambda}{}_{;\lambda;\mu}u^{\mu} \,, \tag{1.2.12}$$

which may be simplified for static fields in relation to the observer. This can further be treated after going through the right-hand side of the equation of gravitation, *i.e.* its relation to matter, which will give the source of the energy–stress tensor $\tilde{F}_{\mu\nu}$ related to curvature. Furthermore, the relations derived and definitions given here are of special relevance for a further definition of the energy density of gravitation in Chapter 7.3, the results of which may be found under [23].
In Chapter 6.4, equation (6.4.8) shows the relation of the field strength $\tilde{F}_{\mu\nu}$ to gravity, given the relation of the Ricci tensor $R_{\mu\nu}$ to u_{μ} as field variable in (1.2.6). The Einstein tensor may be derived through variation from the Ricci scalar R and a cosmological constant in the action. The Hilbert–Einstein action, entailing both terms and a Lagrange density of matter, leads to the equations of gravitation as

$$G_{\mu\nu} = -\kappa_N T_{\mu\nu}, \tag{1.2.13}$$

with the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda_0 g_{\mu\nu}, \tag{1.2.14}$$

the metrical tensor $g_{\mu\nu}$ and a cosmological constant Λ_0. Equation (1.2.13) gives the Einstein or Einstein–Hilbert equations of General Relativity (GR). They may be found derived in the Appendix A.4. $T_{\mu\nu}$ is the energy-stress tensor which may be derived from the Lagrange density of matter. κ_N is the coupling constant. Further, the metrical tensor is presented in Appendix A.1 while Lorentz transformations and the local gauge transformations of the Lorentz group, including connection terms and curvature tensors are presented in Appendices A.2 and A.3, respectively. These show the geometrical meaning of gravitation within GR, which may be related to (1.2.6). Equation (6.4.8), however, gives a more general approach from a more general action which is presented in Chapter 6.1. Yet, for the special case of vanishing scalar fields (see later), the GR formulation is valid.

Chapter 2

Elementary particles and the Standard Model

– Generalities of elementary quantum interactions of physics and quantum states are discussed in view of Yukawa's theory of mesons and Yang–Mills theories which lead to the SM of electroweak and strong interactions of physics. Special attention is paid to fermionic fields, types of matter and composite matter and especially to the dark sectors of matter and energy. Part of this work may be found under [24] as result of this work. Details on the quantum mechanical state and the theory of elementary particles may be found in Appendix B. –

2.1 Quantum interactions and the idea of Yang–Mills theories

Modern particle-physics theories have their beginning and interpretation basis in the early 20^{th} century. Back then, H. Yukawa [246] proposed that nuclear particles were held together against electromagnetic repulsion by mediation of particles he proposed (mesons[1]). Within his model there should exist a nuclear force between nucleons which is greater than electromagnetic repulsion. According to the principle that forces should not act at distance, this force should be related to mediation of a particle as carrier of the properties of the interaction. Within nuclear physics then, Yukawa proposed in 1935 the particles we now know as pi-mesons or pions π. These particles are massive (with about 140 MeV/c^2 $\approx 2.5{\cdot}10^{-28}$ kg of mass) and do indeed mediate short-range interactions within the nucleus, according to Heisenberg's uncertainty principle.

Yukawa's theory ultimately states that as consequence of mediation of mesons between nucleons, stable nuclei appear. Hence, the type of interaction between them is generally called of Yukawa-type. Such interactions can be described through a potential given by the product of a Dirac field ψ and a scalar (or pseudo scalar) field ϕ as follows,[2]

$$V \sim g\bar{\psi}\phi\psi, \tag{2.1.1}$$

[1]From $\mu\epsilon\sigma o$: In the middle, intermediate, and hence with mesons as *per-definitionem* intermediate particles.

[2]For π mesons, actually, ϕ is a pseudoscalar, *i.e.* $\bar{\psi}\gamma_4\psi\phi'$, with the projector operator $\gamma_4 = i\frac{1}{4!}\varepsilon_{\alpha\beta\mu\nu}\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu$ with Dirac matrices γ^μ, the Levi–Civita tensor $\varepsilon_{\alpha\beta\mu\nu}$ and the fermionic state ψ.

with g as a coupling constant, and with V which gives a pseudoscalar quantity which is characteristic of the mesons described by the potential. Further, $\bar{\psi}$ is the adjoint conjugate of the Dirac field, *i.e.* of the quantum mechanical state entailing the whole information for a measurement within the quantum mechanical system for particles with spin (see Appendix B.1).
The Dirac field is a spinor or spin vector. It is thus to be used for fermions. Its adjoint conjugate is defined by usual hermite conjugation coupled with γ^0,

$$\bar{\psi} = \psi^\dagger \gamma^0 \,, \tag{2.1.2}$$

so that antimatter states are described only in terms of a changed sign in relation to matter. γ^0 is one of the Dirac matrices.
Nucleons are fermions, and the pseudoscalar mesons described by Yukawa's model are the pions. Originally, the muons or μ particles (which possess a similar mass to the one of pions and which are elementary, indeed), were assumed to be the Yukawa particles. However, they do basically not interact within nuclei and hence do not represent mesonic particles [57]. Muons are leptons and hence massive isotopic to electrons.[3]
Pions were first discovered in 1947 by Lattes *et al.* [149] For the prediction and for the development of experimental techniques which resulted in their discovery, Yukawa and Powell were awarded with the Nobel prize in 1949 and 1950, respectively.

The theory of Yukawa may be generalized so that other interactions are described. Before the rising of elementary-particle physics, it was further used in attempts to unify nuclear forces with gravitation, again assuming Yukawa mesons as mediators but within a higher dimensional spacetime (*sc.* Kaluza's and Klein's theory. See Chapter 5) [142].
Classically, a Yukawa interaction may be written in terms of a Yukawa potential which may be constructed starting from Coulomb potentials for long-range interactions, the mediators of which are massless. These interactions are of $1/r$-type. Yukawa potentials further possess a mass term and may be written as follows,

$$V(r) \sim -\frac{g^2}{r} e^{-mcr/\hbar}, \tag{2.1.3}$$

with m as the mass of the mediation particle, *i.e.* of the pion in terms of nuclear interactions. $mc/\hbar$ is the reciprocal (Compton) length scale related to the mass m which gives the range of the interaction.
Pions possess an inner structure and decay in leptons [156]. They are thus not fundamental. They mediate only residual interactions. Further, the nucleus-conforming particles (nucleons, but also hadrons in general, see Chapter 2.3) possess a finite diameter of about 10^{-15}m and also an inner structure [131]. Furthermore, they possess magnetic momenta [6]. In this context, Gell-Mann [100] and Zweig [252], independently of each other, interpreted a nonelementarity and introduced constituent particles of hadrons back in 1964. These particles are known as quarks.[4] The experimental evidence of these [92], finally, was acknowledged with the Nobel prize in 1990.
Elementary particle physics describes dynamics on the basis of quantum field theories and hence of quantum mechanical states as property carriers for measurements. Hence, states may be given by Dirac spinors such

[3]It is not irrelevant to mention and further to emphasize that here lies an error in earlier literature and newer one copied from the latter (see Chapter 2.3, *cf.* [155] against [84] with "μ mesons") when stating that μ particles are mesons. Be stressed that mesons are to transmit nuclear (effective) interactions and muons do not have such property (as electrons do neither).

[4]"Three quarks for Muster Mark! Sure he has not got much of a bark. And sure any he has it's all beside the mark".– James Joyce, *Finnegans Wake*

that they be related to constituent particles. The spinor possesses the following general standard form,

$$\psi_{a,A_{L/R}} = \begin{pmatrix} \psi_{1,A} \\ \psi_{2,A} \\ \dots \\ \psi_{N,A} \end{pmatrix}_{L/R} ,$$

Index a give the "generalized" isospin and A the spin. L and R be the subscript for left-handed and right-handed states, respectively.

Be N the dimension of the symmetry group of transformation. N is to give the amount of isotopic particles within the interaction given by the gauge group, *i.e.* it is to give the amount of particles which are indistinguishable within given interactions.

Isospin space depends on the group defined for the interaction. For instance, an effective nuclear theory of nucleons possesses a dimension $N = 2$ where a nucleonic state possesses the neutron and the proton as isotopic elements given by the state ψ. Within a theory of strong interactions where it is differentiated between three different color-quarks for each flavor or family, then there is $N = 3$ with an isospin index a counting each color. Furthermore, electroweak interactions (where electrons and neutrinos for different families are isotopic between each other) are given by a quantum mechanical state composed by two isospin components.

Modern theories are based on Yang's and Mills' field theory of 1954 [245], utilizing N-dimensional wave functions in isospin space. N^2 further, is the amount of components of the transformation matrix including unity. If every component of the state is to be related to physical particles, then the amount of components of this matrix related to U(N) minus unity should give the amount of particles which mediate interactions. These particles are bosons and are called gauge bosons.[5] They possess analogous properties to those of mesons in Yukawa's earlier theory. Furthermore, gauge bosons and mesons (such as pions) possess an integer spin. However, gauge bosons are assumed fundamental and they are related to potentials (photons are related to the electromagnetic potentials, for instance). Furthermore, gauge fields interact with isomultiplets in a universal way [4]. Hence, they appear in the same way for all gauge groups of the different interactions. The Yang–Mills theory is a non-abelian (non-commutative) theory with SU(N) transformations and thus with self-interactions that generalize the Maxwell equations of (abelian U(1)-) electrodynamics to the so-called and analogous Yang–Mills equations (see Appendix B.2, especially equations (B.2.1), (B.2.7) and (B.2.9))

$$\mathcal{D}_\lambda \mathcal{F}_{\mu\nu} + \mathcal{D}_\mu \mathcal{F}_{\nu\lambda} + \mathcal{D}_\nu \mathcal{F}_{\lambda\mu} = 0 \tag{2.1.4}$$

and

$$\mathcal{D}_\nu \mathcal{F}^{\mu\nu} = -4\pi\hbar c g \mathcal{J}^\mu(\psi_a) \tag{2.1.5}$$

with an isotensorial (adjoint) field-strength tensor and current $\mathcal{F}_{\mu\nu}$ (following Ricci identities) and $\mathcal{J}_\mu$, respectively (see Appendix B.2), and isotensorial gauge potentials (matrices) $\mathcal{A}_\mu$ with isospin components a and b in the component notation,

$$(\mathcal{A}_\mu)_a{}^b = A_{\mu i}(\tau^i)_a{}^b, \tag{2.1.6}$$

following local gauge of the transformation group SU(N). By means of SU(N) transformations, new forces appear for states (and their related particles), with gauge fields A_μ as potentials of elementary interactions

[5] Real interactions are described by operators, *i.e.* field operators ψ, A_μ, *etc.*; see Appendix B.

(*cf.* Appendix B.2). However, although Yang–Mills equations reduce to electrodynamics for $N = 1$ and then fermionic mass may be added by means of a mass term of the Lagrangian, the more complex form of quantum mechanical interactions generally prohibits the simple addition of mass terms. Weak interactions, for instance, show parity-symmetry breaking [244] so that addition of a massive Lagrangian term leads to contradictions with experimental facts within Dirac equations, given that right- (left-) handed states couple to mass through the source with left- (right-) handed states. Furthermore, such terms as simply added masses lead to singularities. A per-hand massive Yang–Mills theory is not renormalizable. To achieve a physical theory, it seems necessary to introduce scalar fields and the concept of symmetry breaking so that masses appear in an indirect way by means of new parameters (see Chapter 3, *cf.* [157]).

2.2 Wave function and the Standard Model

The paradigm within elementary-particle theories is the unifying Glashow–Salam–Weinberg Model of the Quantum Asthenodynamics (QAD)[6] of electroweak interactions [96], Nobel-prize awarded in 1971. Together with so-called Quantum Chromodynamics (QCD)[7] of the strong interactions of Gell-Mann and others, it leads to the Standard Model (SM) of elementary particles under the symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The constituents are part of a multiplet or isovector which is conformed by those particles which are indistinguishable within a specific interaction (*i.e.* they are isotopic to each other). The group dimension N (and subscript), hence, is given by the particles represented in each group: three differently "colored" (C) quarks for the strong interactions, as well as electrons and neutrinos (leptons L) for the weak interactions, and electrons for electromagnetism. Y stays for the "hypercharge", which is related to the usual electromagnetic charge of electrons by the so-called Gell-Mann–Nishijima formula.
This information is introduced into the (N dimensional) Yang–Mills theory, and the latter is then further changed empirically. The fundamental first step, however, is defining the properties of the wave function ψ for each transformation group.
The SM, as a quantum field theory of interacting fundamental fields, is based on the so-called gauge principle or gauge invariance, which leads to the covariant derivatives, parallel transport and gauge principle. These make it possible for derivatives to maintain their tensorial character, and they can be introduced in terms of parallel transport (and holonomy) in curved space (*e.g.* a sphere). There, a usual derivative leads outside of the manifold. An additional term is needed as correction, that is to move parallel to the surface of the sphere during the derivation. This additional term is related to so-called connections, such as Christoffel symbols $\Gamma^{\mu}_{\nu\lambda}$ in GR or gauge fields (or potentials) A^{μ} in usual gauge theories of elementary particles. Furthermore, in addition to simple Yang–Mills theory, the SM has built in Gell-Mann's [100] and Zweig's [252] idea of quarks as fundamental constituents of hadrons.[8] Interactions between quarks, then, are understood as mediated through the gauge fields, with the so-called gauge bosons as the field quanta of the interactions. Summarizing, there are these especially relevant interactions which are given by the Standard Model of elementary particles:

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[6]Astheno: $\alpha\sigma\theta\epsilon\nu\iota\alpha$ = weak (soft), lack of strength. weakness. The model was proposed by Sheldon Lee Glashow (1931), Abdus Salam (1926-1996) and Steven Weinberg (1933); Nobel prize 1979.

[7]Chromo: $\chi\rho\acute{\omega}\mu\alpha$ = color.

[8]Hadrons ($\acute{\alpha}\delta\rho\acute{o}\varsigma$ = stout, thick): Protons, neutrons and more massive analogue particles (the hyperons), as well as mesons, see Chapter 2.3.

- QED with the symmetry group U(1), which is an abelian unitary group which then leads to Maxwell's equations. All three ($f = 1 \dots 3$) electron-like particles (electrons e, muons μ and tauons τ, and antimatter analogues) are the one isotopic component, e^f, of the QED isoscalar ψ^{fA} whereas all electron-like particles are electromagnetically indistinguishable (aside from their mass). Further, A is the spin so that the QED isoscalar is a vector in spin space.
  Within QED, the isospin index a counts only 1. Commutators of gauge potentials coming from covariant derivatives (the coupling to interaction) vanish, and their corresponding gauge bosons, the gauge photons, thus, do not self-interact. QED remains an abelian theory.

- Within "nuclear forces" there exists charge-independence of nucleonic interactions [84]. Hence, protons and neutrons are interpretable as different states of a nucleon particle. They differ in the isotopic spin or isospin. This nucleon may be described within "old" nuclear physics as a 2-dimensional isospin vector. As already clear, in elementary-particle physics, this concept is generalized for elementary interactions. Yang–Mills theories treat N dimensional mathematical objects which in principle possess some kind of elementary particles as isospin components. These components belong to the same unitary group related to an interaction. If there are N different particles which are indistinguishable and yet isotopic to each other for given interactions, such are given by an isovector or multiplet ψ^{afA}. a count the isospin as vector index, and f count the family membership. The symmetry group yields U(N) which entails electrodynamics since the group may be decomposed as U(N)=U(1)⊗SU(N). The amount of gauge bosons as intermediate particles other than photons is given by $N^2 - 1$.

- Electroweak interactions are given by an SU(2) ⊗ U(1) doublet for the weak and for the electromagnetic parts of the interactions, such that the wave function is isoscalar for U(1) transformations and isovectorial for SU(2) transformations (for right-handed states, though; *v.i.*). Further, the fact of the fermionic multiplet possessing the dimension of the given gauge group entails that it have two components for electroweak interactions.
  Electroweak interactions *per se* do not distinguish between leptons and quarks. Hence, they form an isospinor of electroweak interactions (hence an isospin index m) with the isodoublet ψ^{mf}. It can be distinguished between leptonic ($m = 1 = l$, for electrons, muons, tauons and neutrinos) and quark dynamics ($m = 2 = q$ for the elementary constituent particles of nucleonic-like matter) where each component is an isodoublet on its own, *i.e.*
  $$\psi_{L/R}{}^{mf} = \begin{pmatrix} \psi^{lf} \\ \psi^{qf} \end{pmatrix}_{L/R} ,$$
  with $m = 1 = l$ for leptons and $m = 2 = q$ for quarks.
  For $m = 1$, on the one hand, left-handed electroweak states of the gauge group SU(2)$_L$ are given by the isospin vector
  $$\psi_L{}^{lf} = \begin{pmatrix} \nu_f \\ e_f \end{pmatrix}_L ,$$
  while right-handed states, with gauge group U(1), are given by the isoscalar
  $$\psi_R{}^l = e_R{}^f . \tag{2.2.1}$$

On the other hand, the quark-isodoublet of $m = 2$ is represented by the following isospin vector (see Appendix B.3)

$$\psi_{L/R}{}^{qf} = \begin{pmatrix} u_f \\ d_f \end{pmatrix}_{L/R} .$$

- Strong interactions are parity conserving and are described within Quantum Chromodynamics (QCD). This is a theory of quarks dynamics and within which all bare quarks are indistinguishable if mass is let aside.

  Neutrinos couple only weakly while electron-like particles couple electroweakly, *i.e.* weakly and electromagnetically. Quarks, on the other hand, couple electroweakly but also strongly. Hence, quarks have to appear as an isovector within strong-interaction transformations also. The isospin of the strong-interaction state is given by a new "strong" property named color which shall be carried by all quarks and takes the place of a sort of generalized charge of SU(3)$_C$ [101]. Hence the name "chromo" in QCD with the analogy to QED with (strong) color charges. Other than within QED, however, this charge exists in three sorts named blue, red and green (like the primary colors for additive combinations in color theory) and three "anti-sorts" (anti-blue, anti-red and anti-green, hence in the analogy cyan, yellow and magenta in the subtractive color mixing). Since differently colored quarks are isotopic to each other, the gauge group is of the dimension $N = 3$. The gauge group is called SU(3)$_{color}$ = SU(3)$_C$. The state then possesses three isospin components for each quark-type which is given by the so-called flavor f (see Appendix B.4). The isospin components are quarks of each color-charge. Hence, the state may be written as follows,

$$\psi_a{}^f = \begin{pmatrix} r_a{}^f \\ g_a{}^f \\ b_a{}^f \end{pmatrix} ,$$

  with the subscript a counting the color charge ($a = 1, 2, 3$) and the index f counting the flavor ($f = 1, \ldots, 6$). The three quark states, only differing by their color charge, thus form a triplet within strong interactions. The 3×3 matrices related to the transformation group SU(3)$_C$, generators of the group, are called Gell-Mann matrices.

  Color shall have analog properties to charge, and the eight gauge bosons called gluons[9] shall be analogue to gauge photons. They shall especially stay massless so that SU(3)$_C$ is an exact symmetry and both QCD- and QED- interactions are long-ranged. However, QCD is not abelian and, hence, gluons, unlike photons, self-interact. Additionally, they carry and mediate both a color- and an anti-color charge (see Appendix B.4).

  Physically, gluodynamics change the color of the constituent particles of hadrons in a way that, in the end, the "total" color of hadrons is vanishing (according to color theory) [101]. This is the process of confinement by which gluons are thought to self-interact in such a way that composite gluon states (glueballs) appear. These composite states acquire a dynamical mass which then leads to an effective, short-ranged, nuclear strong force although gluons themselves are massless [8]. In consequence, quarks move almost freely within hadronic ranges (asymptotic freedom) but cannot be detected as free particles since strong-interaction (color) forces should augment with distance. In Chapter 4.3, a method for explaining confinement using symmetry breakdown is introduced as part of the research

[9] From "glue" (or in scientific tradition from the Latin *glūten*), given that gluons are to be the particles which hold nucleons together.

within this work.

2.3 The types of matter and the dark sector problem

Experimental matter:

Before scalar fields are introduced, it is better to make at least some comments about the types of matter which are known experimentally together with their relation to the elementary particles of the SM. These particles are:

- **Quarks**: elementary and constituent fermions which appear in QCD under the SU(3)$_C$ triplet and under the isospin index $m = q$ of SU(2). There are 36 kinds of them counting matter, antimatter and different handedness.

- **Leptons** ($\lambda\epsilon\pi\tau\acute{o}\varsigma$: thin, light[10]): elementary fermions which appear for $m = l$ on SU(2). They do not interact within strong interactions. They appear in 18 elementary forms counting matter, antimatter and different handedness. The first experimental demonstration of higher-generation electron-like particles (the μ leptons) was achieved in 1937 [182], while the existence of neutrinos was first demonstrated in 1956 [60]. The existence of different generations of the latter was demonstrated in 1962 [63], and nonexistence of right-handed neutrinos (as well as of left-handed antineutrinos) within the SM follows from parity-conservation and conjugation (CP) violation.

- **Gauge bosons**: Interactions are given by elementary particles related to the gauge group. These are the gauge bosons. Hence, the elementary bosonic particles which carry the properties of interactions are related to gauge potentials and thus to transmission of forces on an elementary-particle level. There are 12 experimentally confirmed different kinds of gauge bosons. Electroweak gauge bosons were demonstrated experimentally in 1983 [10]. The first direct experimental evidence of gluons was found in 1979 (*e.g.* [40] along with other experiments at DESY[11]).

Quarks and gauge bosons, especially of strong interactions, are elementary constituent particles of hadrons. They may be categorized as **partons**. However, what is generally measured is macroscopic matter which is usually composite. Particles composed especially by quarks are called **hadrons**. However, hadrons may be further under-classified in baryons and mesons, which gives their statistics (Fermi–Dirac and Bose–Einstein).

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[10] Term which should denote the small mass of these particles in relation to that of nucleons; however, tauons (of the 3^{rd} and most massive generation of leptons) have *ca.* twice the mass of a proton.

[11] DESY: German Electron Synchrotron in Hamburg.

Class of composite	Constituents (partons)	Examples
HADRONS (H)	**QUARKS and GLUONS**	
H1) **Baryons**	**3 quarks OR 3 antiquarks**	
H1.1) *Nucleons*	up- and down- quarks (antiquarks) and gluons	proton neutron antiproton antineutron
H1.2) *Hyperons*	Strangeness$\neq$ 0 or Charm$\neq$ 0 or Topness$\neq$ 0 or Bottomness$\neq$ 0	Ω^- (3 strange-quarks) Λ_C^+ (up, down, charm)
H2) **Mesons**	**ONE quark and ONE antiquark**	
H2.1) *Flavorless*	up-, down- quarks and antiquarks and gluons	pion π^+ (up, anti-down)
H2.2) *Flavored*	up-, down, top-, bottom-, strange- quarks and antiquarks and gluons	kaon K^+ (up, anti-strange)

Nuclear matter constitutes only of quarks of the first (and less massive) generation (u and d). Such combinations are generally preferred energetically, and hyperons hence decay weakly onto nucleons plus mesons and leptons with a lifetime of the order of magnitude of 10^{-10}s.
Flavored mesons, further, decay onto normal (flavorless) mesons plus photons and leptons with analogue lifetimes. Hence, the main type of baryonic matter is nucleonic, and the main type of mesonic matter is the flavorless one.
However, neither baryonic nor hadronic matter in general, are the only types. There are photons and gauge bosons in general as well as leptons. These may appear as a non-baryonic class of nonhadronic matter as leptons may bound in composites such as **leptonia** (electron–antielectron-pairs, for instance). At the same time, gluons, for example, bound in so-called **glueballs** which are to acquire dynamic mass and may explain within the SM the short range of (effective) nuclear forces (*ca.* $2.5 \cdot 10^{-15}$m, in contrast to pure strong interactions, which are long-ranged, since gluons do not possess mass).[12]
The elementary nonhadronic (and thus nonbaryonic) matter is listed below:

[12] Through tunneling, Helium nuclei (α particles) split from the nuclear parent passing the potential barrier given by nuclear forces.

Class of matter	Constituents	Frequent symbol	Some properties
Bosonic matter	photons	A or γ	Mediate electromagnetism. Uncharged. No mass
	gluons	G_i (eight types)	Mediate strong interactions. Possess color- charge and -anticharge, (m=0!)
	weakons	W^+, W^-, Z^0	Mediate weak interactions. Lead to β-decay. Massive
Leptonic matter	electron- (positron-) analogues	$e^\pm$, $\mu^\pm$, $\tau^\pm$	Massive. Three leptonic generations with $m_e < m_\mu < m_\tau$
	neutrinos (antineutrinos)	ν_i and $\bar{\nu}_i$ $(i = e, \mu, \tau)$	Only (gravitationally and) weakly interacting. Nonvanishing small mass

The dark sector and supersymmetric particles:

- **Quantum gravity particles:**
  Within an elementary-particle physics theory of gravitation, there would exist another kind of gauge bosons which is the one of **gravitons** as field quanta of gravitation [195]. These are, however, not yet experimentally discovered.
  There is by now no complete quantum mechanical theory of gravitation. Yet, as gravitation appears to be a long-range interaction, analogy tells that gravitons are to be assumed as massless gauge bosons of gravity.
- **Supersymmetric particles:**
  However, there are other particles which might be by now not of experimental nature. Some of these might indeed be of special relevance in astrophysical contexts and do lead to astrophysical consequences by means of large particle masses. An especially relevant assumption is that there exists a symmetry between mesons and baryons [164], or yet more generally, between fermions (particles with odd spin) and bosons (particles with integer spin) [103, 109, 233, 237]. This symmetry (supersymmetry or SUSY) would relate every boson to a fermion and every fermion to a boson (so-called superpartners). Quark states would be related to (new) bosonic states called squarks while leptons would be related to (also new) bosonic states called sleptons. Hence, there would be more elementary bosons which would, further, not be gauge bosons. Bosons, on the other hand, would be related to fermionic states called bosinos (such as "gauginos" for the supersymmetric partners of gauge bosons, "gravitinos" for the partners of gravitons *etc.*).
  All supersymmetric particles, although strongly analyzed within the subject of supersymmetry in elementary particle physics and superstring theories, are not yet of experimental nature. Their physical status is yet to be clarified by experiments as the ones in process at the LHC. Yet, massive supersymmetric particles may be a class of matter necessary to comprehend dynamics correctly. Within the minimal supersymmetric extension of the SM, for instance, if the supersymmetric parity is preserved, the lightest supersymmetric particle will not decay. This particle, assuming it exists, may account for the observed missing mass of the Universe (*v.i.*).
- **Dark Matter phenomenology and baryonic DM:**
  Actually, it was in 1933 that Zwicky gained first evidence that according to standard dynamics,

new, non-luminous (dark), types of matter were necessary to explain the dynamics of the Coma cluster [253]. Missing matter was further determined in the years after, first for our Local Group of galaxies [138] and then for all giant galaxies [79, 187]. Furthermore, independent determination of rotation velocities of galaxies at large distances from galactic centers [211,212] confirmed the interpretation: the presence of nonluminous (dark) matter halos around galaxies. Its nature, though, is unclear, although it may in principle be some kind of hot gas [138] or possess a stellar origin [180]. It might further consist of a pregalactic generation of (very massive) stars [43]. Modern data, however, indicate that stellar dark matter cannot be dominant in dark-matter (DM) phenomenology [229], and gaseous halos cannot dominate either [89, 145, 220, 228]. Baryonic-gas DM cannot consist of neutral gas and ionized gas. Further, although present as indicated by X-ray analysis, it is not sufficient in galaxies to explain their flat rotation curves. Hence, mass-to-luminosity ratios of galaxies still indicate far higher masses than the one of visible matter. There seems to be some kind of matter which is nonbaryonic.

- **Leptonic DM:**
There is also the possibility to encounter leptons as DM. The possibility of heavy stable leptonic DM was examined in the early seventies [116]. Such candidates for DM dominance were, however, rejected in the year that followed [230]. Still, another kind of relevant non-baryonic, yet experimental leptonic DM type had started being considered from the early 70s on [61]: *neutrinos* as dark-matter candidates. As non-baryonic, further, they would help explain small temperature fluctuations of the cosmic microwave background radiation (CMB) [55].
Neutrinos and their antimatter counterpart comprise indeed a relevant category of physical particles which is especially relevant in an astrophysical description of matter towards DM phenomenology. They possess special rights for the category of DM since the crucial discussion is which kind of matter may be perceived (almost) only gravitationally, and neutrinos interact only very weakly, with a cross-section $\sigma_{\nu+n} \approx 7.1 \cdot 10^{-43}\text{cm}^2$. Neutrinos do not couple electromagnetically and are thus very difficult to detect directly. In an astrophysical context they are therefore called **Hot Dark Matter** (HDM). "Dark" because they lack electromagnetic coupling (which makes them very difficult to detect –after all, 25 years passed since their prediction by Pauli in 1930 [190], which happens even before neutrons were discovered, until their 1995 Nobel-prize awarded discovery by Reines and Cowan in 1956 [60]); "hot" because of the high velocity of neutrinos related to their almost, but according to neutrino oscillations [77] not vanishing, mass of maximally a few eV/c^2 [8]. However, given too low masses of neutrinos, they cannot be the dominant DM contribution either.[13]

- **Cold Dark Matter:**
Under the category of Dark Matter, it can thus only be acknowledged that it may be baryonic or nonbaryonic. A category of nonbaryonic DM is HDM. However, within the SM none of these types of DM explains the problem of the phenomenology of missing mass. Further, there may exist other types of exotic DM which are some kind of as-yet undiscovered matter. This matter

[13] Neutrinos appear especially in nuclear reactors (about 9 MeV of total 200 MeV per fission of ^{235}U) or proceed from the Sun. They appear in weak processes *via* β decay or electron capture. Hence, they can accompany ionizing radiation as α and β particles in decay chains. Screening of ionizing radiation depends on the shield's cross section for scattering and absorption as well as on its thickness. To avoid harmful exposure to radiation (*sc.* the LNT hypothesis) and minimize exposure dose when handling radioactive material, a screening of β particles needs of light material together with heavy one for shielding bremsstrahlung (high-energetic photons) from slowdown of the ionizing particles. Neutrinos cannot be shielded but they do not ionize any substance either as they are electrically neutral and hardly interact with other particles. Hence, they do not represent harmful radiation with biological consequences.

is generally called **Cold Dark Matter** (CDM). Particularly important likely candidates of it are axions[14] or light supersymmetric particles as neutralinos or gravitinos [30, 33].
Within supersymmetry, gravitinos are superpartners of gravitons of a quantum theory of gravitation, and neutralinos are quantum theoretical superpositions of the superpartners of the Z-bosons, of photons (neutral gauginos) and of neutral Higgs particles of supersymmetric theories (higgsinos). The latter are assumed to mix due to the effects of electroweak symmetry breaking (when both electromagnetic and weak become independent interactions, leading to massive weakons characterizing the broken symmetry). As heavy, stable particles, neutralinos, in particular, seem to be good candidates for Cold Dark Matter (CDM) as very weakly interacting massive particles (WIMPs). They are assumed to decay finally especially in τ-leptons, although decay channels including supersymmetric particles as neutral higgsinos, for instance, are also expected [159]. The neutralino mass is expected to be of over $100\,\mathrm{GeV/c^2}$, and evidence of annhihilation of such particles in regions which are expected to be highly "dark-densed" is hoped will be found in γ-ray and neutrino telescopes. The experimental mass constraint of neutralinos lie at masses higher than 46 GeV/c^2 for $m_{\tilde{\xi_1}^0}$, $m_{\tilde{\xi}_2{}^0} > 62.4\,\mathrm{GeV/c^2}$, $m_{\tilde{\xi}_3{}^0} > 99.9\,\mathrm{GeV/c^2}$ and $m_{\tilde{\xi}_4{}^0} > 40\,\mathrm{GeV/c^2}$, according to [8]. Charginos would have masses higher than $94\,\mathrm{GeV/c^2}$.

- **Theoretical viewpoint and exotic particles:**
  From the theoretical point of view, not only possibly still unobserved supersymmetric particles should be taken into account. There are also cosmological relics from symmetry-breaking processes which are predicted by high-energy physics that should be included in a list of Universe's components [144]. All these particles and fields, as far as they do really exist in the physical world, should have played a role in structure formation. They therefore imply the existence of an exotic part of the dark components of the density of the Universe (that is, of the components such as of dark matter which we do not directly see, or the nature of which is still unclear). However, cold dark-matter candidates are yet to be found in high-energy experiments and their nature has to be clarified in view of a demonstration that they are indeed capable of leading to DM phenomenology.

- **DM dominance and modified dynamics:**
  The conclusion within standard GR dynamics, citing [79], yields: "all giant galaxies have massive coronas [halos], therefore dark matter must be the determining component in the whole Universe (at least 90% of all matter)". On the other hand, though, although DM dominates at long ranges, locally, usual types of matter dominate [104, 146, 147, 185, 186]: there is no evidence for the presence of large amounts of dark matter in the disk of the Galaxy. If there exists dark matter near the galactic plane, then it is probably baryonic [80]. This complexity and non-local distribution of DM has been discussed as an indication to deeper, new physics, better described by more general models. Without knowing the nature of CDM particles, CDM cosmology in fact reproduces phenomenological data but does not have predictive power apart from the bare CDM halos themselves if the effects of normal matter on CDM are neglected [163]. Hence, alternative models have been discussed with the idea that Dark Matter phenomenology rather reflects deeper phenomena which are not yet rightly given within standard theories. Sanders'

[14] Strong CP symmetry shouldn't in principle have to be conserved. However, a breaking would be related to a yet unobserved neutron's electric dipole moment. In order to explain strong CP conservation, the axions, as (pseudo-) Goldstone particles (*cf.* Chapter 3.1) from the spontaneous breaking of the (global) Peccei–Quinn symmetry, are related to the effective strong CP-violating term which vanishes under the existence of these particles which then acquire mass by means of QCD vacuum effects (*h.t.* [191]).

model [215] (FLAG), for instance, adds a Yukawa potential to the newtonian potential, and reproduces rotation curves of galaxies ranging sizes from 5 to 40 kpc. Furthermore, Milgrom's model (MOND, MOdified Newtonian Dynamics) takes the phenomenology of missing mass as a signal of a breakdown of newtonian gravity [14], and it assumes a modification of Newton's law below a critical acceleration a_0 so that

$$F = m\mu(a/a_0)a \tag{2.3.1}$$

is valid for Newton's second law of motion, with $\mu(a/a_0) = 1$ for high accelerations a but with $\mu(a/a_0) = a/a_0$ for lower accelerations $a < a_0$. Herewith, the critical acceleration reads $a_0 = 1.2 \cdot 10^{-10}\mathrm{ms}^{-2}$, which is very close to the cosmological value provided by the Hubble rate H with $a_H = Hc$. It is also close to the observed acceleration a_Λ gotten from the expansion rate of the Universe [163]. Furthermore, subsequently to Milgrom's approach, there is the constant tangential velocity

$$v_t = \sqrt[4]{G_N M_1 a_0} \tag{2.3.2}$$

for rotation curves of galaxies (with mass M_1) outside of their luminous cores [15]. Hence, phenomenology of Dark Matter appears as consequence of new dynamics.

There have been approaches to further generalize modification-approaches into covariant formalisms. For instance, Tensor–Vector–Scalar gravity (TeVeS) reproduces MOND in the nonrelativistic limit with the possibility to explain gravitational lensing. TeVeS incorporates various dynamical and non-dynamical tensor, vector and scalar fields [15]. A further approach is Moffat's Scalar–Tensor–Vector gravity (STVG) [165] or Modified Gravity (MOG), which postulates the existence of a vector field while elevating the three constants of the theory to scalar fields. In the weak-field approximation, this theory produces a Yukawa-like modification of the gravitational force due to a point–source so that far away from a gravitational body, gravity be stronger than according to newtonian law. At shorter distances, gravity is to be counteracted by a repulsive force from the vector field. STVG has been successfully used to reproduce flat rotation curves of galaxies among other phenomena without the necessity of Dark Matter [39]. It further leads to non-singular spherically symmetric solutions (grey stars) [167] and to non-singular cosmologies with a bouncing universe without cosmological constant [166, 168]. Furthermore, there are formal analogies to further approaches as [70] which has also been used to account to the phenomenology of Dark Matter [20, 179]. Further, following [71], Higgs particles, which are expected to be found in the LHC in Geneva, would decouple and remain stable. In this case, negative results from high-energy experiments would sign to such a changing of dynamics (see Chapter 6 and later).

In short, the nature of DM is still unclear and a matter of discussion. Cold Dark Matter candidates are still no experimental reality, and alternative models of altered dynamics have been able to successfully account for explanations of phenomenology.

2.4 Dark-energy density and density parameters

Cold Dark Matter is usually defined within the dark sector of energy density of the Universe. Another sector is given by baryonic matter which contributes to about 10% of total matter density. Other, however small contributions to matter density would come from neutrino masses, leptons and so on. A universe in which

matter density gives the total energy there is, is known as Einstein–deSitter Universe. There, the energy density equals exactly the energy density needed for the universe to be flat (ϵ_c) (*cf.* Chapters 2.4 and 8.4). However, Einstein himself introduced back in 1917 the concept of the cosmological constant [82] which would act against gravitational attraction if the constant were positive. Einstein's cosmological constant acts against gravity, or equally, as having a negative pressure. The idea was to get a closed universe which would be static also. For this, Einstein replaced $G_{\mu\nu}$ in his equations (A.4.3) by $G_{\mu\nu} + \Lambda_0\, g_{\mu\nu}$. Λ_0, further, is interpretable as the energy density of vacuum.
If we define density parameters

$$\Omega_i = \frac{\varrho_i}{\varrho_c} = \frac{\epsilon_i}{\epsilon_c}, \tag{2.4.1}$$

whereas $\epsilon_i = \varrho_i c^2$ is the energy density to the mass density ϱ_i, and $\varrho_c = 3H_0^2/(8\pi G)$ is a critical density defined in terms of G, c and the Hubble constant H_0 (which, on the other hand, is a measure of the cosmic expansion), then we have

$$\Omega_{total} = \Omega_{Baryons} + \Omega_{CDM} + \Omega_\Lambda + \dots, \tag{2.4.2}$$

whereas $\Omega_{Baryons}$ and Ω_{CDM} give the most relevant terms of matter density Ω_M. Ω_Λ gives the density parameter of the cosmological constant / energy of vacuum, with the energy density ϵ_Λ. When finite, ϵ_Λ is to represent an energy of non-electromagnetical nature. Thus, it can be denoted as "dark". Further, its nature is not clarified. Hence, its entitled **Dark Energy**, and it is the second dark sector of cosmology. Furthermore, if non vanishing, then it is possible that it be constant, exactly as within Einstein's approach (Λ_0), or a function of time with a more complex nature (Λ).
A particular candidate for Dark Energy is the scalar field commonly known as Quintessence or "cosmon" field [192, 238] as a theoretical carrier. This is generally coupled minimally to gravitation in modern standard theories, or with a scalar field coupling to R which stays almost constant (*cf.* [124]). Cosmologies containing a barotropic fluid plus a scalar field may lead to late-time attractors (*cf.* [58]), and a coupled system of gravity and a scalar field may induce a further time-dependent term in the energy–momentum tensor which would adjust itself dynamically [239]. Hence, there appears a composition-dependent gravity as a long-range force [242] mediated by the quintessence particles. Quintessence particles can further behave similarly to relativistic gases [241] and be associated to DM [240].
Quintessence is related to the cosmological constant (*sc.* [78]). The latter, however, represents a special case of Dark Energy that does not change with time (*cf.* [193]) but which should also be explained within a quantum theory of gravitation.
Dark Energy is related to the phenomenon of cosmic acceleration (see Chapters 2.4 and 8), and some theories as Supergravity lead naturally to antigravity indeed [217]. Antigravitative interactions would lead to a repulsion of matter after the Big Bang.
A universe with positive spatial curvature ($K = 1$) with a nonvanishing cosmological constant is known as Lemaître's universe [152]. The expansion parameter in such a universe is always increasing but there is a period in which it remains practically constant. Thereafter, a further period of expansion follows. During the 1970s, this model invoked to explain the apparent concentration of quasars at a redshift of $z \approx 2$ [56]. However, given that subsequent data falsified this assumption, for a long time Dark Energy became strongly believed to be vanishing. Actually, Einstein himself called the cosmological constant his biggest blunder ("die größte Eselei meines Lebens"). Yet, already works as [29] and [129] propose a nonvanishing, however over-abundant cosmological constant for a slightly closed ($K = 1$, $\Omega_{total} \gtrapprox 1$) baryonic-matter dominated Universe. Still, until the late decade of the 1990s, there were only few strong empirical data which would

point to antigravitation. Further, most experimental data up to that point actually preferred an exactly vanishing Λ and an Einstein–deSitter (closed) Universe (*sc.* [197]).
It was only in the last decade of the 20^{th} century that the assumption of a vanishing Λ began to fall apart. A nonvanishing value for Dark Energy was measured within the context of GR for Super Novae of type Ia (SNeIa) as extragalactic distance indicators [98, 198, 208].[15] In the years that followed, the results were corroborated. Thus, cosmic expansion seems to be accelerated, indeed. However, by now it is unclear whether the value of this dark energy (as antigravitative component) stays constant in time, as a true cosmological constant Λ_0, or whether today's dark-energy component is a remainder of some cosmological function. This function should contribute as $\Omega_\Lambda \approx 0.7$ today to the total density parameter Ω_T of the hodiernal Universe.
Nowadays' standard measured values of the models are

$$\Omega_M = 0.127h^{-2} \tag{2.4.3}$$

for matter, including

$$\Omega_B = 0.0223h^{-2} \tag{2.4.4}$$

for baryons, and

$$\Omega_{DM} = 0.105h^{-2} \tag{2.4.5}$$

for Dark Matter. $h = 0.73$ gives the normalized modern Hubble expansion rate.
For neutrinos, the constraint lies at

$$\Omega_\nu < 0.007h^{-2}, \tag{2.4.6}$$

and the cosmological-constant density reads

$$\Omega_\Lambda = 0.76. \tag{2.4.7}$$

According to the three-year results of WMAP, the total energy density parameter lies around [223][16]

$$\Omega_T = 1.003^{+0.013}_{-0.017}. \tag{2.4.8}$$

An exact value of 1 means a curvature $K = 0$ of a flat universe, while higher values mean a closed universe with $K = 1$, and lower ones indicate an hyperbolic universe with $K = -1$. Hence, observational values point to a dark-energy dominant Universe with almost only dark sectors and with an (almost) flat geometry. Furthermore, ideas of a very highly accelerated (inflationary) phase of the Universe which explain horizon and flatness problems of cosmology do account to this interpretation.
The concept of primeval, cosmic Inflation was first proposed by Alan Guth in 1981 [118], based on ideas of Starobinsky's work [224]. It was later improved by Albrecht, Steinhardt [1] and Linde [154]. Often, an hypothetical scalar field, namely the inflaton field, is proposed in this context. Further, it can be reproduced with induced gravitation also [47–49]. In all ways, this phase is interpretable as a phase in which

[15] SNe are variable stars which (simplified) result from a violent explosion of a white dwarf star which has completed its normal stellar life and where fusion has ceased. After having ignited carbon fusion, the released energy and subsequent collapse has unbound the star in the supernova explosion. For the type Ia especially, the spectrum shows a lack of hydrogen lines but indicates singly-ionized silicon.

[16] The best fit of WMAP reads for the five-year results: $\Omega_T = 1.099^{+0.100}_{-0.085}$. The best fit of WMAP plus Super Novae (SNe) and baryon acoustic oscillations reads $\Omega_T = 1.0050^{+0.0060}_{-0.0061}$ [128].

negative pressure dominates so that a deSitter epoch appears (see also Chapter 8.7). However, it is still unclear whether the pressure term of dark-energetic sectors is constant or not. If Dark Energy components should change in time, though, the scalar field of Quintessence might be one that acts on local planetary [95] or at galactic scales [160]. Moreover, if coupled nonminimally to gravity, such massive fields might even account to both the phenomenology of Dark Matter [20, 210] and Dark Energy [21, 179]. Actually, the cosmology of scalar–tensor theories, *i.e.* theories with curvature coupled nonminimally to scalar fields, leads naturally to cosmic acceleration [44]. This makes scalar fields of such theories the natural candidates to be quintessential-like fields [7, 32, 42].

Chapter 3

Symmetry breaking and scalar fields

– The concept of symmetry breaking in its different modes and especially the Higgs Mechanism of mass generation are discussed in their relevance for the different subjects of physics, especially in relation with the Standard Model (SM) of elementary particle physics. Higgs and Goldstone fields are presented together with unitary gauge and mass terms of the SM. This Chapter is related to the work published in [22]. –

3.1 Symmetry breaking and breaking modes

The question of whether scalar fields exist at all is still open. However, approaches for primeval Inflation and of Quintessence of Dark Energy ground on some kind of scalar fields which, therefore, may contribute to some kind of dark sector of density. Furthermore, the Higgs field, a special kind of scalar field, is necessary for symmetry breaking (SB) indeed, as Yang–Mills theories for elementary interactions are non-physical without some kind of breakdown of symmetry which may lead to the appearance of mass in accordance to empirical data (*viz* weak CP breaking, as in [244]).
There are three main modes of symmetry breaking, depending on the properties of the field's ground state. These are [115]

(i) the Wigner–Weyl mode, usually called only Wigner mode,

(ii) the Nambu–Goldstone or Goldstone mode,

(iii) the Higgs–Kibble or Higgs mode.

About the symmetry-breaking modes:

- The Wigner–Weyl mode:[1]
  In particular, the Wigner–Weyl mode is the most usual symmetry-breaking mode in quantum mechanics (QM), with a real invariant vacuum which can be identified with the classical one as follows in virtue of the Dirac vector $|0>$ for vacuum and a unitary transformation (time evolution) U acting on the same,

$$U|0>= |0> . \tag{3.1.1}$$

[1]After Eugene Paul Wigner (1902-1995); Hermann Klaus Hugo Weyl (1985-1955).

The Wigner–Weyl mode is indeed related to the existence of degeneracy among particles in the multiplet structure of spectra. The violation of symmetries involves here explicit symmetry-breaking terms in the Hamiltonian H or in the Lagrangian which lift the multiplet degeneracies. Such situation appears in the Zeeman effect: given a spherical symmetric system such as an atom, in the absence of external fields the wave functions form degenerate SO(3) multiplets as a consequence of the conservation of angular momentum. If we now place a magnetic field along an axis, the rotational symmetry is lost since a preferred direction has been selected in space. The corresponding nondegenerate multiplet structure is the Zeeman effect, and when it appears, SU(2) symmetry has been broken down to U(1) since the system is still invariant under rotations about a single axis.[2]
Another case of a Wigner mode may be given by the SU(N) isovector which may be (for instance) the SU(2) multiplet structure of isospin. It is the rest-group of the SU(3) flavor multiplet. SU(2) breaks from that symmetry due to effects of hypercharges. Furthermore, this symmetry of isospin is also broken to U(1) charge symmetry by terms of Coulomb interactions that select a preferred direction in isospin space. However, the U(1) symmetry remains unbroken because of current conservation law [115].

- The Nambu–Goldstone and Higgs–Kibble modes:[3]
  Further, in the Nambu–Goldstone and Higgs–Kibble modes, the symmetry is actually not lost but camouflaged and hidden in the background of the mass generation by scalar fields. It is usually spoken about spontaneous breaking of the symmetry. However, on detail, it is sometimes differentiated between a dynamical and a spontaneous symmetry breaking (SSB) by virtue of the nature of the scalar field which leads to the breaking. Both kinds of these symmetry breakdowns (SB) through scalar fields differ in the following way:

  - Dynamical SB: The Higgs field is a composite particle such as a meson, for instance, or a Cooper pair as within superconductivity.
  - Spontaneous SB: The Higgs field is elementary.

  Both symmetry-breaking processes which belong to the Nambu–Goldstone and to the Higgs–Kibble mode or mechanisms of symmetry breaking are very important within many aspects of physics, such as condensed-matter physics (where they first appeared) and elementary-particle physics (where it is spoken about elementary fields). For instance, within QCD, SB leads to the Peccei–Quinn mechanism (*v.s.* in Chapter 2.3). Furthermore, the Higgs mode of spontaneous symmetry breaking is of special relevance as a basis for the SM of particle physics as a whole. Further, the differentiation between fundamentality and compositeness of Higgs fields is usually not declared specifically. Hence, the terminology of SSB is usually used for both dynamical and truly spontaneous SB. Both may be explained analogously to each other, be on the grounds of a fundamental mechanism or of an effective one.
  For the understanding of the concept of spontaneous breakdown of symmetry, let us consider a system whose Lagrangian $\mathcal{L}$ possesses a particular symmetry, which means that its Lagrangian is invariant under the corresponding symmetry transformations. $\mathcal{L}$ may, for instance, be spherically symmetric, *i.e.* invariant under spatial rotation. Two situations are then possible when classifying energy levels

[2] The Zeeman effect (Nobel prize 1902) is related to a momentum–field-strength coupling, ESR and NMR, *cf.* Appendix B.1

[3] After Jōichirō Nambu (1921), Nobel prize 2008; Jeffrey Goldstone (1933); Peter Higgs (1929) and Thomas Walter Bannermann Kibble (1932).

of this system [157]: if a given energy level is non-degenerate, the corresponding energy eigenstate is unique and invariant under the symmetry transformations of $\mathcal{L}$. On the other hand, the level may be degenerate and the eigenstates not invariant but able to transform linearly amongst themselves under symmetry transformations of the Lagrangian. Let us further consider the lowest energy level of the system. If it is not degenerate, the state of the lowest energy of the system (the ground state) will be unique and possess the symmetries of $\mathcal{L}$. In the case of degeneracy, there will not be a unique eigenstate to represent the ground state. Arbitrarily selecting one of these degenerate states as ground state will lead to the ground state not sharing the symmetries of the Lagrangian. The symmetry will be broken for the ground state. We have spontaneous breakdown of symmetry (which may be dynamical, though; *v.s.*). The asymmetry is, however, not due to adding a non-invariant asymmetric term to $\mathcal{L}$ but to the arbitrary choice of one of the degenerate states.
A further example of dynamical or spontaneous symmetry breaking may be found in ferromagnetism [157]: In a ferromagnetic material, the forces which couple the electronic spins and hence the Hamiltonian of the system are rotationally invariant. However, in the ground state the spins are aligned in some definite direction resulting in a finite magnetization $\vec{M}$. The orientation of it is arbitrary. Thus, we have a case of degeneracy. Furthermore, excited states obtained from the ground state by small perturbations also display this asymmetry.
In quantum field theory, the state of lowest energy is the vacuum, and spontaneous symmetry breaking is only relevant to field theory if the vacuum state is non-unique (else, there is a Wigner mode). Y. Nambu [169–171] recognized in the context of superconductivity that in models exhibiting spontaneous breakdown of continuous symmetries new particles had to appear. For this discovery, Nambu was awarded the Nobel prize in 2008. Furthermore, J. Goldstone [107, 108] recognized the same soon-after and systematically generalized the concept into quantum field theory. It implies that some quantity in the vacuum is non-vanishing, not invariant under symmetry transformations of the system, and can therefore be used to characterize a particular vacuum state as *the* ground state [157]. Usually, this quantity is taken as the vacuum expectation value of a quantized field. This field, further, must be a scalar field ($\phi(x)$) so that the vacuum states are invariant under Lorentz transformations. Further, the vacuum expectation value must be constant, so that

$$< 0|\phi(x)|0 >= \phi_0 = const \neq 0 \tag{3.1.2}$$

is valid for the mean value with a ground-state configuration ϕ_0. The appearing particles are spinless bosons which correspond to the broken internal symmetry generators. Some are massive (generally called Higgs) and the others, usually called Nambu–Goldstone bosons or simply Goldstone bosons, are massless. Their vanishing mass is a consequence of the degeneracy of the vacuum, and such bosons frequently occur in theories with spontaneous symmetry breaking:
Goldstone Theorem*: If a continuous global symmetry is broken spontaneously, for each group generator there must appear in the theory a massless particle [115].*
However, no Goldstone bosons are observed in nature, and it is hence of crucial interest that gauge theories with spontaneous (or dynamical) symmetry breaking do not generate them [157]. This is achieved *via* the Higgs mode. The two modes using Higgs fields (composite or elementary) differ from each other through their gauge symmetry while both of them are given by the vacuum defined as follows,

$$U|0 >\neq |0 > . \tag{3.1.3}$$

The Nambu–Goldstone mode, however, works globally while the Higgs–Kibble mode acts locally in view of gauge invariance. As a consequence, the main difference between them is that in the Nambu–Goldstone mechanism both massive (Higgs) and massless (Goldstone) particles appear, while in the Higgs mechanism only the massive particles are present and the mass acquisition of gauge bosons is at the cost of the Goldstone particles, which are to gauge away unitarily. The degrees of freedom of the massive particles, however, won't disappear from the physical spectrum of the theory. In general sense, the gauge fields will absorb the Goldstone bosons and become massive while the Goldstone bosons themselves will become the third state of polarization for massive vector bosons [115].
The elimination of Goldstone bosons from the theory giving mass to the gauge quanta was independently worked out by P. Higgs himself [126] as well as by R. Brout and F. Englert [85] and by G. Guralnik, C.R. Hagen and T. Kibble [117] (hence, the Higgs–Kibble mechanism is sometimes called Brout–Englert–Higgs mechanism). The mass generation by Higgs mechanism, however, can further be identified in the Meißner (or –as following– Meissner) effect of conventional superconductivity (hence applicable in nonrelativistic theories [112] in form of a dynamical breaking) [178]. Goldstone bosons can be made to disappear in the presence of long-range forces [9]. An analogy between the Higgs mechanism and the Meissner effect may be explained in terms of the Yukawa–Wick interpretation of the Higgs mechanism where long-range forces as Coulomb interactions are mediated by massless exchange particles. The long-range force, then, is shielded by the Goldstone field and becomes short-ranged. Transcribed by means of Yukawa's theory, an effective mass of the gauge boson was generated. The condensed electron-pairs (the Cooper pairs) in the ground state of a superconductor may then be identified with a Higgs field for dynamical symmetry breaking. The Higgs field then leads to the magnetic flux expulsion with a finite range given by the penetration depth, which further gives the reciprocal effective mass acquired by the photons [115] (*cf.* Chapter 4).

In the processes of symmetry breaking, the symmetry group G breaks down to a rest-symmetry group $\tilde{G}$ (*i.e.* $G \to \tilde{G}$) with

$$\tilde{G} = \bigcap_{r=1}^{n} \tilde{G}_r, \tag{3.1.4}$$

where $n > 1$ is valid in case of more than one breaking process. In the SM of particle physics, for instance, the following breaking processes are valid,

$$\text{SU(3)}_C \otimes \text{SU(2)}_L \otimes \text{U(1)}_Y \to \text{SU(3)}_C \otimes \text{U(1)}_{em}, \tag{3.1.5}$$

while for the grand unified theory (GUT) under SU(5) (Georgi–Glashow model, see [102]), to give a further example of theoretical approaches, another breaking process takes place at energies of about 10^{15}GeV,

$$\text{SU(5)} \to \text{SU(3)}_C \otimes \text{SU(2)}_L \otimes \text{U(1)}_Y. \tag{3.1.6}$$

Within GUTs, at high energies, all elementary (quantum) interactions are to unify into one interaction which relies on the special unitary group with five isotopic particles. The breaking process onto rest-symmetry groups is related to a breaking of symmetry when energy scales are low enough and the ordered state becomes unstable. This process of breaking of symmetry is characterized by the scalar field as identifier of disorder in terms of a (dis-)order parameter identified with the scalar field itself. This parameter

is the Ginzburg–Landau (also Ginsburg–Landau) parameter which gives the most likely state of a system (*cf.* [105]). It becomes nonvanishing when order, symmetry, is broken. Furthermore, this is identified with the appearance of particles which are again related to the scalar field.
The SM is given by a product group entailing color dynamics of QCD, electroweak interactions and a mixed interaction of hypercharges. For low energies, this leads to electromagnetism while weak processes disappear. GUT, on the other hand, describes a unified interaction where the left-hand isovector entails the five elementary fermions, antielectron, neutron and three quarks of different color as indistinguishable, isotopic particles under GUT (very-high energy) interactions. The right-hand state, further, is a matrix. Given the state for GUT under SU(5), apart of gauge bosons, there have to exist in total 24 gauge bosons which have to lead to decay processes from leptons to quarks which are forbidden under the SM. As a consequence, free protons would decay. However, no such signatures have been found and the lower limit of the proton half-time lies at $6.8 \cdot 10^{33}$years [183]. Still, Georgi–Salam's model represents the best pedagogical example of a unifying model with more than one breakdown of symmetry. Furthermore, within GUT, symmetry breaking is spontaneous, as it is within the QAD in Glashow–Salam–Weinberg's model of the SM. This comprises the necessity of addition of the terms of a new particle into the Lagrangian in form of a scalar field $\phi(x)$. For both the Nambu–Goldstone and the Higgs–Kibble mechanism, a new field has to be postulated. This is called Higgs field.

3.2 Higgs fields and Higgs kinds

In general, the simplest way to generate the spontaneous breakdown of symmetry is to introduce a Higgs field Lagrangian term corresponding to a bosonic scalar particle with kinetic energy density $\mathcal{T} = (1/2)\phi^\dagger_{;\mu}\phi^{;\mu}$ and self-interaction given by a potential density $\mathcal{V}(\phi)$,

$$\mathcal{L}_H = \mathcal{L}(\phi) = \frac{1}{2}\phi^\dagger_{;\nu}\phi^{;\nu} - \mathcal{V}(\phi). \tag{3.2.1}$$

The self-interaction potential density is called Higgs potential $\mathcal{V}(\phi)$ with

$$\mathcal{V}(\phi) = \frac{\mu^2}{2}\phi^\dagger\phi + \frac{\lambda}{4!}\left(\phi^\dagger\phi\right)^2 + \breve{\mathcal{V}}\,, \tag{3.2.2}$$

where $\mu^2 < 0$ and $\lambda > 0$. Such theories are called ϕ^4-theories. For

$$\breve{\mathcal{V}} = \frac{3}{2}\frac{\mu^4}{\lambda}, \tag{3.2.3}$$

the minimum of the potential is lowered so that energy density for vanishing scalar fields is defined as zero with

$$\mathcal{V}(\phi_0^\dagger\phi_0) = 0 \tag{3.2.4}$$

for the ground state (ϕ_0) of the scalar field, and with hermitean conjugate $\phi^\dagger\phi = \phi^*\phi$ in case of isoscalar fields ϕ, and with the transpose ^{T} in case of isospinors. The additive term $\mathcal{V}$ does not appear in Chapter 6.1 but, as it will be seen, the choice of the minimum of the potential is related to the election of a vanishing formal cosmological constant which, however, can be avoided in the theory by adding a constant term

$$\mathcal{V}_{\Lambda_0} = -\frac{3\breve{\alpha}}{4\lambda}\mu^2\Lambda_0, \tag{3.2.5}$$

with Λ_0 as the cosmological constant and with a total potential of the form

$$\mathcal{V}_T(\phi) = \mathcal{V}(\phi) + \mathcal{V}_{\Lambda_0}. \tag{3.2.6}$$

A cosmological function $\Lambda(\phi)$ which is dependent on this generalized Higgs potential appears, as it may be seen in Chapter 6.2.[4]
The ϕ^4 term in the potential (3.2.2) is not bilinear, and it is crucial for the apparent symmetry breakdown. The Lagrangian given by equation (3.2.1) is invariant under spatial inversion (*i.e.* $\phi \to -\phi$) with the features of the tachyonic condensation (*i.e.* condensate for an imaginary mass with $\mu^2 < 0$). Such conditions are needed to stay within the Higgs–Kibble mode, which otherwise becomes a Wigner–Weyl mode with classical vacuum where self-interactions lack to produce the necessary Higgs mechanism at the relatively low energies of the hodiernal Universe. Furthermore, these considerations lead to further properties which are essential of Higgs fields in general.
Be a general Higgs field defined as (*cf.* [93])

- a field with a non-trivial, *i.e.* nonvanishing vacuum state.
  This kind of fields have the property of breaking the symmetry of a theory in a group G on the rest-symmetry to the isotropy group $\tilde{G}$ of the vacuum state spontaneously.

- Moreover, every Higgs field in a field theory interacts gravitationally with the particles with which it couples (*sc.* [68, 69]).

  – A usual symmetry group is G_{QAD} =SU(2)$_L\otimes$ U(1)$_Y$ which breaks into U(1)$_{em}$ in the standard model for the electroweak interaction.

  – In this sense, a Higgs field is more generally defined as only the Higgs field within the SM. Furthermore, if a Higgs field is coupled nonminimally to the curvature scalar R, some characteristics may easily differ from the ones of standard Higgs of the SM. Other important characteristics are open and have then to be given through the Lagrange extensions as is the case within the SM, too.

3.3 Symmetry Breaking and the SM

The SM of elementary particle physics has been remarkably successful in providing the astonishing synthesis of the electromagnetic, weak and strong interactions of fundamental particles in nature [151, 199]. In the Glashow–Salam–Weinberg (GSW) theory without symmetry breaking neither Yang–Mills equations nor the Lagrangian itself possess mass terms at all. Only for QCD processes, a mass term with mass m_f may be defined, given conservation of parity symmetry in strong interactions. At this point of considerations, the GSW theory describes massless fermions and leptons. Hence, it cannot describe nature as we know it. Such a mass, further, cannot be achieved adding a new mass term to its Lagrangian. Such would break with phenomenology of electroweak dynamics. These are characterized by parity violation, and with an added mass term, left- and right-handed particles would couple in the same way to vector bosons in order to

[4] There is $V = \int \mathcal{V} d^3x$ and $T = \int \mathcal{T} d^3x$. However, usually no formal difference is made between potential density $\mathcal{V}$ and potential V. From Chapter 4 on, we will no further differ between them explicitly.

preserve gauge invariance (as they do within QCD). Further, if mass is simply added, a massive propagator, which gives the probability amplitude for a particle to travel from one point to another in a given time or to travel with a certain energy and momentum (in this case for massive virtual particles; *cf.* Appendix B.1) would not lose its longitudinal term. The propagator does not transform into a (transversal) massless one in the limit $M_1 \to 0$ for mass M_1 [139]. As a consequence, when adding masses, most Feynman graphs would diverge, and this would lead to the mass-containing GSW theory not to be renormalizable.[5] For instance, for the Procca equation

$$\partial_\nu F^{\mu\nu} - M_1^2 A^\mu = -4\pi j^\mu(\psi), \tag{3.3.1}$$

i.e. for the Yang–Mills equation with mass term, there is the Green function given by the Fourier-transformed

$$G_\sigma{}^\nu(p^\alpha) = \frac{\delta_\sigma{}^\nu - \frac{p^\nu p_\sigma}{M_1^2}}{-p_\lambda p^\lambda + M_1^2} \,. \tag{3.3.2}$$

For $M_1 \to 0$, this Green function diverges and hence, the massless Yang–Mills equation does not possess a Green function. The only known alternative is symmetry breaking for mass to appear as a consequence of symmetry properties of the Lagrangian in vacuo. Yang–Mills theories combined with the so-called Higgs mechanism of symmetry breakdown, grounding on Nambu's work as a mechanism of spontaneous broken symmetry in subatomic physics, lead to the SM of elementary particle physics. The predictions of the latter, such as the existence of weakons and gluons, have been very successful. The only missing piece of the SM are the Higgs particles.

According to the SM, inertial as well as passive gravitational mass[6] are introduced as generated simultaneously with respect to gauge invariance by the interaction with a scalar Higgs field through the SSB. Then, considering the Higgs field for small enough energy scales, the Higgs field couples to matter. By means of this interaction, matter no longer moves as fast as the speed of light. It spontaneously possesses mass. However, the latter is generated or explained in the theory by an interaction between particles (however only within elementary-particle physics and not within GR).

The Higgs mechanism of SSB [126] provides a way for the acquisition of mass by the gauge bosons and fermions in nature, reducing mass to the parameters of the Higgs potential. These parameters and properties can easily be described by means of an isoscalar field. For the SM, though, an isovectorial field has to be defined for the acquisition of different masses for every component of the fermionic state.

- Isoscalar Higgs fields:
  For isoscalar Higgs fields as in Chapter 3.2, the Euler–Lagrange equations without extra term (3.2.3) give (for a hermitean fluid, there is $\phi^\dagger = \phi^* = \phi$)

  $$\left[\partial_\nu \partial^\nu + \mu^2\right] \phi + \frac{\lambda}{3!}\phi^3 = 0\,. \tag{3.3.3}$$

  There is the energy–stress conservation of ϕ. The canonical energy–stress tensor reads as follows,

  $$T_\nu{}^\mu = \frac{\partial \mathcal{L}(\phi)}{\partial \phi_{,\mu}} \phi_{,\nu} - \mathcal{L}(\phi)\delta_\nu{}^\mu, \tag{3.3.4}$$

[5] The proof of this and how to renormalize QAD, *viz* [132], was Nobel-awarded in 1999 for 't Hooft and Veltman.

[6] Inertial mass is defined as a measure of an object's resistance to the change of its position due to an applied force. Passive gravitational mass is a measure of the strength of the gravitational field due to a particular object (see [22], especially in relation with symmetry-breaking modes and the Higgs mechanism). Although conceptually different, Einstein's principle of equivalence asserts that they are equal for a given body, and this has been well-grounded experimentally.

and the energy density ϵ is its 0–0 component,

$$\epsilon(\phi) \;=\; \frac{1}{2}(\partial_0\phi)(\partial_0\phi) + \frac{1}{2}\sum_{a=1}^{3}(\partial_a\phi)(\partial_a\phi) + \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4, \quad a = 1,2,3. \tag{3.3.5}$$

With the possibility of tachyonic condensation, the ground state ϕ_0 becomes twice degenerate and $\phi_z = 0$ has a maximal value for the energy density ϵ. The ground state for the Higgs potential without $\breve{\mathcal{V}}$ is given by

$$\epsilon_0 = \epsilon(\phi_0) = -\frac{3}{2}\frac{\mu^4}{\lambda} \equiv \epsilon_{min} = -\breve{\mathcal{V}}, \tag{3.3.6}$$

and

$$\phi_0^{(\pm)} = \pm\sqrt{-\frac{6\mu^2}{\lambda}} = v. \tag{3.3.7}$$

v is the vacuum expectation value. Regions with different ϕ_0-values are called topological defects. Those changing the values $\phi = v \leftrightarrow -v$ are termed interface domains.
In fact, the energy of the system is low and ϕ lies near the minimum of energy. It is, therefore, possible to expand the scalar field around its minimal state with its excited values $\hat{\phi}$ in the following form:

$$\phi = v + \hat{\phi}. \tag{3.3.8}$$

The Lagrangian (3.2.1) may now be given in isoscalar form (only up to second-order terms) as follows,

$$\mathcal{L}(\hat{\phi}) = \frac{1}{2}\hat{\phi}^{\dagger}_{,\nu}\hat{\phi}^{,\nu} - \frac{M_H^2}{2}\hat{\phi}^2 - \frac{\lambda}{3!}v\hat{\phi}^3 - \frac{\lambda}{4!}\hat{\phi}^4 \;\neq\; \mathcal{L}(-\hat{\phi}). \tag{3.3.9}$$

The first term in the Lagrangian (3.3.9) corresponds to the kinetic energy of the Higgs field while the second one represents the mass term of the standard Higgs field (*i.e.* $M_H^2 \equiv -2\mu^2$). In fact, due to the presence of the term for the excited field (*i.e.* $\hat{\phi}^3$) in the Lagrangian (3.3.9), the symmetry is suddenly broken as the Lagrangian (3.3.9) is not spatially invariant anymore.

- Isovectorial fields:
  In the case of an isovectorial Higgs field with isocomponent ϕ_a, the treatment of Higgs fields is analogous. The Lagrangian (without constant terms (3.2.3) and (3.2.5) of the potential) is given by

$$\mathcal{L}(\phi_a) = \frac{1}{2}\left(D^{\dagger}_{\mu b}{}^{a}\phi^{\dagger b}\right)\left(D^{\mu}{}_{a}{}^{c}\phi_c\right) - \left(\frac{\mu^2}{2}\phi^{\dagger a}\phi_a + \frac{\lambda}{4!}(\phi^{\dagger a}\phi_a)^2\right). \tag{3.3.10}$$

The Higgs field is now complex (an isospinor) and by means of the covariant derivative

$$D_{\mu a}{}^{b}\phi_b \equiv \delta_a{}^{b}\partial_\mu\phi_b + igA_{\mu i}(\tau^i)_a{}^{b}\phi_b, \tag{3.3.11}$$

it couples with the gauge field A_μ.
The Higgs field equation reads

$$D^{\mu}{}_{a}{}^{b}D_{\mu b}{}^{c}\phi_c + \mu^2\phi_a + \frac{\lambda}{3!}(\phi^{\dagger b}\phi_b)\phi_a = 0. \tag{3.3.12}$$

Analogously to the isoscalar form, the ground state $\phi_{(0)}$ reads

$$\phi^{\dagger a}_{(0)}\phi_{(0)a} = -\frac{6\mu^2}{\lambda} \tag{3.3.13}$$

with the the VEV as the length,

$$\tilde{v} = \sqrt{-\frac{6\mu^2}{\lambda}}e^{i\alpha} \equiv ve^{i\alpha} \neq 0, \tag{3.3.14}$$

and $\phi_{(0)a} \equiv vN_a$ (we will take $\alpha = 0$). N_a is a unit vector with $N^{\dagger a}N_a = 1$. It is used for gauge fixing, *i.e.* to set which fermions couple to Higgs particles, or, what is the same, which particles are to acquire mass.

It is possible to choose $\alpha = 0$ without making any restriction to the system since this does not demand any kind of physical changes. However, this choice does not allow mass to go through the phase transitions without changing its vacuum value. Therefore, even if the Lagrangian is invariant under phase transitions, it must suffer the loss of invariance explicitly through its ground state, and the particles that fall in this state interact with the Higgs bosons and slow down. In particular, in view of Special Relativity (SR), the massless particles travel with the speed of light c, and massive ones have as speed $v < c$. So the mass generation of the particles may be interpreted in relation to their interaction with the Higgs field.

The isospin Higgs field component ϕ_a may be decomposed in a ground ($\phi_{(0)a}$) and an excited state (ϕ'_a) as

$$\phi_a = \phi_{(0)a} + \phi'_a. \tag{3.3.15}$$

The minimum energy is then given by the non-vanishing Higgs ground-state value (*i.e.* $v \neq 0$) in the following form, analogous to the isoscalar case:

$$\epsilon(\phi_{(0)a}) = -\frac{3}{2}\frac{\mu^4}{\lambda}. \tag{3.3.16}$$

After symmetry breaking, the Lagrangian takes the following form,

$$\begin{aligned}\mathcal{L}(\phi) =& \mathcal{L}(\phi'_a) + const. = \frac{1}{2}(\partial_\mu \delta_a{}^b - igA_{\mu i}(\tau^i)_a{}^b)(\phi^{\dagger a}_{(0)} + \phi'^{\dagger a})(\partial^\mu \delta_b{}^c + igA^{\mu i}(\tau_i)_a{}_b{}^c)(\phi_{(0)c} + \phi'_c) \\ &- \frac{\mu^2}{2}(\phi^{\dagger a}_{(0)} + \phi'^{\dagger a})(\phi_{(0)a} + \phi'_a) - \frac{\lambda}{4!}\left((\phi^{\dagger a}_{(0)} + \phi'^{\dagger a})(\phi_{(0)a} + \phi'_a)\right)^2. \end{aligned} \tag{3.3.17}$$

Up to the second order in the field variables A_μ and ϕ', and without the constant term (which has no physical relevance), the latter equation gives a kinetic term of the scalar field, a mass term of the coupled gauge bosons and a mass term of the particle related to the scalar field, *i.e.* the Higgs field,

$$\mathcal{L}(\phi') = \frac{1}{2}\phi'^{\dagger a}{}_{,\mu}\phi'_a{}^{,\mu} + \frac{1}{2}g^2 A_{\mu i}(\tau^i)_a{}^b \phi^{\dagger a}_{(0)} A^{\mu i}(\tau_i)_b{}^c \phi_{(0)c} - \frac{\lambda}{4!}(\phi^{\dagger a}_{(0)}\phi'_a + \phi'^{\dagger a}\phi_{(0)a})^2. \tag{3.3.18}$$

The second term gives the masses of gauge bosons in a theory of elementary particle physics. The mass term may be rewritten so that the mass-square matrix, which is symmetric and real, be the following (using Bach parenthesis),

$$(M^2)^{ij} = g^2\phi^\dagger_{(0)}\tau^{(i}\tau^{j)}\phi_{(0)} = 4\phi\hbar c g^2 v^2 N^\dagger \tau^{(i}\tau^{j)}N = (M^2)^{ji}. \tag{3.3.19}$$

The coupling constant α_{YM} is related to the coupling g^2 as follows:

$$\alpha_{YM} = \frac{g^2}{4\pi\hbar c}. \tag{3.3.20}$$

the diagonal elements of the mass-square matrix read

$$M^{(i)} = 2\sqrt{\pi\hbar c}\alpha_{YM}v\sqrt{(\tau^i N)^\dagger(\tau^{i\dagger}N)}. \tag{3.3.21}$$

This is the mass of gauge bosons coupled to the Higgs fields. For SU(2), for instance, the generators τ^i are related to the Pauli matrices, and skew-diagonal elements of the mass-square matrix vanish. Within electroweak interactions, inhomogeneous Yang–Mills equations then obtain a mass term of 4-currents. With gauge-coupling constants of U(1) and SU(2) as g_i and $g_i = 2g$ (with $i = 1$ and $i = 2$ for U(1) and SU(2) respectively), the gauge-boson square mass of weakon-fields W is then simply

$$(M^2_{(2)})_{ij} = \pi g_2^2 \hbar c v^2 \delta_{ij} = M_W^2 \delta_{ij}. \tag{3.3.22}$$

Furthermore, *sc.* there is

$$\left.\begin{array}{r} \left(M^2_{(1,2)}\right)_i = \pi g_1 g_2 \hbar c v^2 \delta_i{}^3 = M_W{}^2 \delta_i{}^3 g_1/g_2 \\ \text{and} \quad M^2_{(1)} = \pi g_1^2 \hbar c v^2 = (g_1/g_2)^2 M_W{}^2 \end{array}\right\} . \tag{3.3.23}$$

Above mass terms couple to the gauge fields such that electroweak currents (Yang–Mills equations) acquire a $M_{(2)} - M_{(1,2)}$ term

$$(1+\varphi)^2 M_W^2 \left(\delta_{ij} W^{\lambda j} + \frac{g_1}{g_2} \delta_i{}^3 A^\lambda\right), \tag{3.3.24}$$

with scalar-field excitation φ (*cf.* [74]). Further, U(1) currents acquire an $M_{(1)} - M_{(1,2)}$ term

$$(1+\varphi)^2 M_W^2 \left(\frac{g_1}{g_2} \delta_i{}^3 W^{\lambda i} + \left(\frac{g_1}{g_2}\right)^2 A^\lambda\right). \tag{3.3.25}$$

Both terms (3.3.24) and (3.3.25) are non-diagonal, which is in contradiction to their interpretation as mass squares of physical real particles.
Furthermore, both mass terms may be taken as components of a vector X. It defines a total mass-square matrix of electroweak gauge fields such that

$$X = \mathcal{M}^2 W^{\lambda\kappa}, \tag{3.3.26}$$

with $\kappa = 1, \ldots 4$ with the U(1) gauge current as $A^\lambda = W^{\lambda 4}$. The mass-square matrix is non-diagonal and possesses a vanishing determinant, *i.e.* its eigenvalue is zero. The vanishing eigenvector is related to photons as non-massive particles. Further, the mass matrix is to be diagonalized in order to acquire mass of physical particles for $i = 3$ and $i = 4$. An orthogonal transformation is to be fulfilled such that the mass eigenstates of the gauge fields yield

$$Z^\mu \equiv W_3{}^\mu \cos\vartheta_W + A^\mu \sin\vartheta_W\,, \quad B^\mu \equiv -W_3{}^\mu \sin\vartheta_W + A^\mu \cos\vartheta_W. \tag{3.3.27}$$

In this representation, physical Z bosons are represented by the field Z^μ, and photons are represented by B^μ. ϑ_W is called Weinberg angle, and the transformation is called Weinberg mixture. There is $\tan\vartheta_W = g_1/g_2$ with $g_1 \cos\vartheta_W = g_2 \sin\vartheta_W = e$ for the (positive) electric charge e. Experimentally, the Weinberg angle has a value of $\vartheta_W \approx 0.50$ with $\sin^2\vartheta_W \approx 0.23$. Hence, by means of the Weinberg mixture, neutral weakons possess a stronger coupling to mass such that $M_Z = M_W/\cos\vartheta_W > M_W$ is valid.

- Massive fermions:
  The vector N gives gauge fixing and is dependent on the form of the fermionic state ψ. Within electroweak interactions, for instance, if the first component gives the neutrino state, N_1 is to be chosen

as 0. If the second component of ψ is the electron, N_2 is 1. Furthermore, given that $\phi_{(0)a} = vN_a$ is valid for the ground state, $N_1 = 1$ then leads to $\phi_{(0)1} = 0$ while there is $\phi_{(0)2} = v$ for the electron component. Hence, electrons couple to Higgs fields and become massive, with a mass $m_e \sim v$. Meanwhile, neutrinos remain massless. However, with only kinetic and potential terms of Higgs particles only gauge-boson masses are actually generated. Leptonic and quark masses are not yet given. For leptons and quarks to acquire mass *via* Higgs mechanism, a further term of the Lagrangian is needed. This term couples the fermionic state ψ to the Higgs field ϕ and is hence to depend on both fields. The related term of the Lagrangian is called Yukawa coupling and it is of the form

$$\begin{aligned}\mathcal{L}(\phi,\psi) &= -k_f(\bar{\psi}^A \phi^{\dagger a}\hat{x}\psi_{aA} + \bar{\psi}^{aA}\hat{x}^\dagger \phi_a \psi_A) \\ &\equiv m_f(\bar{\psi}^A N^{\dagger a}\psi_{aA} + \bar{\psi}^{aA} N_a \psi_A).\end{aligned} \tag{3.3.28}$$

Within the GSW theory, the subscript f denotes the different generation of quarks while within QCD it denotes the flavor for a color triplet of SU(3)$_C$. k_f is a coupling constant related to the family and to the fermionic mass $m_f \sim k_f v$ after symmetry breaking. Further, $\hat{x}$ is called Yukawa matrix. It gives the mass of leptons and quarks by

$$m_f = k_f v \left(N^\dagger \hat{x} + \hat{x}^\dagger N\right). \tag{3.3.29}$$

With Yukawa coupling, the propagator for the exchanged boson (*i.e.* Higgs boson) *via* the Higgs interaction of two fermions turns out to be in the lowest order of the amplitude equal to the propagator derived from a Yukawa potential (*i.e.* a screened Coulomb potential). The propagator or Green function of such Klein–Gordon equation of a massive particle itself is enough to demonstrate that the Higgs interaction is of Yukawa-type. In fact, the scalar field (ϕ_a) couples with fermions (ψ_A) through the Yukawa matrix $\hat{x}$ and the mass of the fermions.
Such Higgs coupling to fermions is model-dependent, although its form is often constrained by some symmetries. However, to have an accurate picture, quantum mechanical radiative corrections are to be added also in order to have an effective potential $\mathcal{V}_{eff}(\phi)$. Since the coupling is also dependent on the effective mass of the field, the $\lambda\mu^2\phi^2$ and $\lambda^2\phi^4$ terms from a vacuum-energy contribution are caused by vacuum fluctuations of the ϕ field and must be incorporated in the system to have a correct physical description. Furthermore, there are additional quantum gravitational contributions and temperature dependence so that $\mathcal{V}_{eff}(\phi) \to \mathcal{V}_{eff}(\phi, T) \sim \mathcal{V}_{eff}(\phi) + M^2(\phi)T^2 - T^4$ is valid. As a consequence, symmetry must be restored at high energies (or temperatures), especially in the primordial Universe [47], which is contrary to the present state of the Universe.

- Goldstone bosons and unitary gauge:
The scalar multiplet in the SM belongs to a doublet representation of the gauge group in the following form,

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix},$$

which is defined with a non-trivial vacuum state having the properties of symmetry breaking of the gauge group G to the rest-symmetry of the isotropy group $\tilde{G}$. The complex field ϕ^0 can be further rewritten in terms of real fields, *i.e.* $\phi^0 = (\tilde{\sigma} + i\chi)/\sqrt{2}$. With the spontaneous breakdown of gauge symmetry, the minimal value of the energy density u is taken by the ground-state value $\phi_0 = v$ with $<\tilde{\sigma}>= v$. The $\tilde{\sigma}$ and χ fields may be identified with two particles, respectively: the Higgs and the Goldstone particles. The symmetry of the Lagrangian is then broken when particles fall

from their false vacuum (with $\phi = 0$) to the real one ($\phi = v$). In general, for such SSB (assuming elementariness), the least energy is then required to generate a new particle (*i.e.* the Higgs particle) with the associated features of the self-interaction than have it disappear. These particles are expected to be found in high-energy experiments such as in the Large Hadron Collider (LHC), the particle accelerator of the CERN[7] in Geneva, Switzerland (*sc.* the ATLAS detector, which should, among others, help to discover the Higgs particles.). Current constraints are that they should be found at energies less than 250 GeV and higher than 130 GeV (*cf.* [22]).
In the LEP, the predecessor of the LHC, Higgs bosons were expected to appear at electron–positron collisions as

$$e^+e^- \to HZ. \tag{3.3.30}$$

Very massive Higgs particles such as those within the SM are expected to decay into four jets with 60% possibility in the form of heavy hadrons:

$$H \to b\bar{b}$$
$$Z \to q\bar{q}$$

Further, Z particles may also decay into leptons with 6% possibility, and there is another channel in which Higgs particles decay into heavy hadrons and $\tau^-\tau^+$ pairs. However, the decay channel (3.3.30) has to be distinguished from far more probable channels as the following (*cf.* [22]),

$$e^+e^- \to W^+W^-, \quad e^+e^- \to ZZ,$$
$$e^+e^- \to W^+W^-\gamma, \quad e^+e^- \to \gamma\gamma.$$

At energies higher than 110 GeV, though, the cross section for such decay as (3.3.30) is very small in comparison to all others. Yet, in the LHC a Higgs mass of up to twice the Z boson mass may be measured. The production mode is now based on partonic processes, and the greatest rate should come from gluon fusion to form a Higgs particle ($gg \to H$) *via* an intermediate top-quark loop where the gluons produce a virtual top-quark pair which couples to the Higgs particles. Furthermore, the alternatives are the channels of hadronic jets, with a richer kinematic structure of the events. These channels are the quark–gluon scattering ($q(\bar{q})g \to q\bar{q}H$) and the quark–antiquark annihilation ($q\bar{q} \to gH$). Nevertheless, there is still the possibility of more decaying channels, and the generalizations of the SM (such as SuSy) demand the existence of more possible decays with supersymmetric particles. However, experimental evidence is still needed, especially for supersymmetric generalizations.
Higgs particles represent the one still unverified prediction of the SM, which has proven very successful. Still, the SM postulates Higgs fields in order to be renormalizable [232] (*i.e.* especially avoiding divergences in perturbation theory) and so to get a physical description of reality. However, whereas in the SM there is a necessity for Higgs particles to appear, Goldstone bosons are not predicted. Furthermore, their existence would affect astrophysical considerations with some sort of new mechanism for the energy loss in stars.
According to Goldstone's theorem, Goldstone particles have to appear with all global gauge processes. However, the excited Higgs field differs from the ground state by a local transformation that can be gauged away through an inverse unitary transformation U^{-1}. Such unitary transformation contains

[7] CERN: European Organization for Nuclear Research

the Goldstone field $\tilde{\lambda}$ as the generator of unbroken symmetry in the following form,

$$U = e^{i\tilde{\lambda}^a \tau_a} = e^{i\chi_a}. \tag{3.3.31}$$

With such transformation, Goldstone bosons vanish. Hence, the scalar field as well as fermionic fields ψ, field-strength tensors $F^{\mu\nu}$ and gauge fields $A^{\mu\nu}$ are to be gauged unitarily, and a representation of the theory without massless particles of the Nambu–Goldstone mode is gotten.

Chapter 4

QCD, superconductivity and symmetry breaking

– The concept of dual symmetry is presented for electrodynamics in view of magnetic charges, Dirac strings and dyons which are related to Higgs fields. Further, the concept of abelian projection is used in view of QCD and gluodynamics. In the same way that superconductivity is related to Abelian Higgs Mechanism with composite Higgs-fields/Cooper-pairs, dual supeconductivity is introduced as a possible explanation of confinement of quarks in hadrons. This work is partly published in [110, 176] (here we use $\hbar = c = 1$). –

4.1 Dual symmetry, monopoles and dyons

As commonly known, in vacuum, Maxwell equations in geometrical (Gauss) units are the following:

$$\begin{array}{cc} \vec{\nabla}\cdot\vec{B} = 0, & \vec{\nabla}\cdot\vec{E} = 0, \\ \vec{\nabla}\times\vec{B} = \frac{\partial}{\partial t}\vec{E}, & \vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}\,, \end{array} \tag{4.1.1}$$

with the electric field vector $\vec{E}$ and the magnetic field (pseudo) vector $\vec{B}$. There appears a $\mathbb{Z}^2$ dual symmetry of sourceless Maxwell equations, *i.e.* in vacuum there is a dual symmetry between the behavior of electric and magnetic fields. This is an invariance under transformations of the type

$$\vec{E} \to \vec{B}, \quad \vec{B} \to -\vec{E}. \tag{4.1.2}$$

However, the complete equations of Maxwell in derivative form for microscopic systems are the following in the international system of units (SI),

$$\text{First Gauss law: } \vec{\nabla}\cdot\vec{E} = \varrho/\varepsilon_0, \tag{4.1.3}$$

$$\text{Ampère–Maxwell law: } \vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}, \tag{4.1.4}$$

$$\text{Second Gauss law: } \vec{\nabla}\cdot\vec{B} = 0, \tag{4.1.5}$$

$$\text{Maxwell–Faraday induction law: } \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \tag{4.1.6}$$

with the absolute permittivity ε_0 and the permeability μ_0 of free space or vacuum, and the electric charge density ϱ and charge current $\vec{j}$. Within geometrical units, there is $\varepsilon_0 = \mu_0 = 1$.

The term $\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ in equation (4.1.4) is called Maxwell displacement current $\vec{j}_D$, and it finds its dual analogue in $\frac{\partial \vec{B}}{\partial t}$ of equation (4.1.6). This displacement which was found by Maxwell leads to electromagnetic phenomena are being described by (electromagnetic) waves, which could be demonstrated by Hertz.
Maxwell equations, which he re-derived in conjunction with his molecular vertex model of Faraday's Lines of Force [161], show an impressive symmetry between electrical and magnetic phenomena under the insertion of Maxwell's displacement current. However, a lack of symmetry is easily noticed, and it is found in a missing term in Gauss's equations (4.1.5) of a magnetic charge and a magnetic current. Indeed, this appears to be a fundamental difference between electricity and magnetism: It is possible to separate positive and negative electric charges but impossible to separate magnetic poles [110].
The breaking of $\mathbb{Z}^2$ dual symmetry in electrodynamics in the appearance of electric charges is an open issue of physics or at least of the philosophy of the same. However, magnetic poles may be assumed in view of symmetrization of Maxwell's equations, and indeed, this issue lead Dirac in 1931 to introduce quantized singularities of electromagnetic fields, which demonstrate that the existence of a mere monopole can explain the quantization of the whole electric charge in the Universe [75, 106, 202]. For this, Dirac expanded Maxwell's equations with a magnetic charge density σ and a magnetic current density $\vec{k}$ by which dual symmetry is preserved with

$$\vec{\nabla} \cdot \vec{B} = \sigma, \quad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = -\vec{k}. \tag{4.1.7}$$

Hence, expanded Maxwell equations shall further follow this set of transformations (in geometrical units):

$$\left.\begin{aligned} (\vec{E},\vec{B})^T &= R(\vartheta)(\vec{E},\vec{B})^T \\ (\varrho,\sigma)^T &= R(\vartheta)(\varrho,\sigma)^T \\ (\vec{j},\vec{k})^T &= R(\vartheta)(\vec{j},\vec{k})^T \end{aligned}\right\} , \tag{4.1.8}$$

where T denotes the transpose and $R(\vartheta)$ the symmetry operator which is a 2×2 orthogonal matrix as given by [12, 213]

$$R(\vartheta) = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}.$$

ϑ is an arbitrary constant. For $\vartheta = \pi/2$, however, it is easy to notice that the Maxwell's equations in geometrical units are invariant under the afore-mentioned transformations with

$$\left.\begin{aligned} \vec{E} \to \vec{B},\ \vec{B} \to -\vec{E},\ \varrho \to \sigma \\ \sigma \to -\varrho,\ \vec{j} \to \vec{k},\ \vec{k} \to -\vec{j} \end{aligned}\right\} . \tag{4.1.9}$$

Further, a symmetric behavior may also be seen in the energy density of electromagnetic fields which in SI is given by

$$u = \frac{1}{2}\left[\varepsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right]. \tag{4.1.10}$$

Accordingly, if dual symmetry is given, there must exist a particle having a magnetic charge which acts as a source of magnetic fields. This hypothetical particle is called Dirac monopole.
Dual magnetic fields do not satisfy the usual relation $\vec{B} = \vec{\nabla} \times \vec{A}$, demanding a modification in the definition of the magnetic strength $\vec{B}$ in terms of the vector potential $\vec{A}$ in presence of a monopole [53]. Assuming a point-like nature of monopoles enclosed by a volume τ bounded by a closed surface S, there follows [110],

$$\int_\tau \vec{\nabla} \cdot \vec{B} d\tau = \int \sigma d\tau = g, \tag{4.1.11}$$

as an integral form of a dual-extended form of Gauss's law (4.1.7), with a magnetic charge g and a magnetic charge density σ. Further, following the Gauss divergence theorem,

$$\int_\tau \left(\vec{\nabla}\cdot\vec{B}\right) d\tau = \int_S \vec{B} d\vec{s}, \tag{4.1.12}$$

for an infinitesimal element of area $d\vec{s}$, there is, according to [53, 75],

$$\vec{B} = \vec{\nabla}\times\vec{A} + \vec{A}', \tag{4.1.13}$$

with an additional term $\vec{A}'$. With (4.1.12) and (4.1.13), equation (4.1.11) yields in terms of $\vec{A}'$,

$$\int_\tau \left(\vec{\nabla}\cdot\vec{B}\right) d\tau = \int_\tau (\vec{\nabla}\cdot\vec{A}')d\tau = \int_S \vec{A}'\cdot d\vec{s} = g. \tag{4.1.14}$$

In virtue of the usual Maxwell's equations, magnetic fields should be defined in such a way that they be given basically by $\vec{\nabla}\times\vec{A}$. However, $\vec{A}'$ cannot be defined as vanishing since else, the right-hand side of equation (4.1.14) vanishes. Following, Dirac pointed out that one might choose $\vec{A}'$ such that it were zero except at one point on the surface where it is infinite. The additional term would be infinite at one point on each surface bounding any volume τ. Hence, $\vec{A}'$ would have to be infinite on a line joining the monopole to infinity. This line of singularity is called Dirac string [110].

In order to avoid unphysical features of the Dirac string in quantum mechanics, which has an implicit singular behavior, as well as arbitrariness in its localization such that it may be chosen to lie along any direction with a suitable choice of coordinates, Dirac put forward a principle by which no charged particle was to interact with it. On that ground, some ways to define Dirac's monopole without unphysicalities arose. Furthermore, Dirac himself was able to explain the quantization of electric charge based on quantum mechanical principles. Assuming the magnetic monopole as a point–particle like an electron, he showed that when an electron moves around a monopole, there is a change in phase of the wave function of the electron, which corresponds to the magnetic flux and leads to Dirac's quantization condition which is given as follows,

$$e\cdot g = \frac{n}{2}\hbar, \tag{4.1.15}$$

where n is an integer. The existence of monopoles, therefore, indicates that the electric charges in nature are the integral multiple of the electric charge of an electron.

Furthermore, besides the monopole there may also exist a particle having both the electric and magnetic charge. This hypothetical particle is called dyon [216, 250]. Such particles may be understood as a composite of charge and monopole and, although both parts follow Bose–Einstein statistics, dyons are tensorial bosons or spinorial fermions [12]. The generalized Dirac quantization condition on its charges is due to Schwinger, Zwanziger and Saha [12, 213, 216, 250], and indeed, unifying theories of elementary interactions (GUT) do also predict monopoles and dyons, as first pointed out by 't Hooft and Polyakov in 1974 [133, 201]. These monopoles are extremely massive and still of no experimental reality. Further, GUTs usually predict nonconservation of baryon and lepton number, by which free proton decay into leptons plus other parts like mesons and photons is expected. Such decay processes, together with magnetic monopoles, though, have also not been measured. The best cosmic-ray supermassive monopole flux limit lies at less than $1.0\cdot 10^{-15}\text{cm}^{-2}\text{sr}^{-1}\text{s}^{-1}$ [8].

4.2 Covariant form and dyons

Within covariant formalism, the field-strength tensor is defined through Ricci identities by

$$\begin{aligned} F_{\mu\nu a}{}^{b} =& \frac{1}{ig}[D_{\mu a}{}^{c}, D_{\nu c}{}^{b}] \\ &= \left[(A_{\nu i,\mu} - A_{\mu i,\nu}) - gA_{\mu k}A_{\nu l}f^{kl}{}_{i}\right](\tau^{i})_{a}{}^{b}, \end{aligned} \tag{4.2.1}$$

with a 4-potential A^{ν}, and with $f^{kl}{}_{i}$ as a structure constant dependent on the gauge group. Within electrodynamics and hence the unitary group U(1), there is $f^{kl}{}_{i} = 0$ and the covariant homogeneous Maxwell system is valid,

$$F_{\mu\nu,\lambda} + F_{\nu\lambda,\mu} + F_{\lambda\mu,\nu} = F_{(\mu\nu,\lambda)} = 0, \tag{4.2.2}$$

meaning the homogeneous Maxwell equations for non-appearing magnetic sources (monopoles) and electric charges as sources of electric fields following from Bianchi identities.
The inhomogeneous Maxwell equations, dependent on matter and hence on Euler–Lagrange equations, are given in geometrical units by

$$F^{\mu}{}_{\nu,\mu} = 4\pi j_{\nu}, \tag{4.2.3}$$

with the 4-current of density and charge $j_{\mu} = (\varrho, \vec{j})$. The field-strength tensor is an antisymmetric tensor with

$$F^{\mu\nu} = \begin{pmatrix} 0 & -B_z & B_y & E_x \\ B_z & 0 & -B_x & E_y \\ B_y & B_x & 0 & E_z \\ -E_z & -E_y & -E_z & 0 \end{pmatrix}. \tag{4.2.4}$$

Its covariant form yields after transposition using the Minkowski metric,

$$F_{\mu\nu} = \begin{pmatrix} 0 & -B_z & B_y & -E_x \\ B_z & 0 & -B_x & -E_y \\ -B_y & B_x & 0 & -E_z \\ -E_z & -E_y & -E_z & 0 \end{pmatrix}. \tag{4.2.5}$$

The (Lorenz–Joule) 4-force is given by

$$K^{\mu} = F^{\mu}{}_{\nu}j^{\nu} = (c)\frac{d}{ds}p^{\mu}, \tag{4.2.6}$$

with the canonical momentum p^{μ}.
Furthermore, the homogeneous Maxwell system may be rewritten with help of the antisymmetric Levi–Civita tensor $\varepsilon_{\mu\nu\alpha\beta}$. The dual field-strength tensor is given by

$$F^{\mu\nu *} = \frac{1}{2}\varepsilon^{\mu\nu}{}_{\kappa\lambda}F^{\kappa\lambda}, \tag{4.2.7}$$

whereas

$$F^{**}_{\mu\nu} = -F_{\mu\nu}. \tag{4.2.8}$$

The explicit form of the tensor in matrix form is

$$F^{\mu\nu *} \simeq \begin{pmatrix} 0 & -E_z & E_y & B_x \\ E_z & 0 & -E_x & B_y \\ E_y & E_x & 0 & B_z \\ -B_z & -B_y & -B_z & 0 \end{pmatrix}. \tag{4.2.9}$$

The homogeneous Maxwell system in dual form yields

$$F^{\mu\nu}{}^{*}_{,\mu} = 0. \tag{4.2.10}$$

On the other hand, the contravariant inhomogeneous Maxwell system yields (4.2.3)

$$F^{\mu\nu}{}_{,\mu} = 4\pi j^{\nu}.$$

Hence, rupture of dual symmetry between equations (4.2.3) and (4.2.10) for nonvanishing 4-currents is easily seen. Definition of magnetic 4-currents, $j_{g\mu}$, though, leads to dual inhomogeneous Maxwell equations in the following form,

$$F^{\mu\nu}{}^{*}_{,\mu} = 4\pi j^{\nu}_{g}. \tag{4.2.11}$$

4-currents j^{ν}_{g} are related to Dirac's monopole as magnetic charge. Further, a particular solution for the field-strength tensor for the inhomogeneous equation (4.2.11) is given below,

$$F^{\mu\nu *} = (n_{\mu} \cdot \partial_{\mu})^{-1}(n^{[\mu}\, j^{\nu]}_{g}), \tag{4.2.12}$$

whereas n is an arbitrary fixed 4-vector with $n^2 \neq 0$, and $(n_{\mu}\partial^{\mu})^{-1}$ is an integral operator with kernel satisfying following condition according to [251],

$$n_{\mu}\partial^{\mu}_{x} K(x-y) = \delta^{4}(x-y), \tag{4.2.13}$$

or analogously,

$$(n_{\mu}\partial^{\mu}_{x})(n_{\mu}\partial^{\mu}_{x})^{-1} f(x) \equiv f(x). \tag{4.2.14}$$

The general solution to (4.2.11) is given by

$$\left.\begin{aligned} F^{\mu\nu *} &= (\partial^{[\mu}\, A'^{\,\nu]})^{*} + (n_{\mu}\partial^{\mu})^{-1}(n^{[\mu}\, j^{\nu]}_{g}) \\ F^{\mu\nu} &= (\partial^{[\mu}\, A'^{\,\nu]}) - (n_{\mu}\partial^{\mu})^{-1}(n^{[\mu}\, j^{\nu]}_{g})^{*} \end{aligned}\right\} . \tag{4.2.15}$$

The 4-potential A^{μ} depends on the choice of gauge, the choice on n and the determination of $(n_{\mu}\partial^{\mu})^{-1}$. Similarly, there is in [251] the general solution to (4.2.12) which is

$$\left.\begin{aligned} F^{\mu\nu *} &= -(\partial^{[\mu}\, A^{\nu]})^{*} + (n_{\mu}\partial^{\mu})^{-1}(n^{[\mu}\, j^{\nu]}_{e}) \\ F^{\mu\nu} &= (\partial^{[\mu}\, A^{\nu]}) + (n_{\mu}\partial^{\mu})^{-1}(n^{[\mu}\, j^{\nu]}_{e})^{*} \end{aligned}\right\} , \tag{4.2.16}$$

whereas for electric charges a subscript e has been written. Further, A'^{μ} is another 4-potential.
At this point it is better to partially introduce an index-free formalism using (antisymmetric) wedge operators of the inner product in the following form:
A tensor $\omega^{\mu\nu}$ is related to the form $\omega dx^{\mu} \wedge dx^{\nu}$, with differential forms d. Hence, $(a \wedge b)$ give a 2-form

$\omega(x,a,b) = \sum_{ij} \omega(x,e_i,e_j)a_i b_j$ with the standard basis $e_1,\dots,e_n$. ω is a function of both sets a_i and b_j so that there is

$$(a \wedge b) \simeq (a \wedge b)^{\mu\nu} \equiv a^\mu b^\nu - a^\nu b^\mu = 2a^{[\mu} b^{\nu]}. \tag{4.2.17}$$

Further, there is

$$\left.\begin{array}{c} (a \cdot G) \simeq (a \cdot G)^\nu \equiv a_\mu G^{\mu\nu} \\ a \cdot (b \wedge c) = a \cdot bc - a \cdot cb \\ a \cdot (b \wedge c)^* = a_\mu \varepsilon^{\mu\nu}{}_{\kappa\lambda} b^\kappa c^\lambda \end{array}\right\} . \tag{4.2.18}$$

Hence, for instance, there is (4.2.16) as

$$\begin{array}{c} F^* = -(\partial \wedge A')^* + (n \cdot \partial)^{-1}(n \wedge j_e), \\ F = -(\partial \wedge A) - (n \cdot \partial)^{-1}(n \wedge j_e)^*. \end{array} \tag{4.2.19}$$

The 4-potential $A'_\mu \simeq A'$ leads to the dual field-strength tensor dual-equivalently to the way A^μ leads to $F^{\mu\nu}$,

$$n_\mu F^{\mu\nu} \simeq n \cdot F = n \cdot (\partial \wedge A), \quad n \cdot F^* = n \cdot (\partial \wedge B). \tag{4.2.20}$$

Hence, the dual field-strength tensor may be given by A'^μ analogously to electric charge densities are given as the divergence of the electric field. This implies the existence of magnetic monopoles related to A'^μ if dual symmetry is given.
Further, every antisymmetric tensor $G^{\mu\nu}$ follows the following identity with $a_\mu G^{\mu\nu} = (aG)^\nu$,

$$G = \frac{1}{n^2} \left\{ [n \wedge (n \cdot G)] - [n \wedge (n \cdot G^*)]^* \right\}. \tag{4.2.21}$$

So, equation (4.2.20) leads to the field-strengths with the index-free form

$$\begin{array}{r} F = \frac{1}{n^2} \left(\{n \wedge [n \cdot (\partial \wedge A)]\} - \{n \wedge [n \cdot (\partial \wedge A')]\}^* \right), \\ F^* = \frac{1}{n^2} \left(\{n \wedge [n \cdot (\partial \wedge A)]\}^* + \{n \wedge [n \cdot (\partial \wedge A')]\} \right). \end{array} \tag{4.2.22}$$

With them, Maxwell's equations may be written in terms of the potentials [251]:

$$\begin{aligned} &(1/n^2)(n \cdot \partial n \cdot \partial A^\mu - n \cdot \partial \partial^\mu n \cdot A - n^\mu n \cdot \partial \partial \cdot A + \\ &\qquad + n^\mu \partial^2 n \cdot A - n \cdot \varepsilon^\mu{}_{\nu\kappa\lambda} n^\nu \partial^\kappa A'^\lambda) = j_e^\mu, \end{aligned} \tag{4.2.23}$$

$$\begin{aligned} &(1/n^2)(n \cdot \partial n \cdot \partial A'^\mu - n \cdot \partial \partial^\mu n \cdot A' - n^\mu n \cdot \partial \partial \cdot A' + \\ &\qquad + n^\mu \partial^2 n \cdot A - n \cdot \varepsilon^\mu{}_{\nu\kappa\lambda} n^\nu \partial^\kappa A^\lambda) = j_g^\mu. \end{aligned} \tag{4.2.24}$$

For any field-strength tensor $F^{\mu\nu}$ to Maxwell's equations there exist potentials A^μ and A'^μ satisfying the Maxwell equations (4.2.24).
For (4.2.24), a Lagrangian may be given with the form

$$\begin{aligned} \mathcal{L} = &-\frac{1}{2n^2}[n \cdot (\partial \wedge A)] \cdot [n \cdot (\partial \wedge A')^*] + \\ &+ \frac{1}{2n^2}[n \cdot (\partial \wedge A')] \cdot [n \cdot (\partial \wedge A)^*] - \\ &- \frac{1}{2n^2}[n \cdot (\partial \wedge A)]^2 - \frac{1}{2n^2}[n \cdot (\partial \wedge A')]^2 + \mathcal{L}_I, \end{aligned} \tag{4.2.25}$$

with the interaction term

$$\mathcal{L}_I = -j_{e\mu} A^\mu - j_{g\mu} A'^\mu, \tag{4.2.26}$$

which adds to the total action for the partition or propagator function (see Chapter Appendix B.1). Herewith, there is the electric (magnetic) charge e (g).
The integral over the 4-volume is called Zwanziger action. It gives the dynamics for an electrodynamic system with Dirac monopoles.
A possible approach is to take the partition function following from the Zwanziger action since partition functions encode the statical properties of systems in thermodynamical equilibrium and a partition function is nothing less than the Wick rotation ($t \to it$) of Feynman's path integral (propagator). The path integral resembles the partition function of statistical mechanics defined in a canonical ensemble with temperature $1/(T\hbar)$ (*cf.* Appendix B.1).
The so-called Zwanziger partition function yields

$$Z_{Zw}[A_\mu, A'_\mu] = \int \mathcal{D}A_\mu \mathcal{D}A'_\mu exp[-S_{Zw}[A_\mu, A'_\mu]], \tag{4.2.27}$$

where $\mathcal{D}$ denotes the integration over all paths for the kernel.
Let the vacuum now be nontrivial under the incorporation of a scalar field Φ which leads to spontaneous symmetry breaking. The potential of the scalar field be given by

$$V(\Phi) = \lambda \left(|\Phi|^2 - \Phi_0^2\right)^2, \tag{4.2.28}$$

i.e. a Higgs potential. The kinetic energy term of the action be given by

$$T(\Phi) = \frac{1}{2}|D_\mu \Phi|^2, \tag{4.2.29}$$

whereas the covariant derivative be given here as

$$D_\mu = \partial_\mu - ieA_\mu - igA'_\mu. \tag{4.2.30}$$

With this terms, a partition function for dyons with a Higgs field Φ can be given as [178]

$$Z_D[A_\mu, A'_\mu, \Phi] = \int \mathcal{D}A_\mu \mathcal{D}A'_\mu \mathcal{D}\Phi e^{S_D[A_\mu, A'_\mu, \Phi]}, \tag{4.2.31}$$

with dyon action

$$S_D[A_\mu, A'_\mu, \Phi] = S_{Zw}[A_\mu, A'_\mu] + T(\Phi) + V(\Phi). \tag{4.2.32}$$

Here, the Higgs field acquires dyonic properties with an electric (e) and a magnetic (g) charge given by the covariant derivative in (4.2.30).
These ideas have been successful in U(1) for understanding superconductivity so that they have been extended to non-abelian models in view of elementary interactions and especially color confinement by symmetry breaking. That makes it crucial to formulate the theory in terms of its relevant abelian degrees of freedom, which are color-magnetic monopoles, color-electric charges and photons. In non-abelian theories, a gauge field can be Cartan decomposed into the diagonal A^μ and the off-diagonal part a^μ. Hence, a formulation in terms of abelian degrees of freedom is achieved by fixing to a gauge in which the gauge freedom of the maximal abelian subgroup remains (abelian projections) [143]. As a consequence, magnetic monopoles emerge with necessity as degrees of freedom in abelian projections for the dynamics of gluons [114].
Diagonal gluon fields transform as abelian gauge fields, whereas off-diagonal gluons transform as adjoint matter fields. From a non-abelian gauge theory, an abelian one is obtained by neglecting the off-diagonal gauge fields, although they can be taken into account by integration in sense of a Wilson renormalization

group, reducing to the normalization of the effective abelian gauge theory. These are abelian projections and in those of gluodynamics for QCD, magnetic monopoles necessarily emerge as degrees of freedom. An abelian-projected effective gauge theory is then considered as the low-energy effective gauge theory of the original non-abelian gauge theory, *e.g.* QCD. The latter especially because the off-diagonal gluons become massive after the maximal abelian gauge.
Further, if the vacuum is not assumed to be trivial and spontaneous symmetry breaking is incorporated into Zwanziger's formalism, then unphysical singularities arise. They however vanish following some formulation of the Abelian Higgs Model (AHM), *i.e.* the Higgs model within electrodynamics (abelian). In the abelian projection (*i.e.* in principle taking only abelian contributions) and taking elementary-particle processes in scope, quarks are electrically charged particles, and if monopoles are condensed, the dual Abrikosov string carrying the electric flux is formed between quarks and antiquarks. Due to a non-zero string tension the quarks are confined by the linear potential [5].
According to Akhmedov, for the (anti-)self-dual fields the abelian monopoles become abelian dyons. Further, the infrared properties of QCD in the abelian projection can be described by the AHM in which dyons are condensed [5].
Let us consider a linear transformation of the gauge fields as $(\tilde{A}_\mu, \tilde{A}'_\mu)^T = R(\vartheta)(A_\mu, A'_\mu)^T$, where T denote the transpose with

$$R(\vartheta) = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix}.$$

such that $\vartheta = g/e$. The integration of the dyon partition function (4.2.32) over the transformed dual-electric gauge potential $\tilde{A}'_\mu$ then leads to the partition function of the AHM of QCD,

$$Z_{AHM} = \int \mathcal{D}\tilde{A}'_\mu \Phi e^{-S_{AHM}[\tilde{A}'_\mu, \Phi]}. \tag{4.2.33}$$

The AHM action with transformed magnetic gauge field $\tilde{A}'_\mu$ is given by

$$S_{AHM}[\tilde{A}'_\mu, \Phi] = \int d^4x \left\{ -\frac{1}{4}\tilde{C}_{\mu\nu}\tilde{C}^{\mu\nu} + |\left(\partial_\mu - i\sqrt{e^2+g^2}\tilde{A}'_\mu\right)\Phi|^2 + \lambda(|\Phi|^2 - \Phi_0^2)^2 \right\}, \tag{4.2.34}$$

with a generalized "magneto-charge" $Q = \sqrt{e^2+g^2}$ and the dual field strength as follows,

$$\tilde{C}_{\mu\nu} = \partial_\mu \tilde{A}'_\nu - \partial_\nu \tilde{A}'_\mu = \tilde{A}'_{\nu,\mu} - \tilde{A}'_{\mu,\nu}. \tag{4.2.35}$$

$\partial_\mu - iQ\tilde{A}'_\mu$ gives the covariant derivative showing that Φ in action (4.2.34) is dyonic in nature. For Φ, there is

$$D^\mu D_\mu \Phi - 4\lambda\left(|\Phi|^2 - \Phi_0^2\right)\Phi = 0. \tag{4.2.36}$$

The tensor $C_{\mu\nu}$, further, is dual to the usual field-strength tensor $F_{\mu\nu}$ and of the same structure with gauge potential $\tilde{A}'_\mu$. Its field contents are color-electric fields $\tilde{\vec{E}}$ and color-magnetic fields $\tilde{\vec{B}}$ [174]. Hence, the action given by (4.2.34) coincides with the Ginzburg–Landau action of superconductivity, however in dual form for gluodynamics.

4.3 Superconductivity, dual superconductors and the Higgs field

- **Superconductivity and Higgs fields:** In words of Stephen Weinberg, a superconductor is more or less a material in which a particular symmetry of the laws of nature, electromagnetic gauge invariance, is

spontaneously broken. The symmetry group here is the group of two-dimensional rotations. These rotations act on a two-dimensional vector whose two components are the real and imaginary parts of the electron field, the quantum mechanical operator that in quantum field theories of matter destroys or creates electrons [236].
The symmetry breaking in a superconductor leaves unbroken a rotation by 180°, which changes the sign of the electron field. In consequence, products of any even number of electron fields have non-vanishing expectation values in a superconductor. A single electron field, however, does not [236]. Phenomenologically, electrons are said to be bound into a composite which is known as BCS (Bardeen–Cooper–Schrieffer) or simply Cooper pair.[1] Consequently, all experimental phenomena such as the Meißner–Ochsenfeld (or simply Meissner) effect (ME), zero electrical resistance, the expelling of magnetic fields and so on appear following the assumption that electromagnetic gauge invariance is broken.
Superconductivity is traced back to an order parameter which is the nonvanishing value of the product of two electron fields. This order parameter, further, is related to the Higgs field of spontaneous symmetry breaking. The scalar field as an order parameter gives the order of the system in terms of the broken symmetry and the unbroken subgroup since a nonvanishing expectation value of the Higgs field accompanies a broken mode of symmetry. The appearing Higgs bosons of the field, further, are related in condensed-matter physics to the appearing bosonic state of electron (BCS) pairs. Field-theoretically, then, electrons are bound together by mediation of virtual photons which acquire an effective mass following symmetry breaking, in analogy to Higgs mass, given the fact that Cooper pairs are phenomenological analogues of Higgs bosons for condensed-matter physics. They possess an effective mass which is then related to the penetration depth of magnetic fields in superconductors (Meissner effect).

- **Ginzburg–Landau and Meissner effect:** A phenomenological approach to superconductivity is given by the Ginzburg–Landau model. Further, the Ginzburg–Landau action may be stated by the following Lagrangian,

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) + \frac{\mu^2}{2}\phi\phi^* - \frac{\lambda}{4}(\phi\phi^*)^2, \tag{4.3.1}$$

with a covariant derivative as follows,

$$D_\mu = \partial_\mu - ieA_\mu, \tag{4.3.2}$$

and a field-strength tensor given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{4.3.3}$$

Thus, ϕ is a Higgs field which is here coupled to QED (AHM). For the static case, for which there is $\partial^0\phi = \partial^0\vec{A} = 0$ and $A_0 = 0$, the field equation for the potential $\vec{A}$ can be given as below,

$$\vec{\nabla} \times \vec{H} = \vec{j} = ie\frac{1}{2}\left[\phi^\dagger(\vec{\nabla} - ie\vec{A})\phi - (\vec{\nabla} + ie\vec{A})\phi^\dagger\phi\right], \tag{4.3.4}$$

[1]Following the many-body BCS theory [13], Nobel prize 1972. The solutions of BCS theory in a homogeneous system are found using a linear canonical transformation called Bogoliubov transformation [31], which is often used to diagonalize Hamiltonians, i.e. to make them equivalent to a set of non-interacting harmonic oscillators (*cf.* [219]).

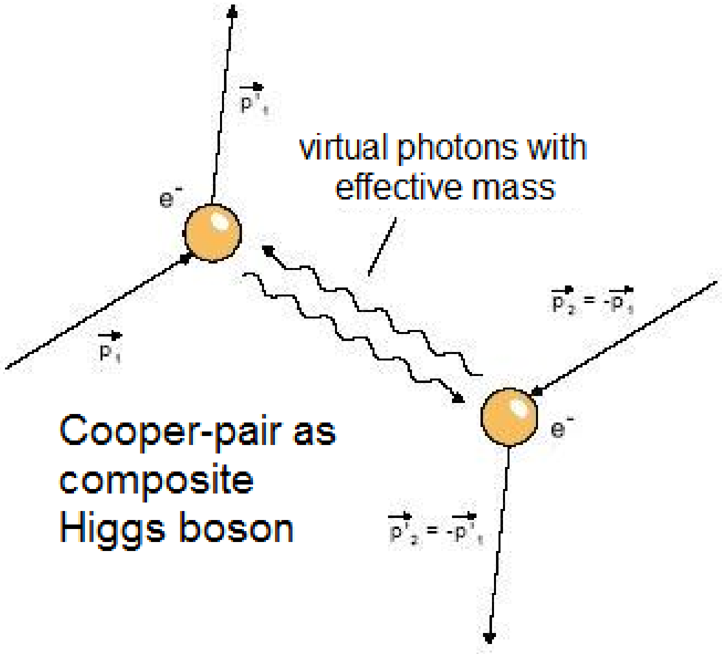

Figure 4.1: Illustration of a Higgs composite of superconductivity with electrons mediating massive gauge bosons according to a Yukawa model and field theories of interaction processes. *N.B.: The appearance of the Higgs boson leads to superconductivity as a broken-symmetry phenomenon.*

where $\vec{H}$ is the magnetic field for a macroscopic system. In the spontaneously broken phase of symmetry, the current satisfies the following local relation which is known as the London equation,

$$\vec{j} = e^2 v^2 \vec{A}, \tag{4.3.5}$$

where $v = \sqrt{\mu^2/\lambda}$. Equation (4.3.5), further, leads to

$$\nabla^2 \vec{H} = e^2 v^2 \vec{H}, \tag{4.3.6}$$

where there is Gauss's equation $\vec{\nabla} \cdot \vec{H} = 0$. Finally, (4.3.6) is solved for $x \geq 0$ by

$$\vec{H}(x) = \vec{H}(0) e^{-x/l_A}, \tag{4.3.7}$$

where $l_A = \frac{\hbar}{c} m_A^{-1} = (ev)^{-1}$ is the penetration depth which is the inverse of the vector gauge field mass. Further, equation (4.3.7) implies the Meissner effect indicating that the magnetic field decays in a distance l_A. Phenomenologically, the Ginzburg–Landau model gives an explanation of the Meissner effect by means of photons acquiring effective mass *via* Higgs fields. Symmetry breaking leads to effective masses related to short ranges of interaction of the particles coupled to the scalar field. Hence, Ginzburg–Landau photons do not enter superconductors more than a distance given by the penetration depth l_A. Magnetic fields are avoided.

- **Dual Meissner effect:** As already shown, the phenomenology of superconductivity may be understood in terms of field theory, and it indeed finds its nature in the concept of symmetry breaking and hence in the appearance of some kind of Higgs field coupled to electrodynamics. Hence, superconductivity is a condensed-matter phenomenon which is actually usual within all ranges of physics finding its roots in elementary-particle physics. Furthermore, as dual symmetry to the Ginzburg–Landau model shows, the Zwanziger formulation may be used to understand issues from nuclear and elementary particle physics if it is interpreted in terms of elementary fields. Actually, color confinement can be understood in terms of a color-magnetic superconductor in which color charges are confined (*cf.* [54]). This picture is dual to ordinary superconductors [143] in which electric charges condense and magnetic monopoles are confined through the Meissner effect. Some concepts about the ideas of dual Quantum Chromodynamics may be found under [177] while the reader may further find a thorough review on Color Confinement in [209].

Zwanziger's formalism allows to consistently describe a photon interacting with magnetic and electric charges [114]. In the dual description, quarks are the electrically charged particles which are confined within hadrons. Gluons as mediated gauge fields acquire effective mass, which is usually understood under the appearance of glueballs as composite state of gluons which self-interact due to non-abelian properties of the gauge group. Hence, a dual description of superconductivity may help understanding dynamics of gluons when QCD is abelian projected. The action given by (4.2.34) can be approximated as follows,

$$H_0 = K_g + \frac{1}{2}Q^2\Phi_0^2\tilde{A}_i'^{\,2}, \tag{4.3.8}$$

where $K_g = \tilde{E}^2/2$ is the gluon-field energy and Q the magneto–electric charge of dyons. Φ be a Higgs field for symmetry breaking while $\tilde{A}'_\mu$ be a transformed dual gauge field of A_μ so to maintain dual symmetry.

In the dual form of the Ginzburg–Landau action we take dual field-strength tensors related to the further potential A'_μ which lead to magnetic monopoles. The scalar field Φ represents the monopole (or dyon) field, and it has a non-zero magnetic charge g (or both e and g). The potential $V(\Phi)$ is the effective potential which generates the mass of the dual gauge field in the broken phase of symmetry and consequently the features of magnetic superconductivity in the condensed mode of QCD vacuum when the model is used for elementary interactions. In fact, as usual within a Higgs mode, the Higgs potential ensures that the average value of the scalar field is nonvanishing ($<\Phi>\neq 0$) in vacuum and that the monopole field plays the role of the Ginzburg–Landau (GL) order parameter in the way the scalar field takes the phenomenological role of macroscopic Cooper-pair wave functions in conventional (electric) superconductivity.

In order to analyze screening currents and their implications on the nature of QCD vacuum, the field equations corresponding to (4.2.35) are derived in the form given below [178] with $\hbar = c = 1$,

$$\partial^\nu \tilde{C}_{\mu\nu} - i\frac{Q}{2}\left(\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right) - Q^2\left(\Phi\Phi^*\right)\tilde{A}'_\mu = 0, \tag{4.3.9}$$

$$D^\mu D_\mu \Phi - 4\lambda\left(|\Phi|^2 - \Phi_0^2\right)\Phi = 0. \tag{4.3.10}$$

These equations govern the dynamics of QCD vacuum in the broken phase of symmetry. Furthermore, equations (4.3.9) and (4.3.10) are identical to the GL-type field equations in conventional superconductivity when $\tilde{C}_{\mu\nu} \to F_{\mu\nu}$ and $\tilde{A}'_\mu \to A_\mu$.

Since the macroscopic description of the formulation involves a number of dyons, it is better to specify the mass modes and other crucial parameters in terms of the density of the condensed dyons or monopoles. The scalar field Φ would be such that it remain effectively unperturbed by the color-electric field, and the density of superconducting dyons or monopoles must be defined by its constant modulus given in terms of Φ_0. In the dual QCD vacuum, the parameters specifying the confining mechanism of vacuum are, indeed, closely related to such density profile of dyon/monopole pairs. The vacuum, as a coherent condensate of all such pairs [4], may then be normalized to

$$n_s(\Phi) = |\Phi|^2 = \Phi_0^2. \tag{4.3.11}$$

The density of condensed dyons given by (4.3.11) cannot be defined in this way in the large perturbative sector of QCD as the VEV of dyon fields would disappear completely in the ultraviolet region. The density profile along with other confinement parameters in the non-perturbative infrared sector can be therefore used for the correct physical explanation of the confining behavior of QCD vacuum. Let us consider the variations in the dyon field such that $\partial_\mu \Phi = 0 = \partial_\mu \Phi^*$ since it has a finite value at each spacetime point. Equation (4.2.35) then leads to [176]

$$(\Box + m_V^2)\tilde{A}'_\mu - \partial_\mu(\partial^\nu \tilde{A}'_\nu) = 0, \tag{4.3.12}$$

where $m_V = Q\Phi_0$ is the mass of dual gauge fields.
Equation (4.3.12) is of massive vector type and may be identified with that of the condensed mode of QCD vacuum. For this formulation, two mass modes may be given, *i.e.* of a vector and a scalar mass,

$$m_V = Q\sqrt{n_s(\Phi)}, \quad \text{and} \quad m_\Phi = 2\sqrt{\lambda n_s(\Phi)}. \tag{4.3.13}$$

These mass modes appear as in any standard Higgs mechanism, and the massive vector equation (4.3.12) shows that QCD vacuum, as a result of symmetry breaking, acquires properties similar to those of a relativistic superconductor where quantum fields generate a non-zero VEV. The interaction between the macroscopic field Φ and $\tilde{A}'_\mu$ leads to a color-flux screening arising because of a screening current due to the strong correlation among the dyonic or the pure magnetic charges. Further, let us make some comments about the relation between the mass modes and the superconducting phase: In usual semiconductors there exists the GL parameter κ which describes the type of superconductor one has. This parameter is given by the ratio of the penetration depth and the coherent length ξ which is a natural length scale for spatial variations of the order parameter. For dual superconductors, ξ may be related to the coherent length of monopole condensates and thus to the reverse of the scalar-field mass m_Φ. Hence, the dual GL parameter may be defined as

$$\kappa = \frac{m_\Phi}{m_V}. \tag{4.3.14}$$

QCD vacuum thus behaves as a type-II superconductor for $m_\Phi > m_V$ while it behaves as a type-I superconductor for $m_\Phi < m_V$. Further, both masses possess an equal value for $Q = 2\sqrt{\lambda}$. In that case, the QCD vacuum undergoes a transition from a type-II to type-I superconducting state [4].
In QED, type-I superconductors are those which cannot be penetrated by magnetic flux lines, according to the Meissner–Ochsenfeld effect. They have only a single critical temperature at which the material ceases to superconduct. Elementary superconductors are of this type, which generally is exhibited by materials with a regularly structured lattice. This allows electrons to be coupled over a relatively large distance onto Cooper pairs. On the other hand, type-II superconductors of QED are characterized by a gradual transition from the superconducting to the normal state within an increasing magnetic field. Typically, they superconduct at higher temperatures and magnetic fields than type-I

superconductors. In the dual picture, then, only type-I dual superconductors lead to a strict confinement of color-electric fields. This is the case for smaller penetration depths l_A in relation to the dyon charge Q.

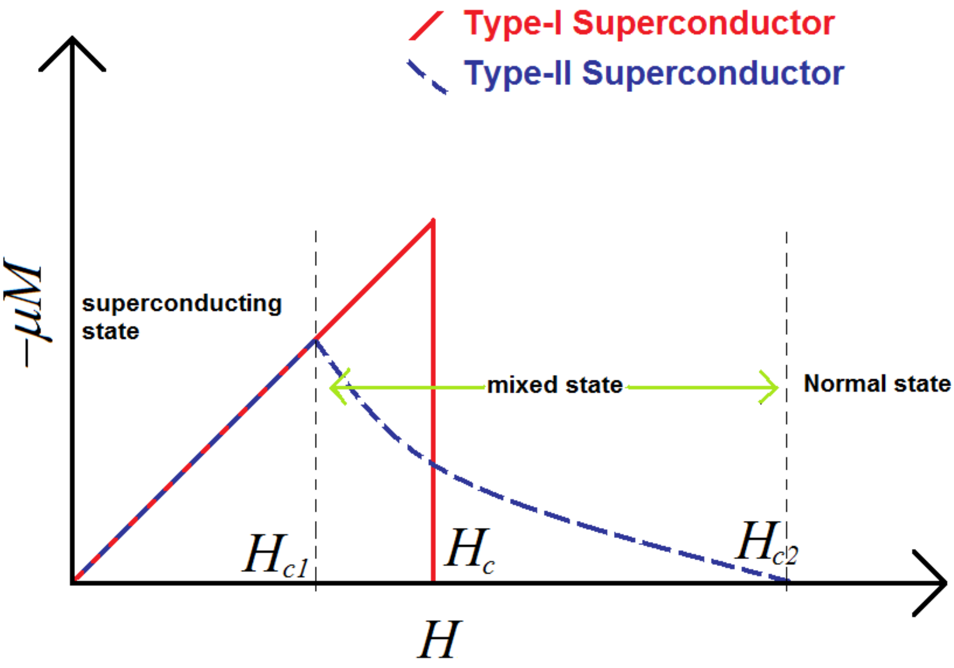

Figure 4.2: Behavior of the different types of QED superconductors in dependence of the field strength H with critical fields H_c and H_{c1} and H_{c1}.

The divergence of equation (4.3.12) leads to $\partial^\mu \tilde{A}'_\mu = 0$ for $m_V \neq 0$ [176]. The massless dual gauge quantum which propagates in the dyonically condensed QCD vacuum then satisfies

$$\Box \tilde{A}' = j_s^\mu, \tag{4.3.15}$$

where j_s^μ is the screening current that resides in vacuum. Comparing (4.3.12) and (4.3.15) using the Lorentz condition, there is

$$j_s^\mu = -m_V^2 \tilde{A}'^\mu, \quad (m_V = Q\sqrt{n_s(\Phi)}), \tag{4.3.16}$$

which reduces in the static case to the London equation which in QED gives $\vec{j} \propto nv^2 \vec{A}$ (*viz* 4.3.5). The simplest solution of (4.3.12) may be derived in the half-space of all space ($x \geq 0$, $y = z = 0$). The dual gauge field has then only a dependence on x and $\tilde{A}'_\mu$ as follows,

$$\left(\partial_x^2 - m_V^2\right) \tilde{A}'_\mu = 0, \tag{4.3.17}$$

which then results in

$$\tilde{A}'_\mu = \tilde{A}'_{0\mu} e^{-m_V x}, \tag{4.3.18}$$

where $\tilde{A}'_{0\mu}$ is a constant vector.
In analogy to QED, applying Ampère's law, for color-magnetic ($\tilde{\vec{B}}$) and color-electric ($\tilde{\vec{E}}$) fields, $\tilde{\vec{E}}$ satisfies $\vec{\nabla} \times \tilde{\vec{E}} = \vec{j}_s$ with $\tilde{\vec{E}} = \vec{\nabla} \times \tilde{\vec{A}}'$. Under such considerations, one can obtain (in analogy to (4.3.6))

$$\nabla^2 \tilde{\vec{E}} - \vec{\nabla}(\vec{\nabla} \cdot \tilde{\vec{E}}) - m_V^2 \tilde{\vec{E}} = 0. \tag{4.3.19}$$

If one takes a vector field $\tilde{\vec{E}} \equiv (0, 0, \tilde{E}_z(x))$, the condition $\vec{\nabla} \cdot \tilde{\vec{E}} = 0$ is satisfied so that a dual form of (4.3.5) is achieved. One can continue analogously to QED. Equation (4.3.19) reduces to

$$\left[\partial_x^2 \tilde{E}_z(x) - m_V^2 \tilde{E}_z(x)\right] = 0, \tag{4.3.20}$$

which is a dual Helmholtz equation of QCD. It possesses the general solution (*viz* (4.3.7))

$$\tilde{E}_x(x) = D_1 e^{-m_V x} + D_2 e^{m_V x}, \tag{4.3.21}$$

where D_1 and D_2 are integration constants. The initial conditions are $\tilde{E}_z(0) = E_0$ at $x = 0$ while E_z cannot increase to infinity far from x. Hence, there is $D_1 = E_0$ and $D_2 = 0$. The color-electric field thus penetrates the vacuum up to a finite depth given by m_V^{-1}, equivalently to QED [176]. Equation (4.3.21) indicates that the electric field is screened out a distance $l \cong m_V^{-1}$ which is the penetration depth where m_V is the dual gauge field mass. This equation guarantees a dual Meissner effect (DME). With increasing density of the condensed dyons, the electric field dies off more rapidly.
In the case $e = 0$, the dyonic vector mass mode goes through the pure magnetic dual counterpart of electric charges, *i.e.* to magnetic monopoles. Therefore, the dyonic mass mode is always greater than its pure magnetic counterpart.
Let us take the dimensionless quantity

$$\gamma = \frac{Q}{g} \tag{4.3.22}$$

It has the value 1 for $e = 0$, and $\gamma > 1$ for $e \neq 0$. These cases correspond to monopole and dyon condensation, respectively. In case of dyon condensation, the decay of color flux is always faster than that of monopole condensation (*cf.* (4.3.21)). The color-electric flux thus constricts itself more rapidly in a smaller region. We can consider the radius of such flux tube as the inverse of the vector mass [54, 172, 173, 189, 227]. For it, there is

$$r_1 = m_g^{-1} > r_2 = m_V^{-1}. \tag{4.3.23}$$

In order to have a comparison of the role of pure magnetic and dyonic condensation on the confining mechanism, the string tension of the flux tube may be another guiding parameter. Hence, let us consider the spin (J) and mass (M_J) relationship of a flux tube as $J = \alpha_0 + \alpha' M_j^2$ where $\alpha' = (2\pi\sigma)^{-1}$ is the Regge slope parameter, and σ is the string tension of the flux tube. Since the dual GL free energy given by (4.3.8) is always greater for the dyonic case than for the monopole case, the string tension for the latter will be naturally less than the previous one. The dyonic case may, therefore, lead to the lowest lying states of the Regge trajectories for hadrons (for more details, see [174, 175]).

Furthermore, following notions of magneto-statics for a dual representation, there is a magnetic current $\vec{j}_s = \vec{\nabla} \times \tilde{\vec{E}}$ for the dual electric field $\tilde{\vec{E}}$ and assuming electric vacuum so that $\vec{\nabla} \cdot \tilde{\vec{E}} = 0$ is valid, together with a constant scalar-pressure analogue P which is identified to a color-force density $\vec{\tilde{E}} \times \vec{j}_s$ of the flux tube,

$$\vec{\nabla} \cdot \vec{\tilde{P}} = \vec{j}_s \times \vec{\tilde{E}}, \tag{4.3.24}$$

[176] shows that the minimization of kinetic energy leads to a quantization of color-electric charge such that

$$\Psi_E = \int \left(\vec{\nabla} \times \vec{\tilde{A}} \right) dS = \int \vec{\tilde{E}} \cdot dSn\Psi_0 \tag{4.3.25}$$

is valid in order to maintain an equilibrium between the condensate and color force. Here $\Psi_0 = 2\pi/Q$. This shows that the electric flux is quantized in terms of the dyonic charge.

Further, as shown in [176], the presence of magnetic dyonic charges in QCD imparts a dielectric nature to it due to their vacuum polarization. QCD vacuum behaves as a perfect dielectric medium independently of the type of condensate in the vacuum. Additionally, there exists a phenomenological relation between the flux-tube structures for the monopole and dyon condensation case from the viewpoint of the DME as an onset of screening currents and the dielectric parameters [176], whereas μ is concluded to be dependent on the square of the momentum p and the reciprocal squared value of m_V,

$$\mu(p^2, \Phi_0) = 1 + \tilde{\Pi}(p) = 1 - p^{-2} m_V^2, \tag{4.3.26}$$

while ε is the reciprocal of μ according to $\varepsilon\mu = c^{-2}$. This is achieved by means of a dual magnetic polarization tensor which is given as $\tilde{\Pi}(p^2, \Phi_0) = -m_V^2/p^2$, following [134]. Higher dyonic charges Q lead to smaller dielectric permittivity ϵ and a larger permeability μ. The dual magnetic field $\vec{\tilde{H}} = \vec{\tilde{B}}/(\mu_0\mu)$ and displacement field $\vec{\tilde{D}} = \varepsilon_0\varepsilon\vec{\tilde{E}}$ (given isotropy and nondispersive behavior) are screened. At the same time, this is related to denser flux-tube structures between the charges, related to higher dual polarizations ($\sim -m_V^2/p^2$) and smaller flux-tube radii.
Hence, some properties needed for confinement of quarks in hadrons may be given using a dual approach with symmetry breakdown using Higgs fields. Nevertheless, the mechanism which is actually responsible for the vanishing of color-dielectric function of a color-confining medium is still unclear and a subject of discussion.

Part II

Induced gravity theories with scalar fields

Chapter 5

Alternative theories of gravity and historical overview

– Scalar–tensor theories are introduced historically in view of the Jordan–Brans–Dicke theory and the Bergmann–Wagoner class together with Higgs gravitation and broken-symmetric theories of gravitation. This may be partly found in [24] (here we set $\hbar = c = 1$). –

5.1 Jordan's theory

In modern quantum theories, interactions between equally charged particles mediated by bosons with odd spin are repulsive, and interactions between differently charged particles with the mediation of odd-spined bosons are attractive:

- In QED, photons possess spin 1 and equally charged particles repulse each other.
- QCD confinement derives from an attractive force which acts between differently color-charged quarks in hadrons.

Interactions which are mediated by even-spin bosons are attractive:

- Higgs particles possess spin 0 and thus mediate attractive forces between particles which couple to them.
- Pions, as spin-0-bosons, mediate an attractive effective nuclear force between isotopic particles.

Classically, to describe gravitational interaction, the gravitational Lagrangian of the theory (which obeys the Euler–Lagrange equations for a field) describes the propagation and self-interaction of the gravitational field only through the Ricci scalar R (see (A.4.6)). From Einstein's GR (in analogy to quantum theories) it follows that the gravitational interaction is, in its quantum-mechanical nature, mediated by massless spin-2-excitations only [88]. This is expected to be related to the still-hypothetical gravitons as intermediate particles of a quantum theory of gravity. Scalar–tensor theories (STTs), on the other hand, postulate in this context the existence of more complex dynamics from further mediating particles, named in this case *graviscalars* within the context of quantum theories. This means that STTs modify classical GR by the addition of scalar fields to the tensor field of GR. They further demand that the "physical metric" $g_{\mu\nu}$ (coupled to ordinary matter) be a composite object of the form

$$g_{\mu\nu} = A^2(\hat{\phi}) g^*_{\mu\nu}, \tag{5.1.1}$$

with a coupling function $A(\hat{\phi})$ of the scalar field $\hat{\phi}$ [62].
The first attempts of a scalar–tensor theory were started independently by M. Fierz in 1956 [90] and by Pascual Jordan in 1949 [136]. The latter noticed through his isomorphy theorem that projective spaces as Kaluza–Klein's (five-dimensional) can be reduced to usual Riemannian 4-dimensional spaces and that a scalar field as fifth component of such a projective metric can play the role of a variable effective gravitational "constant" $\tilde{G}$, which is typical for STTs and by which it is possible to vary the strength of gravitation [87] (thus, obviously violating in some account the strong equivalence principle (SEP)). The same gravitational interactions might not hold on all physical systems. Furthermore, this kind of general-relativistic model with a scalar field is conform equivalent to multi-dimensional general-relativistic models [59]. Many theories involve this physics (*e.g.* string theories or brane theories), but scalar–tensor theories are typically found to represent classical descriptions of them [113].
In his theory, Jordan introduced two coupling parameters of the scalar field. One parameter produced a variation of the gravitational constant. The other one would break the energy conservation through a non-vanishing divergence of the energy–momentum tensor to increase the mass in time, in accordance with the ideas of Jordan and Dirac [137]. However, the cosmic microwave background radiation (CMB) as a real black-body radiation discovered in 1965 [196][1] forces to accept general energy conservation as experimental fact (see [130]).
In Jordan's theory, there appears a g^{55} component of the metrical tensor which is in general dependent on a scalar quantity. In a more usual formulation, however, the latter is equivalent to having κ (or the gravitational coupling G) as a field quantity. Hence, in this sense he spoke about an augmented or generalized gravitational theory (*erweiterte Gravitationstheorie*) which he also derived following the variation principle. Therefore, he explicitly assumed $\kappa \neq const.$ and allowed derivatives

$$\kappa_{,\mu} = \frac{\partial \kappa}{\partial x^\mu} \neq 0 \tag{5.1.2}$$

of the same. He assumed, in absence of matter, an action[2]

$$S_J = \int \kappa^\eta \left(R - \zeta \frac{\kappa_{,\mu}\kappa_{,\nu} g^{\mu\nu}}{\kappa^2} \right) \sqrt{-g} dt. \tag{5.1.3}$$

Here, we have the determinant g of the metrical tensor, the (Ricci) curvature scalar R, the gravitational coupling κ and η and ζ as empirical values.
Taking matter into account, Jordan derived generalized Einstein equations as follows,

$$\begin{gathered} \eta R + \zeta \left\{ (\eta - 2) \frac{\kappa^{,\mu}\kappa_{,\mu}}{\kappa^2} + 2 \frac{\kappa^{,\mu}\kappa_{,\mu}}{\kappa^2} \right\} = 0, \\ g_{\nu\lambda} \left(\frac{1}{2} R + \frac{(\kappa^\eta)^{,\mu}{}_{;\mu}}{\kappa^\eta} - \frac{\zeta}{2} \frac{\kappa^{,\mu}\kappa_{,\mu}}{\kappa^2} \right) - R_{\nu\lambda} - \frac{(\kappa^\eta)_{,\nu;\lambda}}{\kappa^\eta} + \zeta \frac{\kappa_{,\nu}\kappa_{,\mu}}{\kappa^2} = \kappa T_{\nu\lambda}. \end{gathered} \tag{5.1.4}$$

For central symmetry, there appears a correction for $e^\lambda = e^{-\nu}$ as well as a nonstatical coupling

$$\kappa = \kappa_0 \tau^{\beta_0/B} \tag{5.1.5}$$

[1] Work which resulted in 1984 in the Nobel prize for physics for A.A. Penzias and R.W. Wilson. Further, the exact analysis and corroboration of the qualities of CMB, together with the small anisotropy present in it, led to the Nobel prize award to John C. Mather and George F. Smoot in 2006.

[2] J after Ernst Pascual Jordan (1902-1980).

for vacuum, with the eigentime τ, a constant β_0 and $B = 1 + 2\eta\beta_0$. Time-dependence of κ, he concluded, however, would be especially weak, with

$$0 \neq \beta_0 \approx 0 \tag{5.1.6}$$

according to solar-relativistic effects. Furthermore, he concluded $\eta = 1$ and $|\zeta| \gg 1$ according to empirical data within some approximation of the theory.

5.2 Brans–Dicke theory

Jordan's theory was worked out independently by Brans and Dicke in 1961 [41] without breaking energy conservation, but again introducing a scalar field with an infinite length scale which now explicitly played the role of a variable gravitational coupling. The generalization to GR's action (A.4.6) was then proposed as follows, [3]

$$S_{JBD} = \int \left[\hat{\phi} R + (16\pi/c^4)\, \mathcal{L}_M - \frac{\omega}{\hat{\phi}} \left(\frac{\partial \hat{\phi}}{\partial x_\mu} \frac{\partial \hat{\phi}}{\partial x^\mu} \right) \right] \sqrt{-g}\, d^4x. \tag{5.2.1}$$

Here, we have the determinant g of the metrical tensor, the (Ricci) curvature scalar R, the matter Lagrangian $\mathcal{L}_M$ and a scalar field $\hat{\phi}$ which plays the role of the reciprocal newtonian constant G^{-1}. The first term of (5.2.1) couples the scalar field and gravitation given by R, while the third term represents the kinetic energy of $\hat{\phi}$, since the Lagrange density $\mathcal{L}$ (conceptually derived from the Lagrange function of mechanics) is usually defined in terms of the subtraction of the potential from the kinetic energy of the analyzed system. Other than in the original theory of Jordan, Brans' and Dicke's theory in equation (5.2.1) does not contain a mass-creation principle. The wave equation of $\hat{\phi}$ can be transformed so as to make the source term appear as the contracted energy–momentum tensor of matter alone. In other words, the inhomogeneous part of the wave equation is only dependent on the trace T of the tensor $T_{\mu\nu}$, and this is in accordance with the requirements of Mach's principle: $\hat{\phi}$ is given by the matter distribution in space.

In 1968, P. Bergmann [18], and in 1970 R. Wagoner [234], discussed a more general scalar–tensor theory which possesses an additive cosmological function term $\Lambda(\hat{\phi})$ in the Lagrangian. Furthermore, the latter may now possess a functional parameter $\omega = \omega(\hat{\phi})$ for a scalar field $\hat{\phi}$. This general kind of theories, now often called *Bergmann–Wagoner* (BW) class of STTs, possesses the Jordan–Brans–Dicke (JBD) class as a special case for $\omega = const.$ and $\Lambda(\hat{\phi}) = 0$.

In physics a theory is said to be in a canonical form if it is written in a paradigmatic form taken from classical mechanics (as ideal which is in principle, however, freely eligible and a matter of definition).[4] The equation (5.2.1), called to be in *Jordan frame*, is not in this form. The Bergmann–Wagoner-formed models are not canonical. However, STTs can be transformed conformally into a canonical form (*Einstein frame*) in which a cosmological function still appears, but $\hat{\phi}$ is minimally coupled.

Canonical form is achieved by changing from the Jordan frame (with mixed degrees of freedom of metric and scalar field) to the einsteinian one (with unmixed degrees of freedom). In the four-dimensional case, such is fulfilled through $g_{\mu\nu} \to \hat{\phi}^{-1} g_{\mu\nu}$ (*cf.* (5.1.1)) and a redefinition of the scalar field and cosmological

[3] JBD after Jordan and Carl Henry Brans (1935) and Robert Henry Dicke (1916-1997).

[4] *Cf.* an interesting analysis about the historical origin and meaning of the concept of "canon" by J. Assmann in [11], which he further relates to Halbwachs' "mémoire volontaire" of a society.

function. However, it is still subject of discussion which frame is best. The Jordan one, though, is usually called the physical frame [45].

The scalar field in the Jordan–Brans–Dicke's theory is massless. However, a generally covariant theory of gravitation can accommodate a massive scalar field in addition to the massless tensor field [2, 120]. Thus, a version of the JBD or BW theory with massive scalar fields may be postulated [94]; indeed, A. Zee incorporated as first the concept of SSB to gravity within a STT [247], suggesting that the same symmetry-breaking mechanism was responsible for breaking a unified gauge theory into strong, weak and electromagnetic interactions (mediated by their corresponding gauge bosons). Spontaneous symmetry breaking (SSB) causes some scalar field to have a vacuum expectation value v, thus generating the mass of the intermediate bosons and of fermions, relating them to the ground state of the scalar field after the breakdown of symmetry. Zee attributed the smallness of Newton's gravitational constant G_N (of the order of magnitude of about 10^{-11} $\mathrm{m^3kg^{-1}s^{-2}}$) to the massiveness of some particle (this may be compared with the result of [68]) with Newton's coupling constant $G_N \sim (10^{19}\mathrm{m}_N)^{-2}$ as

$$G_N \sim 1/v^2, \tag{5.2.2}$$

where

$$v^2 = \frac{\sqrt{2}}{(8\pi G_F)} \approx 6.07 \cdot 10^4 (\mathrm{GeV})^2. \tag{5.2.3}$$

Thus, SSB generates the mass of the intermediate boson such that for Fermi's coupling constant,

$$G_F \approx \frac{1}{2\pi(294\,\mathrm{GeV})^2} \sim (300\,\mathrm{m}_N)^{-2}, \tag{5.2.4}$$

with weakon mass M_W ($\sim 80\,\mathrm{GeV/c^2}$) and elementary charge e ($\sim 10^{-19}$C)) there is

$$G_F \sim e^2/M_W^2 \sim 1/v^2, \tag{5.2.5}$$

which may be compared with (5.2.2). Since SSB has proven extraordinarily faithful in many areas of physics, Zee considered it worthwile to incorporate this mechanism into gravitation [247] and explain the smallness of Newtons's constant through the mass of Higgs particles.[5] Through the incorporation of SSB, the scalar field is anchored in a deep potential well $V(\phi)$. The physical consequences are then indistinguishability between Einstein's model and Zee's STT except under extreme conditions of spacetime curvature [247]; in sharp contrast to earlier work of Brans and Dicke.

Zee's mechanism includes a self-interaction of the scalar field and, thus, a potential V as part of the cosmological function of the BW class of STTs which, however, lacks in usual JBD theories. As a result of the missing potential, Brans–Dicke's theory is inconsistent with observation unless a certain parameter is very large [235]. In fact, from measurements of radio-signal current time delay with Viking probes form Mars, the coupling parameter ω of the usual JBD theory in (5.2.1) (a measure of the strength of the scalar field coupling to matter) is required to being greater than about 500 [205].

In any sensitive theory, Brans and Dicke proved in their original work [41], the dimensionless constant ω must be of the general order of unity. For $\omega \to \infty$, however, GR is obtained, which entails that the JBD theory leads nearly to the same results as GR. In contrast, however, in a STT with the scalar field anchored by the SSB potential, this strength of the scalar-field coupling may naturally be smaller. Thus, new physics

[5]The smallness of G can also, as will be seen, be explained through a high expectation value of the scalar field as well as through a strong coupling of the scalar field to gravitation, analogously to [69].

of higher order is possible. Given the gravitational properties of Higgs-like fields (see Chapter 3), for instance, it seems natural to couple them to gravitation and analyze new properties of the model. This kind of Higgs fields (because of their coupling to SSB and the possession of a nontrivial vacuum state) may then be relevant in view of a gravitational theory which might entail long-range changes in the dynamics to explain dark components, anchored or not with elementary particle physics.
Indeed, the simplest "Higgs-field model" beyond the standard model consists in the addition of a singlet particle that only interact with the Higgs sector of the SM, in which the sector does not couple directly to vector bosons. With a fundamental gauge-invariant construction block $\phi^\dagger \phi$, the simplest coupling of a particle to a Higgs or Higgs-like field is [127]

$$\text{Lagrangian term of Higgs sector } = \tilde{\lambda} X \phi^\dagger \phi, \tag{5.2.6}$$

where X is a scalar field and † represents the hermitean conjugation, the transposition of a tensor for real-valued components, and complex conjugation for purely scalar quantities.
The Higgs field develops a vacuum expectation value and, after shifting it, the vertex (5.2.6) leads to a mixing between the scalar field and the Higgs field. Thus, it may give rise to new effects that do not involve the scalar explicitly [127]. Furthermore, the X-field may be considered as not fundamental, but an effective description of an underlying dynamical mechanism, and a relation between gravity and the generalized Higgs sector may be assumed. Both gravity and a Higgs particle possess some universal characteristics:

- Gravity couples universally to the energy–momentum tensor and the Higgs particle to mass, which corresponds to the trace of the energy–momentum tensor. This suggests a relation between the generalized Higgs sector and gravity, which is indeed given by Higgs gravity in [69].

- Furthermore, there is a similarity between X and the hypothetical graviton since both are singlets under the gauge group (see [26]).

Because they have no coupling to ordinary matter, singlet fields are not well constrained by experiments. Typically, one can argue that they are absent from the theory because they can have a bare mass term which can be made to be of the order of the Planck mass M_P, making these fields invisible. However, one can take the attitude that the Planck length be not a fundamental constant but rather a property of today's state of the world, which evolve in time and be typically given by a vacuum expectation value of some scalar field [238]. With a Higgs coupling to gravity, then, all masses, including the Planck mass, should be given by SSB. In this case there is a hierarchy of mass scales $M_P \gg v$. Given these similarities, X can be considered to be essentially the graviton and be identified as constant $\cdot R$, with the curvature scalar R (as done by [26]). Moreover, this possibility may be used to explain the naturalness problem, especially since other candidates such as top-quark condensation or technicolor (in which quarks are no longer primordial) have not functioned so far and supersymmetry doubles the spectrum of elementary particles, replacing Bose (Fermi) degrees of freedom with Fermi (Bose) degrees of freedom, whereas all supersymmetric particles are by now beyond physical reality (*cf.* Chapter 2.3).
Making a low-energy expansion [26] and ignoring higher derivative terms, a spontaneous symmetry breaking theory of gravity with a Higgs field as the origin of the Planck mass may be derived [26, 27]. Moreover, this is the theory which was first derived in [70] and [71]. The remnant of originally very strong interactions is the parameter $\breve{\alpha}$, which in Chapter 6.1 will be introduced as the coupling strength of the Higgs field to gravitation. It will essentially give Newton's gravitational constant, and its high value will enable the model to be distinguishable to gravity at low energy scales, other than the case within usual JBD theories.

The class of STTs with massive scalar fields is given within the Bergmann–Wagoner (BW) class with the following Lagrangian,[6]

$$\mathcal{L}_{BW} = \frac{\hat{\phi}}{16\pi}\left\{R + \frac{\omega(\hat{\phi})}{\hat{\phi}^2}\hat{\phi}_{,\lambda}\hat{\phi}^{,\lambda} - 2\hat{\phi}U(\hat{\phi}) + \mathcal{L}_M\right\}\sqrt{-g}, \tag{5.2.7}$$

Further, $\hat{\phi}U(\hat{\phi}) = \tilde{\Lambda}(\hat{\phi})$ gives a cosmological function and $_{,\lambda}$ the derivative in respect to the λ-coordinate. Within the Bergmann–Wagoner class, there is a wide account of analyses, although most of them focus on $U(\hat{\phi}) = 0$ as special case. However, analyses within the general BW class such as on the existence of black holes as well as global properties of static, spherically symmetric configurations can be found, for instance, in [36–38], and on deSitter and warm inflation models in the framework of STTs in [19], and with Higgs potential in [48, 49]. Friedmann–Lemaître–Robertson–Walker (FLRW or simply RW) models for Friedmann–Lemaître Universes for cosmology, further, are analyzed in [200], obtaining a class of separable Wheeler–deWitt equations after a quantization of the models. That is, we obtain equations which a wave function of the Universe should satisfy in a theory of quantum gravity.

[6] For purposes of completeness, the BW class can be given in an even more general form for D dimensions and with a non-minimal coupling $f(\phi)R$ (see [38]).

Chapter 6

Scalar–tensor theory with Higgs potential

– A theory of induced gravity with Higgs potential is introduced parting from the Bergmann–Wagoner class of scalar–tensor theories and Spontaneous Symmetry Breaking. The field equations of gravity before and after symmetry breakdown are presented together with Maxwell-like equations for gravity with a gravitational energy density. Parts of this work may found published especially in [20, 23, 24]. –

6.1 Lagrange density and models

Let us take a closer look at a Bergmann–Wagoner (BW) model with an in general nonvanishing cosmological function. Let then the scalar field be defined through a U(N) isovector which is a scalar field also, with

$$\hat{\phi} = \breve{\alpha}\phi^\dagger\phi \quad \text{and the definition} \quad \omega = \frac{2\pi}{\breve{\alpha}} = const., \tag{6.1.1}$$

with the gravitational strength $\breve{\alpha}$ (as remnant of strong interactions, *cf.* [26]), and the cosmological function of the BW class given by

$$U(\hat{\phi}) = U(\breve{\alpha}\phi^\dagger\phi) = \frac{1}{\breve{\alpha}\phi^\dagger\phi}\left[8\pi V^*(\phi^\dagger\phi)\right] = \frac{8\pi}{\hat{\phi}}V^*\left(\frac{\hat{\phi}}{\breve{\alpha}}\right), \tag{6.1.2}$$

whereas $V^*(\phi) \equiv V^*(\phi^\dagger\phi)$ be the potential (density) of the scalar field.[1]

The model parting from equations (6.1.3) and (6.1.6) does not possess solely gravitative vertices as an einsteinian quantum theory would. This lacking of only gravitative vertices should further exclude the appearance of outer gravitational lines (as long as no primordial gravitational constant is assumed[2]) [93]. If a primordial gravitational constant appeared, gravitational source terms (vertices) would follow, and then the renormalization arguments would not apply anymore. As can be easily seen, such a model does not possess a dimension-loaded coupling constant as G, which is the main problem for renormalizing Einstein's

[1]For a relation between Zee's [247] and Dehnen's models, one may later take $\breve{\alpha}/\epsilon = 8\pi$ and isoscalar fields $\phi = \varphi$ with $\breve{\alpha}$ and $\epsilon = const$. Further, there is $\lambda_{\text{Zee}} = (1/3)\lambda$.

[2]This would be the case in a generalization with many scalar fields. Then, R^2-terms would be necessary for renormalization, as is the case in the work of K. Stelle [225]

theory. Through (6.2.4), G will be replaced with the reciprocal dimensionless constant $\breve{\alpha}$ multiplied with $\phi^\dagger\phi$. Thus, the dimension problem for renormalization disappears. DeWitt's power counting criterion [243] for normalizability may be used [93] and the theory should be renormalizable [70].

The scalar field shall couple nonminimally with the Ricci curvature scalar R with the gravitational strength $\breve{\alpha}$. In this way, we can give the Lagrangian of a scalar–tensor theory in Jordan frame of the form

$$\mathcal{L} = \left[\frac{1}{16\pi}\breve{\alpha}\phi^\dagger\phi R + \frac{1}{2}\phi^\dagger_{;\mu}\phi^{;\mu} - V^*(\phi) + \mathcal{L}_M\right]\sqrt{-g}\,, \tag{6.1.3}$$

whereas $\hbar = 1$ and $c = 1$ are set, and $_{;\mu}$ mean the covariant derivative with respect to all gauge groups.The subscript $_{,\mu}$ represent the usual derivative (see discussion in relation with the Lagrangian (5.2.7)). The Lagrangian (6.1.3) postulates possible gravitational interactions not only mediated by massless spin-2-excitations as is postulated on the one hand in usual GR, but takes into account gravitational interactions of massive scalar fields. Further, let the potential $V^*(\phi)$ of the scalar field be of the form of the one of the Higgs field of elementary particle physics, that is a ϕ^4-potential with

$$V(\phi) = \frac{\lambda}{24}\left(\phi^\dagger\phi + 6\frac{\mu^2}{\lambda}\right)^2 = \frac{\mu^2}{2}\phi^\dagger\phi + \frac{\lambda}{24}\left(\phi^\dagger\phi\right)^2 + \frac{3}{2}\frac{\mu^4}{\lambda}\,. \tag{6.1.4}$$

The potential in (6.1.4) possesses the additive factor $3\mu^4/(2\lambda)$ of equation (3.2.3). The additive term is thus related to the election of a vanishing formal cosmological constant which, however, can be inserted in the theory by adding the constant term (3.2.5),

$$V_0 = -\frac{3\breve{\alpha}\mu^2}{4\pi\lambda}\Lambda_0$$

with Λ_0 as a true cosmological constant so that the total Higgs potential is given by equation (3.2.6),

$$V^*(\phi) = V(\phi) + V_0 \tag{6.1.5}$$

with a cosmological function $\Lambda(\phi)$ dependent on this generalized Higgs potential, as will be seen in equations (6.2.5) and (6.3.25). The cosmological constant Λ_0 is often expected to be vanishing for physical economy. However, together with Quintessence in general, it is related to our understanding of the nature of gravity. It might indeed be a low-energy appearance coming from primary gravitation in the early Universe, as proposed in [188], but related to dynamical quintessential fields. Nevertheless, the constant part of the cosmological function coming from the Higgs potential (6.1.5) (*i.e.* the true cosmological constant) will further be taken as vanishing and, if written, then only for purposes of completeness.
In principle, equation (6.1.3) represents a model between the JBD and the BW class of STTs, with a constant coupling $\omega = const.$ and $\Lambda(\phi) \neq 0$ (unless for STT $\rightarrow$ GR).
In (6.1.3), $\mathcal{L}_M$ is the Lagrange density of the fermionic and massless bosonic fields,

$$\mathcal{L}_M = \frac{i}{2}\bar{\psi}\gamma^\mu_{L,R}\psi_{;\mu} + h.c. - \frac{1}{16\pi}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - (1-\hat{q})k\bar{\psi}_R\breve{\phi}^\dagger\hat{x}\psi_L + h.c., \tag{6.1.6}$$

while ψ in (6.1.6) are the fermionic fields, and

$$\begin{aligned}\mathcal{F}_{\mu\nu} = \frac{1}{ig}\left[\mathcal{D}_\mu, \mathcal{D}_\nu\right] &= \mathcal{A}_{\nu,\mu} - \mathcal{A}_{\mu,\nu} + ig\left[\mathcal{A}_\mu, \mathcal{A}^a_\nu\right] \\ &= \mathcal{A}_{[\nu,\mu]} + ig\left[\mathcal{A}^a_\mu, \mathcal{A}_\mu\right]\end{aligned}$$

is the matrix representation of the field-strength tensor for the gauge potentials $\mathcal{A}_\mu$ (see (4.2.1), (B.5.1) and (B.5.11)). It is defined by the commutator of the covariant derivative $\mathcal{D}_\mu$ (Ricci identities), analogously to within electrodynamics for the electric and magnetic strengths $\vec{E}$ and $\vec{B}$. The exact form of covariant derivatives, that is of the potentials, however, depends on the chirality and form of the actual fermionic field. For the electroweak interactions, left-handed wave functions are thus described by (iso-)doublets, while right-handed ones are described by (iso-)singlets (*cf.* Appendix B.3).
Within electrodynamics, the homogeneous Maxwell equations are derived using Jacobi identities with covariant derivatives (**Bianchi identities**). The inhomogeneous ones depend on the Lagrangian and thus on the exact system (and thus on the environment, as reflected in the appearance of magnetization $\vec{M}$ and polarization $\vec{P}$ in the field equations). The more general equations of Yang–Mills's theories, for the dynamics for $\mathcal{F}_{\mu\nu}$ and isovectorial ψ, are derived analogously. However, unlike within QED, the commutator $[\mathcal{A}_\mu, \mathcal{A}_\nu] \equiv \mathcal{A}_\mu\mathcal{A}_\nu - \mathcal{A}_\nu\mathcal{A}_\mu$ is not vanishing. It presents self-interactions of the gauge potentials. Through them, in QCD, for instance, gluons do interact with each other, while such interactions vanish within QED given the abelian (commutative) character of the symmetry group $U(1)$. Photons, as gauge bosons in QED, do not self-interact.
In equation (6.1.6), $\hat{x}$ give the Yukawa coupling operator, k (or k_f when taking the different families or flavors) be a constant factor, and the subscripts R and L refer to the right- and left-handed fermionic states of ψ. The index a be the iso-spin index, which counts the N isotopic elements of the multiplet ψ. For matters of complementarity, in addition to $\hat{\phi}$ and ϕ, we have taken a scalar field $\breve{\phi}$ in (6.1.6). However, let us further take $\breve{\phi} = \phi$ in the following; this means the same scalar field coupled with the Ricci scalar R and matter for the case $\hat{q} \neq 1$.
Equation (6.1.6) together with (6.1.3) leads to the field equations as derived first in [26, 27, 70, 71]. The parameter $\hat{q}$ is defined to give the fermionic coupling with the scalar field. It will be essential for the Klein–Gordon equation of the Higgs field of the model as well as for the Dirac equation (*v.i.* in Chapter 6.2).

Concluding, following (6.1.3), (6.1.4), (6.1.5) and (6.1.6), we have the following Lagrangian,

$$\begin{aligned}\mathcal{L} = \Bigg[&\frac{1}{16\pi}\breve{\alpha}\phi^\dagger\phi R + \frac{1}{2}\phi^\dagger_{;\mu}\phi^{;\mu} - \frac{\lambda}{24}\left(\phi^\dagger\phi + 6\frac{\mu^2}{\lambda}\right)^2 + \frac{3\breve{\alpha}\mu^2}{4\pi\lambda}\Lambda_0 + \frac{i}{2}\bar{\psi}\gamma^\mu_{L,R}\psi_{;\mu} + h.c.- \\ &-\frac{1}{16\pi}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - (1-\hat{q})k\bar{\psi}_R\breve{\phi}^\dagger\hat{x}\psi_L + h.c.\Bigg]\sqrt{-g}.\end{aligned} \tag{6.1.7}$$

$\breve{\alpha}$ is the field strength, R is the Ricci curvature scalar, Λ_0 is a cosmological constant, $\mathcal{F}_{\mu\nu}$ is the field-strength tensor in matrix notation for isocomponents a with field variables $\mathcal{A}_\mu$, ψ is the fermionic wave function of the matter Lagrangian $\mathcal{L}_M$, k is the Yukawa coupling, $\hat{x}$ is the Yukawa matrix and g is the determinant of the metric $g_{\mu\nu}$. Subscripts R and L denote the right- and left-handed wave function. ϕ is a field with Higgs potential of parameters $\mu^2 < 0$ and $\lambda > 0$. In general terms, $\breve{\phi}$ may be a further Higgs field added to ϕ. It leads in the case of $\hat{q} = 0$ to mass of elementary particles. $\breve{\alpha}$ is related to the gravitational coupling $\tilde{G}$ which will be induced by symmetry breaking.

6.2 The field equations

Using the Hamilton Principle of Least Action and the Euler–Lagrange equations for relativistic fields, one acquires generalized Einstein field equations and a Higgs field equation with a coupling of the scalar field ϕ

with the curvature scalar R and the symmetric metrical energy–momentum tensor $T_{\mu\nu}$:[3]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda^*(\phi)g_{\mu\nu} = -\frac{8\pi}{\breve{\alpha}\phi^\dagger\phi}T_{\mu\nu} - \frac{8\pi}{\breve{\alpha}\phi^\dagger\phi}\left[\phi^\dagger_{(;\mu}\phi_{;\nu)} - \frac{1}{2}\phi^\dagger_{;\lambda}\phi^{;\lambda}g_{\mu\nu}\right] - \\ - \frac{1}{\phi^\dagger\phi}\left[(\phi^\dagger\phi)_{,\mu;\nu} - \left(\phi^\dagger\phi\right)^{,\beta}{}_{;\beta}\, g_{\mu\nu}\right], \tag{6.2.1}$$

$$\phi^{;\mu}{}_{;\mu} - \frac{\breve{\alpha}}{8\pi}\phi R + 2\frac{\delta V^*(\phi)}{\delta\phi^\dagger} = -2\frac{\delta\mathcal{L}_M}{\delta\phi^\dagger}, \quad \text{with} \quad 2\frac{\delta V(\phi)}{\delta\phi^\dagger} = \mu^2\phi + \frac{\lambda}{6}(\phi^\dagger\phi)\phi, \tag{6.2.2}$$

where $T_{\mu\nu}$ will be given in equation (6.3.16), and with $\Lambda^*(\phi)$ given in (6.2.5).
The term on the right-hand side of scalar-field equation (6.2.2) is the source of the scalar field with

$$2\frac{\delta\mathcal{L}_M}{\delta\phi^\dagger} = 2\left(\frac{\delta\mathcal{L}_M}{\delta\phi}\right)^\dagger = -2k(1-\hat{q})\bar{\psi}_R\hat{x}\psi_L. \tag{6.2.3}$$

Equation (6.2.3) depends on the fermionic Lagrangian and thus on the parameter $\hat{q}$, consequences of which will be discussed in Chapter 6.3.
In analogy to GR (see equation (A.4.3)), we may define in (6.2.1) a gravitational coupling term as follows,

$$G(\phi) = \frac{1}{\breve{\alpha}\phi^\dagger\phi}, \tag{6.2.4}$$

whereas $G(\phi)$ here is a field quantity and thus local. It is dependent on the scalar field ϕ and the gravitational strength $\breve{\alpha}$. Analogously, a general cosmological function was defined in (6.2.1) as

$$\Lambda^*(\phi) := \frac{8\pi}{\breve{\alpha}\phi^\dagger\phi}V^*(\phi) = 8\pi G(\phi)V(\phi) - \frac{6\mu^2}{\lambda}\frac{\Lambda_0}{\phi^\dagger\phi}, \tag{6.2.5}$$

mainly given by the potential of the scalar field and its excitations (*viz* (6.3.2) and (6.3.1)), and related to the cosmological function term $\breve{\alpha}\phi^\dagger\phi U(\phi)$ of the BW class of STTs. The field equations for the fermionic fields and the bosonic Yang–Mills fields are neglected.
Both the Ricci curvature scalar R in the field equations of gravity and of the scalar-field equation are coupled to the scalar field itself. $R = g^{\mu\nu}R_{\mu\nu} \equiv \sum_{\mu,\nu=0}^{3} g^{\mu\nu}R_{\mu\nu}$ can be derived from equation (6.2.1), with the form

$$R = \frac{8\pi}{\breve{\alpha}\phi^\dagger\phi}\left[T + 4V^*(\phi) - \phi^\dagger_{;\beta}\phi^{;\beta}\right] - \frac{3}{\phi^\dagger\phi}(\phi^\dagger\phi)^{;\beta}{}_{;\beta}, \tag{6.2.6}$$

whereas $V^*(\phi) = V(\phi) + V_0$ is valid from (6.1.5) and T is the trace of the tensor $T_{\mu\nu}$.

The field equations of the theory of elementary particles are valid. For instance, the Dirac equations read[4]

$$i\gamma^\mu_{\binom{L}{R}}\psi_{;\mu} - k\begin{pmatrix}\hat{x}\phi\psi_R \\ \phi^\dagger\hat{x}\psi_L\end{pmatrix} = 0, \quad h.c., \tag{6.2.7}$$

with the Yukawa coupling operator $\hat{x}$ and the Dirac matrices γ^μ, which are given by the Clifford algebra

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\underline{1}. \tag{6.2.8}$$

The inhomogeneous Yang–Mills equations for the gauge-field strength read

$$F_a^{\mu\nu}{}_{;\nu} = 4\pi j_a^\mu, \tag{6.2.9}$$

[3] $_{(\ldots)}$ are the antisymmetric Bach parenthesis given by $A_{(i}B_{k)} = \frac{1}{2}(A_iB_k + A_kB_i)$.

[4] As usual, the second term in equation (6.2.7) belongs to the mass of the (other-handed) particles.

with the gauge currents j^μ_a given by

$$\begin{aligned} j^\mu_a =& j^\mu_a(\phi) + j^\mu_a(\psi) \\ =& g\bar{\psi}\gamma^\mu_{L,R}\tau_a\psi + \frac{ig}{2}\phi^\dagger\tau_a\phi^{;\mu} + h.c., \end{aligned} \tag{6.2.10}$$

with a fermionic part $j^\mu_a(\psi)$ and $j^\mu_a(\phi)$ of the Higgs fields. Further, the Higgs currents $j^\mu_a(\phi)$ are gotten through equation (6.1.3) as gauge fields of the inner symmetry group, with the gauge potential A^a_μ:

$$j^\mu_a(\phi) = \frac{\delta\mathcal{L}_M}{\delta A^a_\mu} = \frac{ig}{2}\phi^\dagger\tau_a\phi^{;\mu} + h.c. \tag{6.2.11}$$

Further, the energy–stress tensor is defined in equation (A.4.9). It is of the following form,

$$T^{\mu\nu} = \frac{i}{2}\psi^{(\mu}_{L,R}\psi^{;\nu)} + h.c. - \frac{1}{4\pi}\left(F^{\mu\ a}_{\ \lambda}F^{\nu\lambda}_a - \frac{1}{4}F^a_{\alpha\beta}F^{\alpha\beta}_a g^{\mu\nu}\right). \tag{6.2.12}$$

6.3 Field equations after symmetry breakdown

In the spontaneously broken phase of symmetry, developing the scalar field ϕ around its ground-state value v,

$$\phi_a = vN_a + \phi'_a\,, \tag{6.3.1}$$

the ground-state value of the scalar field is given by

$$\phi^\dagger_{a0}\phi_{a0} = v^2 = -\frac{6\mu^2}{\lambda}\,, \tag{6.3.2}$$

with v real-valued, $\mu^2 < 0$ and $\lambda > 0$. This can further be resolved in general as $\phi_0 \equiv \phi_{a0} = vN_a$ with $N_a = const.$, with the tensor N satisfying $N^{\dagger a}N_a = 1$, with

$$\phi = \rho\, U\, N = \frac{\rho}{v}\, U\,\phi_0\,. \tag{6.3.3}$$

Here, U is a unitary transformation and ρ is a real-valued function which takes into account that no conservation rule is valid for Higgs fields alone. Given the properties of the transformation U and of N, there is $\phi^2 = \rho^2$ so that ρ^2 gives the squared value of the scalar field.

For the ground state ϕ_0, the potential vanishes for the election of no further additive factor Λ_0 of the cosmological function, following (6.1.4) and (6.1.5):

$$V(\phi_0) = u_0 \equiv \frac{1}{8\pi G(\phi_0)}\Lambda_0\,. \tag{6.3.4}$$

This is the energy density of the ground state of the scalar field, which is $\breve{V}^*(\phi_0) = -(3/2)(\mu^4/\lambda) + (1/(8\pi G(\phi_0))\Lambda_0$ if the last factor of equation (6.1.4) is not taken. Such would lead to a formal cosmological constant added to the cosmological function itself, which we want to avoid. Hence the chosen form of the potential in (6.1.4).

According to the usual mechanism, after symmetry breaking, two particles appear: a massless particle, called Goldstone, and a massive particle usually called Higgs (*cf.* [22]). The first of these particles can be "gauged away" through the so-called unitary gauge [20, 69]. *i.e.*

$$\left.\begin{aligned} \phi \;&\to\; U^{-1}\phi = \rho\, U^{-1}U\, N = \rho\, N \\ A^a{}_\mu\tau_a \;&\to\; U^{-1}A^a{}_\mu U\tau_a U^{-1} + \tfrac{i}{g}U^{-1}_{,\mu}U \end{aligned}\right\}\,. \tag{6.3.5}$$

After unitary gauge, there is still $\phi^\dagger\phi = \rho^2$. Further, the scalar field ϕ can be written in terms of the real-valued excited Higgs scalar field ξ (a real-valued scalar variable) in the following form:

$$\phi = \rho\, N \equiv \phi_0\sqrt{1+\xi} = v\sqrt{1+\xi}\, N \quad \text{with} \quad \xi = \frac{\phi^\dagger\phi}{v^2} - 1\,, \tag{6.3.6}$$

with the dimensionless parameter $\breve{\alpha}$ from (6.1.3) and the ground-state value v which, following (6.3.2), is related to the Higgs potential and the Higgs parameters as follows,

$$\begin{aligned} V^*(\xi) &= \frac{3}{2}\frac{\mu^4}{\lambda}\xi^2 - \left(\frac{8\pi\lambda}{6\mu^2}\right)^{-1}\Lambda_0 \\ &= -\frac{1}{4}\mu^2\, v^2\, \xi^2 + \left(\frac{8\pi}{\breve{\alpha} v^2}\right)^{-1}\Lambda_0 \\ &= \frac{\lambda\, v^4}{24}\,\xi^2 + \left(\frac{8\pi}{\breve{\alpha} v^2}\right)^{-1}\Lambda_0 . \end{aligned} \tag{6.3.7}$$

The gravitational strength $\breve{\alpha}$ may further be defined in terms of the ratio

$$\breve{\alpha} \simeq (M_P/M_B)^2 \gg 1\,, \tag{6.3.8}$$

where M_P and M_B are the Planck mass and the mass of the gauge boson, respectively. The mass of the gauge boson is given by

$$M_B \simeq \sqrt{\pi} g v, \tag{6.3.9}$$

where g is the coupling constant of the corresponding gauge group.
In relation to the fermionic mass, thus in the case of a coupling of ϕ to the fermionic Lagrangian (6.1.6), the coupling constants g and the ground-state (vacuum expectation) value are indirectly known from high-energy experiments. From a comparison between current–current coupling within Fermi's theory, low-energetic limits of W^+-couplings and the weakon mass M_W, the ground-state value v can be written dependent on Fermi's constant G_F and be experimentally determined as $v^2 \approx 6\cdot 10^4(\text{GeV})^2$. However, the relation between the vacuum expectation value (v) and the mass M of the particle related to Higgs mechanism is now different to within the SM so that constraints on v affect M in a different way within this scalar–tensor theory (*v.i.* mass and discussion).
Let us discuss the field equations of this model after breakdown of symmetry:

(i) **Dirac equation:**
The Dirac equations (6.2.7) (with Dirac matrices γ^μ) acquire the following form after symmetry breaking:

$$i\gamma^\mu_{\binom{L}{R}}\psi_{;\mu} - (1-\hat{q})\sqrt{1+\xi}\,\hat{m}\psi_{\binom{R}{L}} = 0\,. \tag{6.3.10}$$

The parameter $\hat{q}$ is defined such as to show the fermionic coupling of the scalar field. Hence, there is:

- (i) $\hat{q} = 0$ in the case that this Higgs field couples to the fermionic field (ψ) in the Lagrangian (6.1.6), and
- (ii) $\hat{q} = 1$ when it couples only with curvature R.

In the case (ii), the scalar field may be cosmon-like or else be isovectorial and couple analogously to Higgs fields in GUTs (*cf.* [48]). In the case (i), the scalar field may lead to mass generation analogously to SM (*cf.* [49]). Consequently, $\hat{m}$ is the fermionic mass matrix which is related to the Yukawa coupling operator $\hat{x}$ and to the ground-state value as follow (*cf.* Chapter 3.3),

$$\hat{m} = \frac{1}{2}kv\left(N^{\dagger}\hat{x} + \hat{x}^{\dagger}N\right). \tag{6.3.11}$$

Here, for matters of simplicity we have let the family subscript f aside.
If the scalar field is coupled to the fermionic field (ψ) in (6.1.6) (*i.e.* in the case $\hat{q} = 0$), we have the same structure as within the SM. The diagonal elements (eigenvalues) of the mass-square matrix read (with $c \neq \hbar \neq 1$)

$$\bar{M}^{(i)} \equiv 2\sqrt{\pi\hbar\breve{g}}v\sqrt{\left(\tau^i N\right)^{\dagger}\left(\tau^i N\right)}, \tag{6.3.12}$$

with gauge-coupling constant g and the generators τ^i of the symmetry group (see Chapter 3.3), for which the following algebra relation is valid,

$$\left.\begin{array}{r}[\tau^i, \tau^j] = if^{ij}{}_k\tau^k \\ \{\tau^i, \tau^j\} = c^{ij}\underline{1} + d^{ij}{}_k\tau^k\end{array}\right\}. \tag{6.3.13}$$

(ii) **Scalar-field equation and mass parameter:**
The dynamics of the Higgs particles is given by equation (6.2.2). Insertion of (6.2.6) in the same leads after symmetry breaking to the Higgs field equation which now reads as follows,

$$\xi^{,\mu}{}_{;\mu} + \frac{\frac{4\pi}{9\breve{\alpha}}\lambda v^2}{1+\frac{4\pi}{3\breve{\alpha}}}\xi = \frac{1}{1+\frac{4\pi}{3\breve{\alpha}}}\cdot\frac{8\pi}{3\breve{\alpha}v^2}\left[\hat{T} - \sqrt{\xi+1}\bar{\psi}\hat{m}\psi\right] + \frac{4}{3}\Lambda_0\left(1+\frac{4\pi}{3\breve{\alpha}}\right)^{-1}, \tag{6.3.14}$$

with the energy–momentum tensor $T_{\mu\nu}$ (analogously to the SM) with the trace (*sc.* [71])

$$T = \frac{i}{2}\bar{\psi}\gamma^{\mu}_{L,R}\psi_{;\mu} + h.c. = \sqrt{1+\xi}\bar{\psi}\hat{m}\psi \tag{6.3.15}$$

and fermionic mass matrices $\hat{m}$ as defined according to equation (6.3.11). Further terms from gauge bosons which would appear within the energy–momentum tensor after symmetry breaking have been neglected.
The energy-stress tensor satisfies the following equation law (see discussion below),

$$\hat{T}_{\mu}{}^{\nu}{}_{;\nu} = (1-\hat{q})\frac{1}{2}\xi_{,\mu}(1+\xi)^{-1}\hat{T}. \tag{6.3.16}$$

In equation (6.3.14), which is a Yukawa equation, a gravitational coupling constant

$$G_0 = \frac{1}{\breve{\alpha}v^2} = -\frac{1}{\breve{\alpha}}\frac{\lambda}{6\mu^2} \tag{6.3.17}$$

may be defined (*v.i.* equation (6.3.26)). Further, the (Compton-)length scale of the scalar field under validity of (6.3.11) may be defined using equation (6.3.17). With $\hbar \neq c \neq 1$, the length scale reads

$$l = \left[\frac{1+\frac{4\pi}{3\breve{\alpha}}}{16\pi G_0(\mu^4/\lambda)}\right]^{1/2} = M^{-1}\left(\cdot\frac{\hbar}{c}\right) \tag{6.3.18}$$

and it is (geometrically) the reciprocal of the scalar-field mass M, which in the SM is only given by $\sqrt{|2\mu|^2}$ (*v.s.*). For the mass of the Higgs particles, hence, we have

$$\left.\begin{array}{rl}M^2 = & -\frac{8\pi}{3}\frac{\mu^2}{\breve{\alpha}}\left(1+\frac{4\pi}{3\breve{\alpha}}\right)^{-1}\left(\frac{c}{\hbar}\right)^2 \\ = & \frac{4\pi}{9\breve{\alpha}}\lambda v^2\left(1+\frac{4\pi}{3\breve{\alpha}}\right)^{-1}\left(\frac{c}{\hbar}\right)^2\end{array}\right\}. \tag{6.3.19}$$

The scalar-field mass is dependent on the reciprocal gravitational coupling strength $\breve{\alpha}^{-1}$. Hence, in contraposition to the SM, regardless of high values of μ, M may be small-valued indeed.
After insertion of the length scale (6.3.18), the Higgs potential (6.3.7) reads

$$V^*(\xi) = \frac{3}{32\,l^2}\frac{\xi^2}{\pi\,G_0}\left(1+\frac{4\pi}{3\breve{\alpha}}\right)+(8\pi G_0)^{-1}\Lambda_0. \tag{6.3.20}$$

Further, after insertion of (6.3.18) and (6.3.15) in (6.3.14), the Higgs field equation reads

$$\xi^{,\mu}{}_{;\mu}+\frac{\xi}{l^2} = \frac{1}{1+\frac{4\pi}{3\breve{\alpha}}}\cdot\left(\frac{8\pi G_0}{3}\hat{q}\hat{T}+\frac{4}{3}\Lambda_0\right). \tag{6.3.21}$$

Now, with (6.3.21) and (6.3.16), the following is clear (*cf.* [23]):

- (I) In the case $\hat{q} = 0$, (6.3.21) will not possess a source, and for the SM, the latter means the production of fermionic mass through this Higgs field. This fact leads to a breaking of the conservation law (6.3.16) through a new "Higgs force". If the scalar field (ϕ) couples to ψ indeed, then for $\Lambda_0 = 0$ the source of equation (6.3.21) vanishes exactly (*sc.* [71]).
- (II) for no such coupling ($\hat{q} = 1$) the source is weak (*cf.* [20]) (this means proportional to G_0), and there is no entropy process from the conservation equation (6.3.16); *sc.* [70].

For the physical properties of the particles related to this Higgs field, the case (I) means that the particles, which are responsible for mass of elementary particles, decouple and interact only gravitationally. Hence, they cannot be generated through high-energy collision experiments as expected in the forthcoming LHC experiments. On the other hand, the case (II) means new particles which interact with other particles indeed, however weakly. These particles are related to a dark sector (*cf.* Chapter 2.3).
Furthermore, according to [27] (with $\xi_{Bij} = \breve{\alpha}/(16\pi)$), the Planck scale arises after SSB, thus resolving the discrepancy problem between the Planck and the electroweak scale. Additionally, wave-function renormalization of the scalar field results in the effective coupling of this Higgs field to matter becoming of gravitational strength $O(M/M_P)$ (*loc. cit.*).[5] For this reason, the Higgs becomes essentially a stable particle, which may have some cosmological consequences. We have basically the SM without Higgs particles (*loc. cit.*), especially for $\hat{q} = 0$ but also for $\hat{q} = 1$, for which, however, a further scalar field is to be added for not only astrophysical considerations.

(iii) **Higgs mass:** Cosmological consequences would depend on the length scale of the scalar field [25]. Especially, the scalar-field particles should effectively decouple for a small mass M. Meanwhile, $\breve{\alpha}$, as remnant of an original strong interaction (*cf.* [26]), would be the essential cause for the gravitational coupling G_0 being so small. Particularly in the case $\hat{q} = 0$, the scalar field possesses qualities as in [17] as a candidate of self-interacting DM, and might be in this way related to works like [73].
The way the Higgs is removed from the theory here by making its coupling to matter small is to be contrasted with the usual way where the mass of the Higgs has to be taken to be large.
Further, according to [25], for $\lambda = O(1)$, the mass M of the Higgs becomes very small and this results in a contribution to the gravitational force with a range $\sim 1/\sqrt{\lambda}$.
Additionally (*loc. cit.*), the Higgs particle then behaves as the cosmon of Quintessence. With high

[5] Hence, the model is renormalizable indeed, as already stated in [93].

λ-values, assumed of $O(\breve{\alpha})$, M became of the order of the electroweak scale because the Higgs coupling would have been reduced to gravitational strength.
Above-results point to a small mass of the particles related to the scalar field within induced gravity of a scalar–tensor theory with Higgs potential, and indeed, if the particles related to the scalar field are as massive as indicated in [20, 23, 24, 50], then they hardly decay in less massive particles. Furthermore, with those low masses, they still lie below the accuracy range of 5^{th}-force experiments as discussed in [3]. Such masses would to-date fulfill the strong equivalence principle (SEP). If an effective coupling is further also dependent on stiffness (say internal properties of matter), then the weak equivalence principle (WEP) will also be broken. Experiments in that matter, which try to measure the correlation between inertial and gravitational mass, are known under the concept of Eötvös experiment (*viz* [86], or Adelberger's molybdenum "Eöt-Wash" experiment [140] from which the greatest compactified dimension in string theory has to be smaller than 44 μm).
The squared mass (6.3.19) of the scalar field depends essentially on the gravitational coupling strength G_0, which is very weak. Thus, the Compton length given by $l = M^{-1}$ may at this point be expected to be high-valued. Within the SM, this would mean a very small value of $\sqrt{|\mu^2|}$. The constraints of a Higgs field mass, though, may change here in relation to those in the standard theory. Especially, nonvanishing values of v are possible for small masses M which here may be small-valued without the necessity of a small $|\mu^2|$ parameter. This is the case for vanishing values of both μ and λ leading to $v \cong \mu^2/\lambda \neq 0$ (see discussion about $\hat{q}$ and l and M in [20, 21, 23, 179] also).
In view of the structure of l, relatively large values of the length scale are possible, indeed. Thus, the solution of the field equations for the extreme case $l \to \infty$ is worth analyzing (see Chapter 7.4). Only relatively small values (which should, however, be finite, see Chapter 7.2) seem to be able to help explaining problems as the one of Dark Matter, and early analyses (*viz* [20]) do lead to high values of l to explain cosmological problems through long-range changes of dynamics. The extreme behavior of the limiting case $l \to \infty$ can help in the characterization of the usual one.
For values like in [20], $\frac{1}{l^2}\xi$-terms are negligible indeed, and the strong equivalence principle is then valid even for supra-solar as well as microscopic distances. In the work [20], further, linearization in ν and λ and not in ξ (which is valid), leads for length scales l of the order of magnitude of some galaxy radii (some kiloparsec with 1 kpc $\sim 10^{13}$km) to flattened rotation curves in a model of galaxies with polytropic density distribution with polytropic index $\gamma = 2$, with or without assuming a very massive core [20]. Further, for the strongest bars in isolated galaxies, a similar value of the length scale, of about 10 kpc, is gotten in [50] within the general BW class and with an arbitrary potential (but analogous field equations, with $p = 0$). This value is beyond the accuracy of the experiments presented in [3], and represents a mass $M = \frac{\hbar}{lc} \sim 10^{-26}$eV/c^2.

(iv) **Einstein equations:**
The generalized Einstein field equations (6.2.1) read now as given below,[6]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda^*(\xi)g_{\mu\nu} = -8\pi\,\tilde{G}\,\hat{T}_{\mu\nu} - \frac{\pi}{\breve{\alpha}}\frac{1}{(1+\xi)^2}\,[2\,\xi_{,\mu}\xi_{,\nu} - \\ - \,\xi_{,\lambda}\,\xi^{,\lambda}\,g_{\mu\nu}] - \frac{1}{1+\xi}\left[\xi_{,\mu;\nu} - \xi^{,\lambda}{}_{;\lambda}g_{\mu\nu}\right]. \tag{6.3.22}$$

[6]These may be compared with the field equations in [50, 210] within the BW class with a rescaled potential. The newtonian approximation of it leads to essentially the same equations as here.

Through similitude with the standard theory, an effective gravitational coupling (as screened gravitational strength) was defined as[7]

$$\tilde{G} \equiv G(\xi) = (1+\xi)^{-1} G_0 \,. \tag{6.3.23}$$

The latter is related to the local coupling $G(\phi)$ in (6.2.4) and it reduces to (6.3.17) in the absence of a Higgs-like scalar-field excitation ξ (that is for $\xi = 0$ with the chosen form of Higgs excitations, see (6.3.6)). Hence, there is

$$G(vN) \equiv G(\phi_0) = G_0 . \tag{6.3.24}$$

Further, $G(\xi)$ becomes singular for a vanishing Higgs-like scalar field with $\xi = -1$.
Further in equation (6.3.22) a cosmological function is given by

$$\Lambda^*(\xi) = \frac{8\pi G_0}{1+\xi} V(\xi) + \frac{\Lambda_0}{1+\xi} = \frac{12\pi}{\breve{\alpha} v^2} \frac{\mu^4}{\lambda} \frac{\xi^2}{1+\xi} + \frac{\Lambda_0}{1+\xi} \,. \tag{6.3.25}$$

It is clear that, for the special case of vanishing scalar-field excitations ξ, equation (6.3.22) goes through to the usual Einstein field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda_0 g_{\mu\nu} = -\kappa T_{\mu\nu} \,, \tag{6.3.26}$$

with $\kappa = \kappa_0 = 8\pi G_0/c^4$, given that $G(\xi = 0) = G_0$.
As a result of equation (6.3.17), the gravitational coupling strength given by $\breve{\alpha}$ is very high, so that the second term on the right-hand side of equation (6.3.22), $\pi/(3\breve{\alpha})$-proportional, can be neglected due to the smallness of the term $4\pi/(3\breve{\alpha})$.
Equation (6.3.22) can be rewritten for $\breve{\alpha} \gg 1$. This and insertion of the Higgs-like field equation (6.3.14) in the Einstein field equations lead to the following,

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \frac{1}{l^2}(1+\xi)^{-1}\xi \left(1 + \frac{3}{4}\xi\right) g_{\mu\nu} - \frac{1}{3}(1+\xi)^{-1}\Lambda_0 g_{\mu\nu} \\ = -8\pi\tilde{G}\left(\hat{T}_{\mu\nu} - \frac{\hat{q}}{3}\hat{T} g_{\mu\nu}\right) - (1+\xi)^{-1}\xi_{,\mu;\nu}, \quad (\breve{\alpha} \gg 1) \,. \end{aligned} \tag{6.3.27}$$

The cosmological function $\Lambda(\phi)$ after symmetry breaking (6.3.25) is essentially quadratic in ξ. For $\breve{\alpha} \gg 1$, it yields

$$\Lambda^*(\xi) = \frac{3}{4\, l^2} \frac{1}{1+\xi} \xi^2 + \frac{\Lambda_0}{1+\xi} \,. \tag{6.3.28}$$

Hence, with

$$\xi = \frac{G(v) - \tilde{G}}{\tilde{G}} \tag{6.3.29}$$

it can be written in the form as below,

$$\Lambda^*(\xi) = \frac{3}{4\, l^2}\left(\frac{G(v)^2 + \tilde{G}^2}{G(v)\, \tilde{G}} - 2\right) + \frac{\tilde{G}}{G(v)}\Lambda_0 . \tag{6.3.30}$$

[7] We will use the tilde of $\tilde{G}$ only for the cases when we do not write the ξ dependency explicitly.

In equation (6.3.30), apart from the constant Λ_0 term, the cosmological function is clearly a consequence of the locality of the gravitational function $\tilde{G}$.
Further, the trace of equation (6.3.27) leads to the Ricci scalar,

$$R = \frac{3}{l^2}\xi + 8\pi\tilde{G}\,(1-\hat{q})\,\hat{T} \;= \frac{3}{l^2}\left(\frac{G(v)}{\tilde{G}} - 1\right) + 8\pi\tilde{G}\,(1-\hat{q})\,\hat{T}\,. \tag{6.3.31}$$

R is independent on Λ_0, since it appears in the Higgs-like field equation (6.3.21) and in the Einstein field equations (6.3.22). The trace over Einstein's field equations, using the Higgs-like field equation, leads to a cancelation of the Λ_0-term.
Using equation (6.3.31), (6.3.27) can be rewritten into the form

$$\begin{aligned} R_{\mu\nu} &- \frac{1}{2l^2}\left[\frac{1+\frac{3}{2}\xi}{1+\xi}\right]\xi\, g_{\mu\nu} \;- \frac{1}{3}(1+\xi)^{-1}\Lambda_0\, g_{\mu\nu} \\ &= -8\pi\tilde{G}\left[\hat{T}_{\mu\nu} - \frac{1}{2}\left(1-\frac{1}{3}\hat{q}\right)\hat{T}\, g_{\mu\nu}\right] - (1+\xi)^{-1}\xi_{,\mu;\nu}\,, \quad (\breve{\alpha} \gg 1)\,, \end{aligned} \tag{6.3.32}$$

with

$$\frac{1+\frac{3}{2}\xi}{1+\xi} = \frac{1}{2}\left(3 - \frac{\tilde{G}}{G_0}\right).$$

Obviously, for vacuum and for $\hat{q} = 0$ in general, the Ricci scalar is given only by the scalar field if $1/\breve{\alpha}$ terms are neglected. The matter term of equation (6.3.31) leads, however, to a different right-hand side in the square bracket of the einsteinian field equations where the Ricci curvature has been inserted.

6.4 Maxwell-like equations and gravitational energy density

Let us take a general $\breve{\alpha}$ together with $c \neq \hbar \neq 1$. According to (6.3.22), the generalized Einstein equations after symmetry breaking are given by

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{3}{4l^2}\frac{\xi^2}{1+\xi}g_{\mu\nu} + \frac{\Lambda_0}{1+\xi}g_{\mu\nu} = -\frac{8\pi\tilde{G}}{c^4}T_{\mu\nu} - \frac{1}{1+\xi}\left[\xi_{,\mu;\nu} - \xi^{,\lambda}{}_{;\lambda}g_{\mu\nu}\right] - \\ - \frac{\pi}{\breve{\alpha}}\frac{1}{(1+\xi)^2}\left[2\xi_{,\mu}\xi_{,\nu} - \xi_{,\lambda}\xi^{,\lambda}g_{\mu\nu}\right]. \end{aligned} \tag{6.4.1}$$

Multiplying with $g^{\mu\nu}$ leads to the Ricci scalar,

$$R = \frac{3}{l^2}\frac{\xi^2}{1+\xi} + 4\frac{\Lambda_0}{1+\xi} + 8\pi\tilde{G}T - \frac{3}{(1+\xi)^2}\xi^{,\lambda}{}_{;\lambda} - \frac{2}{\breve{\alpha}}\frac{1}{(1+\xi)^2}\xi_{,\lambda}\xi^{,\lambda}. \tag{6.4.2}$$

The latter takes following form after introducing the scalar-field equation for $\xi^{,\lambda}{}_{,\lambda}$ (*cf.* (6.3.31)):

$$R = \frac{3}{l^2}\xi + 8\pi\tilde{G}T\left(1 - \frac{\hat{q}}{1+\frac{4\pi}{3\breve{\alpha}}}\right) - \frac{2}{\breve{\alpha}}\frac{\xi_{,\lambda}\xi^{,\lambda}}{(1+\xi)^2} + 4\frac{\Lambda_0}{1+\xi}\left(1 - \frac{1}{1+\frac{4\pi}{3\breve{\alpha}}}\right). \tag{6.4.3}$$

Now, it is clear that $\hat{q} = 1$ does not lead to an exactly vanishing coupling between the Ricci scalar and energy–stress. It is, however, weak.
Let us now transform equation (6.4.1) further. Making use of symmetry $g_{\mu\nu} = g_{\nu\mu}$ and orthonormality

$g_{\mu\nu}g^{\mu\lambda} = \delta_{\nu}{}^{\lambda}$ of the metric, multiplying equation (6.4.1) by $g^{\mu\sigma}u_{\sigma}$ leads to the form

$$R^{\sigma}{}_{\nu}u_{\sigma} - \frac{1}{2}Ru_{\nu} + \frac{3}{4l^2}\frac{\xi^2}{1+\xi}u_{\nu} + \frac{\Lambda_0}{1+\xi}u_{\nu} = -8\pi\tilde{G}T^{\sigma}{}_{\nu}u_{\sigma} - \frac{1}{1+\xi}\left[\xi_{;\nu}^{;\sigma}u_{\sigma} - \xi^{,\lambda}{}_{;\lambda}u_{\nu}\right] - \tag{6.4.4}$$
$$- \frac{\pi}{\breve{\alpha}}(1+\xi)^{-2}\left[2\xi^{,\sigma}\xi_{,\nu}u_{\sigma} - \xi_{,\lambda}\xi^{,\lambda}u_{\nu}\right].$$

Equation (6.4.4) may further be transformed using the scalar-field equation onto

$$R^{\sigma}{}_{\nu}u_{\sigma} - \frac{1}{2}Ru_{\nu} + \frac{1}{l^2}\xi\,(1+\xi)^{-1}\left[1+\frac{3}{4}\xi\right]u_{\nu} + \frac{\Lambda_0}{1+\xi}\left[1-\frac{4}{3}\left(1+\frac{4\pi}{3\breve{\alpha}}\right)^{-1}\right]u_{\nu} = -\tilde{\kappa}j_{\nu} + \tag{6.4.5}$$
$$+\left(1+\frac{4\pi}{3\breve{\alpha}}\right)^{-1}\hat{q}\tilde{\kappa}\frac{T}{3}u_{\nu} - \frac{1}{(1+\xi)^2}\xi^{,\sigma}{}_{;\nu}u_{\sigma} - \frac{\pi}{\breve{\alpha}}\frac{1}{1+\xi}\left[2\xi^{,\sigma}\xi_{,\nu}u_{\sigma} - \xi_{\lambda}\xi^{,\lambda}u_{\nu}\right].$$

We use $\tilde{\kappa} = \kappa_0/(1+\xi)$ and $\kappa_0 = 8\pi G_0/c^4$. Furthermore, we have defined the current by means of the energy–momentum density of matter measured by the observer in the following form,

$$j_{\mu} \equiv T^{\sigma}{}_{\mu}u_{\sigma}. \tag{6.4.6}$$

It may be related to an equation analogue to Maxwell's ones of electrodynamics which we know from Chapter 2.1. We know that a set of homogeneous Maxwell-like equation are valid for gravity. Here, we intend to rewrite the generalized Einstein field equation onto a set of inhomogeneous Maxwell-like equations of gravity (*viz* as [64,65], *cf.* [23]). In order to fulfill this, the left-hand side of equation (6.4.5) is to be written as a divergence of the field-strength tensor $\tilde{F}_{\mu\nu}$. Using equation (6.4.6) and (1.2.9), there is

$$\tilde{F}_{\mu}{}^{\sigma}{}_{;\sigma} = -\,2R^{\sigma}{}_{\nu}u_{\sigma} - Q_{\mu}{}^{\sigma}{}_{;\sigma} \tag{6.4.7}$$
$$= 2\tilde{\kappa}\left\{j_{\nu} - \frac{1}{2\tilde{\kappa}}\left[\frac{1}{1+\frac{4\pi}{3\breve{\alpha}}}\left(1-\frac{\hat{q}}{3}+\frac{4\pi}{3\breve{\alpha}}\right)\hat{T} - \frac{1}{l^2}\left(\frac{1+\frac{3}{2}\xi}{1+\xi}\right)\xi - \frac{2}{3}\left(1+\frac{4\pi}{3\breve{\alpha}}\right)\frac{\Lambda_0}{1+\xi}\right]u_{\nu} + \right.$$
$$\left. + \frac{1}{\kappa_0}\xi^{,\sigma}{}_{;\nu}u_{\sigma} - \frac{1}{2\tilde{\kappa}}Q_{\nu}{}^{,\sigma}{}_{;\sigma} + \frac{1}{\kappa_0}\frac{\pi}{\breve{\alpha}}(1+\xi)^{-1}\left[2\xi^{,\sigma}\xi_{,\nu} - \xi_{,\lambda}\xi^{,\lambda}\delta^{\sigma}{}_{\mu}\right]u_{\sigma}\right\}, \tag{6.4.8}$$

The latter defines a tensor s_{μ} with

$$s_{\nu} = -\frac{1}{2\tilde{\kappa}}\left\{\left[\tilde{\kappa}\left(1-\frac{\hat{q}}{3}+\frac{4\pi}{3\breve{\alpha}}\right)\hat{T} - \frac{1}{l^2}\left(\frac{1+\frac{3}{2}\xi}{1+\xi}\right)\xi - \frac{2}{3}\left(1+4\frac{\pi}{\breve{\alpha}}\right)\frac{\Lambda_0}{1+\xi}\right]u_{\nu} + Q_{\nu}{}^{\sigma}{}_{;\sigma}\right\} +$$
$$+\frac{1}{\kappa_0}\left[\xi^{,\sigma}{}_{;\nu} + \frac{\pi}{\breve{\alpha}}(1+\xi)^{-1}\left(2\xi^{,\sigma}\xi_{,\nu} - \xi_{,\lambda}\xi^{,\lambda}\delta^{\sigma}{}_{\nu}\right)\right]u_{\sigma} \tag{6.4.9}$$

so that following Maxwell-like equations are valid:

$$\tilde{F}_{\mu}{}^{\sigma}{}_{;\sigma} = 2\tilde{\kappa}\,(j_{\mu} + s_{\mu}). \tag{6.4.10}$$

The field-strength $\tilde{F}_{\mu\nu}$ has two sources: 4-currents j_{ν} as energy–momentum density of matter and s_{ν} as energy–momentum density of the gravitational field (*cf.* [64]). Consequently, momentum conservation is valid with

$$(j^{\mu} + s^{\mu})_{;\mu} = 0. \tag{6.4.11}$$

The energy–momentum density s^{μ} as defined here depends on the acceleration state of the observer. For $\breve{\alpha} \gg 1$, it simplifies to

$$s_{\mu} = -\frac{1}{2\tilde{\kappa}}\left\{\left[\tilde{\kappa}\left(1-\frac{\hat{q}}{3}\right)\hat{T} - \frac{1}{l^2}\left(\frac{1+\frac{3}{2}\xi}{1+\xi}\right)\xi - \frac{2}{3}\frac{\Lambda_0}{1+\xi} + \right]u_{\nu} + Q_{\mu}{}^{\sigma}{}_{;\sigma}\right\} + \frac{1}{\kappa_0}\xi^{,\sigma}{}_{;\mu}u_{\sigma}. \tag{6.4.12}$$

The energy density measured by an observer is

$$s = s_\mu u^\mu \,. \tag{6.4.13}$$

Hence, for $\breve{\alpha} \gg 1$, the latter leads to

$$s = \frac{1}{2\tilde{\kappa}}\left\{\left[\tilde{\kappa}\left(1-\frac{\hat{q}}{3}\right)T - \frac{1}{l^2}\left(\frac{1+\frac{3}{2}\xi}{1+\xi}\right)\xi - \frac{2}{3}\frac{\Lambda_0}{1+\xi}\right]\delta_\nu{}^\mu + Q_\mu{}^\sigma{}_{;\sigma}u^\mu\right\} + \frac{1}{\kappa_0}\xi^{,\sigma}{}_{;\mu}\delta_\sigma{}^\mu \,. \tag{6.4.14}$$

Further, there is $Q_\mu{}^\sigma{}_{;\sigma}u^\mu$ as in equation (6.4.7). In a statical case, it simplifies to

$$Q_\mu{}^\sigma{}_{;\sigma}u^\mu = -4u_\mu{}^{;\sigma}u^\mu{}_{;\sigma} - 2u^\sigma{}_{;\mu;\sigma}u^\mu. \tag{6.4.15}$$

After rewriting with $\xi^{,\sigma}{}_{;\mu}\delta_\sigma{}^\mu = \xi^{,\mu}{}_{;\mu}$, the gravitational energy–momentum density as measured by the observer then reads for $\breve{\alpha} \gg 1$ as follows,

$$s = +\frac{2}{\tilde{\kappa}}u_\mu{}^{;\sigma}u^\mu{}_{;\sigma} + \frac{1}{\kappa_0}\xi^{,\mu}{}_{;\mu} + \frac{1}{\tilde{\kappa}}\left\{u^\sigma{}_{;\mu;\sigma}u^\mu - \frac{\tilde{\kappa}}{2}\left(1-\frac{\hat{q}}{3}\right)T + \frac{1}{2l^2}\left(\frac{1+\frac{3}{2}\xi}{1+\xi}\right)\xi + \frac{1}{3}\frac{\Lambda_0}{1+\xi}\right\} \,. \tag{6.4.16}$$

For weak dynamical behavior, the metric is nearly constant and for statical fields with

$$u^\nu{}_{;\nu} = \frac{(\sqrt{-g}u^\nu)_{,\lambda}}{\sqrt{-g}} = 0, \tag{6.4.17}$$

the Ricci identities lead to

$$R_{\mu\nu}u^\mu u^\nu = -u^\sigma{}_{;\mu;\sigma}u^\mu, \tag{6.4.18}$$

which may be found in equation (6.4.16). Further (6.4.18) reads explicitly as follows,

$$\begin{aligned} R_{\mu\nu}u^\mu u^\nu &= -\,u^\sigma{}_{;\mu;\sigma}u^\mu \\ &= -\frac{1}{2l^2}\left(\frac{1+\frac{3}{2}\xi}{1+\xi}\right)\xi - \frac{1}{3}\frac{\Lambda_0}{1+\xi} + \tilde{\kappa}T_{\mu\nu}u^\mu u^\nu - \frac{\tilde{\kappa}}{2}\left(1-\frac{\hat{q}}{3}\right)T + \frac{1}{1+\xi}\xi^{,\mu}{}_{;\mu}. \end{aligned} \tag{6.4.19}$$

If we insert the scalar-field equation in (6.4.16) for timelike 4-vectors (with $g_{\mu\nu}u^\mu u^\nu = 1$), there is

$$s = \frac{2}{\tilde{\kappa}}u_\mu{}^{;\sigma}u^\mu{}_{;\sigma} + \frac{1}{\kappa_0}\xi^{,\mu}{}_{;\mu} + \frac{1}{\kappa_0}\xi_{,\mu;\nu}u^\mu u^\nu\delta_\nu{}^\mu + T_{\mu\nu}u^\mu u^\nu - \left(1-\frac{\hat{q}}{3}\right)T \,. \tag{6.4.20}$$

Multiplying with the unity $g_{\mu\nu}g^{\mu\nu} = 1$, there is directly

$$\xi_{,\mu;\nu}u^\mu u^\nu = \xi^{,\nu}{}_{;\nu} \tag{6.4.21}$$

so that the energy density takes the simplified form as follows,

$$s = +\frac{2}{\tilde{\kappa}}u_\mu{}^{;\sigma}u^\mu{}_{;\sigma} + T_{\mu\nu}u^\mu u^\nu - (1-\hat{q})\,T - \frac{2}{\kappa_0 l^2}\xi + \frac{8}{3}\frac{\Lambda_0}{\kappa_0} \,. \tag{6.4.22}$$

The energy density of the gravitational field depends on the geometry, a scalar field Λ_0 and the energy-stress tensor $T_{\mu\nu}$ of matter.

For $l \to \infty$ or low scalar-field excitations, together with $\hat{q} = 0$ and $\Lambda_0 = 0$, equation (6.4.22) gives the usual energy density. For $\hat{q} = 1$, however, there is no term proportional to T. Furthermore, under such circumstances of $l \to \infty$ in $\hat{q} = 1$, we know from earlier works (*e.g.* [20, 179]) that κ_0 is to be rescaled to κ_N. The demonstration of this fact will follow for central symmetry in linear approximation of the theory. Further, in Chapter 7 we will use central symmetry to analyze this model. In Chapter Chapter 7.3, we will use it in relation to the energy density of equation (6.4.22) so to compare the results with those of standard dynamics and to gain constraints to the model presented here (*cf.* [23]).

Part III

Cosmological Consequences of Induced Gravity

Chapter 7

Induced gravity with spherical symmetry

– The model of induced gravity with Higgs potential is analyzed for central symmetry in virtue of its phenomenological consequences for weak fields and Black Holes. This is related to the phenomena of Dark Matter and Dark Energy. The grounding and results presented here may be found partly in [21, 23, 24, 179] as consequence of this work. –

7.1 The exact equations for spherical symmetry

Let us make the assumption of a vanishing true cosmological constant Λ_0 and a gravitational strength $\breve{\alpha} \gg 1$. Further, let us assume that spherical symmetry (x^μ=$\{x^0 = ct, x^1 = r, x^2 = \vartheta, x^3 = \varphi\}$) as

$$ds^2 = e^\nu (cdt)^2 - e^\lambda dr^2 - r^2 d\Omega^2 \tag{7.1.1}$$

is given, with ν and λ as functions of the r and t coordinates only and $d\Omega^2 = \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)$ as the metric of a 2-dim unit sphere. Furthermore, let us take now $c \neq \hbar \neq 1$ in the equations.
For the energy–momentum tensor $T_{\mu\nu}$, let us assume phenomenologically an ideal liquid with the energy–momentum tensor in geometrical form as

$$T_{\mu\nu} = (\epsilon + p)\, u_\mu u_\nu - p g_{\mu\nu}, \quad u^\mu u_\mu = 1, \tag{7.1.2}$$

with pressure p and energy-density distribution ϵ, and with the 4-velocity $u_\mu = (u_0, u_1, 0, 0)$ and $u_1 := u_0 \frac{v_1}{c}$ (velocity v_1) and thus

$$u_0^2 = \left[e^{-\nu} - \left(\frac{v_1}{c}\right)^2 e^{-\lambda}\right]^{-1}. \tag{7.1.3}$$

The nonvanishing components of $T_{\mu\nu}$ as well as the nonvanishing terms of the Ricci tensor and scalar may be found in Appendix C.1. For the line element (7.1.1), with equations (C.1.8) through (C.1.12) and metric components ν and λ, the exact field equations (6.3.27) read now

$$\begin{aligned} & e^{\nu-\lambda}\left(\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\nu'\lambda'}{4}+\frac{\nu'}{r}\right)-\frac{1}{c^2}\frac{\ddot\lambda}{2}-\frac{1}{c^2}\frac{\dot\lambda^2}{4}+\frac{1}{c^2}\frac{\dot\lambda\dot\nu}{4}+\frac{1}{2l^2}(1+\xi)^{-1}\xi\left(1+\frac{3}{2}\xi\right)e^{\nu}= \\ & =\frac{8\pi}{(1+\xi)}\frac{G_0}{c^4}\left[\left(e^{-\nu}-\frac{v_1^2}{c^2}e^{-\lambda}\right)^{-1}\left(\epsilon+\frac{v_1^2}{c^2}p\,e^{\nu-\lambda}\right)-\underline{\frac{1}{2}\left(1-\frac{1}{3}\hat q\right)(\epsilon-3p)e^{\nu}}\right]+ \\ & +(1+\xi)^{-1}\left[\frac{\ddot\xi}{c^2}-\frac{\dot\nu}{2c^2}\dot\xi-\frac{\nu'}{2}e^{\nu-\lambda}\xi'\right], \end{aligned} \tag{7.1.4}$$

$$\begin{aligned} & e^{\lambda-\nu}\frac{1}{c^2}\left(\frac{\ddot\lambda}{2}+\frac{\dot\lambda^2}{4}-\frac{\dot\lambda\dot\nu}{4}\right)-\frac{\nu''}{2}-\frac{\nu'^2}{4}+\frac{\nu'\lambda'}{4}+\frac{\lambda'}{r}-\frac{1}{2l^2}(1+\xi)^{-1}\xi\left(1+\frac{3}{2}\xi\right)e^{\lambda}= \\ & =\frac{8\pi}{(1+\xi)}\frac{G_0}{c^4}\left[\left(e^{-\nu}-\frac{v_1^2}{c^2}e^{-\lambda}\right)^{-1}\left(\frac{v_1^2}{c^2}\,\epsilon+p\,e^{\lambda-\nu}\right)+\underline{\frac{1}{2}\left(1-\frac{1}{3}\hat q\right)(\epsilon-3p)e^{\lambda}}\right]+ \\ & +(1+\xi)^{-1}\left[\xi''-\frac{\dot\lambda}{2c^2}e^{\lambda-\nu}\dot\xi-\frac{\lambda'}{2}\xi'\right], \end{aligned} \tag{7.1.5}$$

$$\frac{1}{c}\frac{\dot\lambda}{r}=-\frac{8\pi}{(1+\xi)}\frac{G_0}{c^4}\left[e^{-\nu}-\frac{v_1^2}{c^2}e^{-\lambda}\right]^{-1}(\epsilon+p)\,\frac{v_1}{c}-(1+\xi)^{-1}\frac{1}{c}\left[\dot\xi'-\frac{\nu'}{2}\dot\xi-\frac{\dot\lambda}{2}\xi'\right] \tag{7.1.6}$$

and

$$\begin{aligned} & e^{-\lambda}\left(1+\frac{r}{2}(\nu'-\lambda')\right)-1+\frac{r^2}{2l^2}(1+\xi)^{-1}\xi\left(1+\frac{3}{2}\xi\right)= \\ & =-\frac{8\pi}{(1+\xi)}\frac{G_0}{c^4}\left[p\,r^2+\underline{\frac{1}{2}\left(1-\frac{1}{3}\hat q\right)(\epsilon-3p)r^2}\right]-(1+\xi)^{-1}re^{-\lambda}\xi'. \end{aligned} \tag{7.1.7}$$

The underlined terms show a difference between both the models $\hat q=0$ and $\hat q=1$ in the presence of mass given by energy density ϵ and pressure p. Further, for $\xi=0$, the original Einstein field equations for central symmetry are restored. For them, the Birkhoff theorem is valid. Thus, for vacuum ($\epsilon=0$) all fields are static and $\nu=\nu(r)$ and $\lambda=\lambda(r)$. For nonvanishing excitations ξ, however, this cannot be stated directly. The Higgs-like equation for the excited Higgs field ξ yields (*cf.* Appendix C.2)

$$\frac{1}{c^2}\ddot\xi e^{-\nu}-\xi''e^{-\lambda}-\frac{1}{c^2}\frac{\dot\nu-\dot\lambda}{2}e^{-\nu}\dot\xi-\frac{\nu'-\lambda'}{2}e^{-\lambda}\xi'-\frac{2}{r}e^{-\lambda}\xi'+\frac{1}{l^2}\xi=+\hat q\frac{8\pi}{3}\frac{G_0}{c^4}(\epsilon-3p)\,, \tag{7.1.8}$$

The coupling of the scalar field to matter is given by $\hat q$, which has special relevance in connection to the source and stability of the Higgs (or Higgs-like, in the sense of the SM) particles of the model.

7.2 Linear equations and static weak-field solutions

A linearization in the potentials ν and λ is expected as a good approximation for many physical circumstances. For very massive centers ($r=0$), however, it may be necessary not to linearize in the scalar field ξ. This is possible because the effective gravitational coupling $\tilde G\sim(1+\xi)^{-1}$ runs to zero for $r\to 0$ (with the scalar-field excitation ξ as a Yukawa solution, given the form of the Higgs field) without the gravitational potential getting singular (the masses decouple from gravitation since their coupling vanishes). Since the gravitational coupling disappears at $r=0$, it is possible to linearize in ν and λ, without the necessity of linearizing in ξ indeed (*cf.* [20]). Under such circumstances, the field equations (7.1.4) through (7.1.8) reduce to the following ones,

$$\frac{1}{2}\Delta\nu + \frac{1}{2l^2}\frac{1+\frac{3}{2}\xi}{1+\xi}\xi - \frac{1}{c^2}\frac{\ddot{\lambda}}{2} + (1+\xi)^{-1}\frac{\ddot{\xi}}{c^4} = \frac{8\pi G_0}{c^2}\left[\left(\epsilon - \frac{1}{2}\left(1-\frac{1}{3}\hat{q}\right)(\epsilon-3p)\right)\right](1+\xi)^{-1}, \tag{7.2.1}$$

$$-\frac{\nu''}{2} + \frac{\lambda'}{r} - \frac{1}{2l^2}\frac{1+\frac{3}{2}\xi}{1+\xi}\xi + \frac{1}{c^2}\frac{\ddot{\lambda}}{2} = \frac{8\pi G_0}{c^4}\left[p + \frac{1}{2}\left(1-\frac{1}{3}\hat{q}\right)(\epsilon-3p)\right](1+\xi)^{-1} + \\ + \xi''(1+\xi)^{-1}, \tag{7.2.2}$$

$$-\lambda + \frac{r}{2}(\nu'-\lambda') + \frac{r^2}{2l^2}\frac{1+\frac{3}{2}\xi}{1+\xi}\xi = -\frac{8\pi G_0}{c^4}\left[p + \frac{1}{2}\left(1-\frac{1}{3}\hat{q}\right)(\epsilon-3p)\right]r^2(1+\xi)^{-1} - \\ - r\xi'(1+\xi)^{-1}, \tag{7.2.3}$$

and for the scalar field, equation (7.1.8) leads to the following:

$$\Box\xi + \frac{1}{l^2}\xi = \hat{q}\frac{8\pi}{3}\frac{G_0}{c^4}(\epsilon-3p), \tag{7.2.4}$$

with the d'Alembert operator[1]

$$\Box = \frac{1}{c^2}\frac{\partial}{\partial t} - \Delta \quad \text{with}\, \Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}. \tag{7.2.5}$$

Herewith, $\epsilon = \varrho c^2$ is the energy density of the massive object, and p the pressure. It would be expected to act as part of the measured mass of what we will call the effective mass later in this Chapter.

For further linearization in the potentials ν, λ and in the scalar-field excitation ξ as well, especially for $r \to \infty$, there is [20]

$$\hat{\xi} = \left[\frac{1+\frac{3}{2}\xi}{1+\xi}\right]\xi \approx \xi. \tag{7.2.6}$$

The field equations now read as follows,

$$\frac{1}{2}\nu'' + \frac{1}{r}\nu' + \frac{1}{2l^2}\xi - \frac{1}{c^2}\frac{\ddot{\lambda}}{2} = \frac{8\pi G_0}{c^4}\left[\left(\epsilon - \frac{1}{2}\left(1-\frac{1}{3}\hat{q}\right)(\epsilon-3p)\right)\right] + \frac{\ddot{\xi}}{c^2}, \tag{7.2.7}$$

$$-\frac{\nu''}{2} + \frac{\lambda'}{r} - \frac{1}{2l^2}\xi = \frac{8\pi G_0}{c^4}\left[p + \frac{1}{2}\left(1-\frac{1}{3}\hat{q}\right)(\epsilon-3p)\right] - \frac{\ddot{\lambda}}{c^2} + \xi'', \tag{7.2.8}$$

$$-\lambda + \frac{r}{2}(\nu'-\lambda') + \frac{r^2}{2l^2}\xi = -\frac{8\pi G_0}{c^4}\left[p + \frac{1}{2}\left(1-\frac{1}{3}\hat{q}\right)(\epsilon-3p)\right]r^2 - r\xi', \tag{7.2.9}$$

$$\xi'' + \frac{2}{r}\xi' - \frac{1}{l^2}\xi - \frac{1}{c^2}\ddot{\xi} = -\hat{q}\frac{8\pi}{3}\frac{G_0}{c^4}(\epsilon-3p). \tag{7.2.10}$$

For $\xi = 0$, the original Einstein equations for central symmetry are restored.

For weak fields, *i.e.* for the newtonian approximation, the metric is well-given through small-valued deviations $h_{\mu\nu}$ with

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1 \tag{7.2.11}$$

of the Minkowski metric $\eta_{\mu\nu} = diag(1,-1,-1,-1)$. A new variable $\psi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}$ leads to an inhomogeneous wave equation

$$\psi_\mu{}^\nu{}_{,\nu} = -2\kappa T_{\mu\nu}. \tag{7.2.12}$$

[1] The d'Alembert operator is defined according to the signature (+,-,-,-) of the metric.

It leads to the existence of gravitational waves corresponding to $h_{\mu\nu}$ which propagate with lightspeed.[2] Further, the newtonian potential Φ is given through

$$h_{00} = 2\frac{\Phi}{c^2}\,, \tag{7.2.13}$$

with $h_{00} = \nu$. Equation (7.2.7) can be written as

$$\begin{aligned}\Delta\Phi + \frac{c^2}{2l^2}\xi =& 8\pi G_0\left[\left(\varrho - \frac{1}{2}\left(1-\frac{1}{3}\hat{q}\right)\left(\varrho - 3\frac{p}{c^2}\right)\right)\right] + \frac{1}{c^2}\frac{\ddot{\lambda}}{2} + \frac{\ddot{\xi}}{c^2}\\ =& 8\pi G_0\left(\frac{2}{3}\varrho + \frac{p}{c^2}\right) + \frac{1}{c^2}\frac{\ddot{\lambda}}{2} + \frac{\ddot{\xi}}{c^2}, \quad (\hat{q}=1),\\ =& 4\pi G_0\left(\varrho + 3\frac{p}{c^2}\right) + \frac{1}{c^2}\frac{\ddot{\lambda}}{2} + \frac{\ddot{\xi}}{c^2}, \quad (\hat{q}=0),\end{aligned} \tag{7.2.14}$$

together with the scalar-field equation as follows,

$$\Delta\xi - \frac{1}{c^2}\ddot{\xi} - \frac{1}{l^2}\xi = -\hat{q}\,\frac{8\pi}{3}\frac{G_0}{c^2}\left(\varrho - 3\frac{p}{c^2}\right). \tag{7.2.15}$$

Equation (7.2.4), which is equal to (7.2.15), is a Klein–Gordon equation. Calculations for the case $\hat{q} = 0$ may be found in Chapter C.1. Furthermore, in the static case, the scalar-field equation (7.2.4) for $\hat{q} = 1$ equals the modified Helmholtz equation derived in other works (*viz* [210] and related) for a normalized gravitational constant $G/c^4 = 1$ and $\alpha_R\varrho_R = (\epsilon - 3p)/3$.
Here, let us take the time independent case of the Yukawa equation for the vacuum solution. Equation (7.2.15) may be solved through the following ansatz (we use the subscript a for $r \geq R$),

$$\xi_a(r) = \frac{a}{r}e^{-r/l} \tag{7.2.16}$$

with the Compton wavelength $l = \frac{\hbar}{Mc}$ of the scalar field and neglecting an anti-Yukawa solution so that the scalar field vanishes for spatial infinity (boundary condition). The value of a depends on whether $\hat{q} = 0$ or $\hat{q} = 1$ is set. The constant a is further dependent on the mass M_1.
The pressure p, which derives from the equation of state in T, is in general a function of the coordinate r, and it often depends on the density $\varrho(r)$. It is often given through the barotropic parameter w with $p = w\varrho c^2$, assuming a linear proportionality between density and pressure. Specifically, it is spoken about dust matter for $w = 0$ and radiation for $w = \frac{1}{3}$. Matter modeled with $w = 1$ is further called ultra stiff, and with $w = -1$ it is called anti-stiff matter.
Further, for the statical case the linear solutions of equations (7.2.1) through (7.2.4) for vacuum are known. They are given by equation (7.2.16) and the following,

$$\nu_a = -\frac{a}{r}e^{-r/l} - \frac{b}{r}, \quad \lambda_a = -\nu - 2\xi_a(1+\frac{r}{l}) = -\frac{a}{r}\left(1+\frac{2r}{l}\right)e^{-r/l} + \frac{b}{r}, \tag{7.2.17}$$

with

$$(\lambda_a' + \nu_a') = r\xi_a'' \tag{7.2.18}$$

[2]Pulsars are highly magnetized, rotating neutron stars that emit a beam of electromagnetic radiation. In some cases, the regularity of their pulsation is as precise as an atomic clock. A double pulsar system shall produce strong gravitational radiation, causing the orbit to continually contract as it loses orbital energy. Indeed, indirect proof of gravitational waves comes from measurements on the double pulsar PSR 1913+16 [135] (Nobel prize 1993) as well as on the Quasar OJ 287 [231]. Laser-interferometry on Earth by GEO600, Virgo or LIGO might prove their existence directly. The best upper limit for the wave amplitude by LIGO lies about $2 \cdot 10^{-26}$ [153]. More likely is a detection by means of the LISA in space, though.

and the integration constants a and b. Boundary conditions are used to reestablish the minkowskian metric at spatial infinity.

The exact physical meaning of the integration constants can be determined by solving the inhomogeneous equations, *i.e.* the linearized equations in the presence of the source. Examples are – a polytropic density distribution as shown in [20], a Freeman-disc profile as in [52] or a homogeneous mass distribution which then gives the solution for a point–particle when the radius R_0 of the gravitating body is taken as $R_0 \to 0$. Let us take:

(i) **Polytropic density:**
A polytropic equation of state is given by $p = w_P \epsilon^\gamma$ with a polytropic amplitude w_P and a polytropic index γ. Density is given by [66]

$$\varrho = \left(\frac{1}{w_P}\right)^{\frac{1}{\gamma-1}} \left[\frac{1}{2}\frac{\gamma-1}{\gamma}\left(\nu_s - \nu\right)^{\frac{1}{\gamma-1}}\right], \tag{7.2.19}$$

with $\nu_s = \nu(R_0)$.
For $\gamma = 2$, the Einstein equations become linear in ν concerning ϵ, in accordance with the linearized Einstein theory. Then, the polytropic equation becomes

$$p = w_P \epsilon^2, \quad \epsilon = \frac{1}{4 w_P}(\nu_s - \nu). \tag{7.2.20}$$

Therefore, pressure p can be neglected (*cf.* [20]).
As a result of this assumption, the inner fields ν_i, λ_i and ξ_i for $r \leq R_0$ lead to boundary conditions which further lead to flattened tangential-velocity curves. Hence, for large distances, Dark Matter is partly given, assuming or not a very massive center of the galaxies (*sc.* [20]). In this work, however, we will use a different approach.

(ii) **Constant densities and negligible pressure:**
A simple model for the inner fields may be given through a homogeneous sphere of radius R_0, with an **inside-constant density distribution** $\varrho = const. = 3M_1/(4\pi R_0^3) = \epsilon/c^2$ and a **negligible pressure** p. For this inner solution, we take $\hat{q} = 1$. Accordingly, we will write $a = A_1$, $b = B_1$ and so on for the integration constants.
The inner scalar field ($r \leq R_0$) is then given by (we use the subscript i for $r \leq R_0$)

$$\xi_i = \frac{1}{r}\left(C_1 \sinh\frac{r}{l} + C_2 \cosh\frac{r}{l}\right) + \frac{8\,\pi\, G_0}{3\,c^4}\epsilon\, l^2, \quad (\hat{q} = 1)\,. \tag{7.2.21}$$

Using continuity conditions at $r = R_0$ with the vacuum field, one gets for the integration constants the following condition:

$$C_1 = -C_2 - \frac{8\,\pi\, G_0}{3\,c^4} l^2 \epsilon (R_0 + l) e^{R_0/l}, \tag{7.2.22}$$

whereas C_2 is dependent on the existence of a point–mass in the center of the sphere. For $C_2 = 0$ (*i.e.* no such mass), the integration constant A_1 reads

$$A_1 = \frac{2 M_1 G_0}{c^2}\frac{l^2}{R_0^2}\left[\cosh\frac{R_0}{l} - \frac{l}{R_0}\sinh\frac{R_0}{l}\right], \tag{7.2.23}$$

with the mass M_1 of the gravitational object with radius R_0.
For large length scales in relation to the radius of the gravitational source, there is

$$\left(\frac{l}{R_0}\right)^2\left(\cosh\frac{R_0}{l} - \frac{l}{R_0}\sinh\frac{R_0}{l}\right) \longrightarrow \frac{1}{3} \text{ for } l \gg R_0, \tag{7.2.24}$$

Further, the scalar field is given by

$$\xi_a = \frac{2M_1 G_0}{3c^2 r} e^{-r/l}. \tag{7.2.25}$$

The mass M_1 is given by the integral over density in the gravitational object with $M_1 = 4\pi R_0^3 \epsilon/3$. The amplitude of the Yukawa term is of the order of magnitude of unity for $r \ll l$. The coupling constant G_0, however, was defined here in analogy to the field equations of GR and to newtonian theory. Its value as a coupling constant, though, has to be measured in the laboratory, and its relation to the constant of newtonian dynamics is dependent on the actual theory used. Furthermore, its value is here constrained by the length scale l. The coupling constant is important for all calculations of astrophysical masses, and it cannot be given through astrophysical considerations.
The actual value of the gravitational constant within a model can be gained only through experiments. This measurement within the induced gravity, as will be seen below, should lead to a rescaling (*cf.* (7.2.30)) for $l \to \infty$ in equation (7.2.29). Further, the exact value of the coupling should be material-independent, and it should be given in good approximation by the potential for point-like particles, for which the inner structure is negligible (see equation (7.2.29)). The effective value of the coupling ($\tilde{G}$), however, is material-dependent, and it changes with distance (it is not of first order in ξ, though). The coupling is given with $G_0/(1+\xi)$. The variability of the gravitational coupling decreases in view of equation (7.2.25) with decreasing distance from a mass (since $\xi \sim 1/r$ in vacuum). The cosmological function Λ, further, increases its absolute value with decreasing distance from a mass. However, it is of second order in ξ and therefore not yet contained in the linear approximation given through equation (7.2.29).

Analogously to A_1, the integration constant B_1 is given by

$$B_1 = \frac{2M_1 G_0}{c^2}\left[1 + \frac{l}{R_0}\left(1+\frac{l}{R_0}\right)\left(1 - e^{-R_0/l}\right)\left(\cosh\frac{R_0}{l} - \frac{l}{R_0}\sinh\frac{R_0}{l}\right)\right]. \tag{7.2.26}$$

It is easily seen that for a vanishing length scale (*i.e.* very high masses of the scalar-field particle), A_1 disappears and B_1 reads $B_1 = \frac{2M_1 G_0}{c^2} = \frac{2M_1 G_N}{c^2}$ and gives both the potentials λ and ν exactly as within GR. For large length scales $l \to \infty$ (*i.e.* vanishing masses of the scalar field particle), from (7.2.24) it is seen that $A_1 = \frac{2M_1 G_0}{3\,c^2}$ is valid, and $B_1 = \frac{2M_1 G_0}{c^2}$.

The inner gravitational potential Φ_i reads in linear form

$$\Phi_i = -\frac{M_1 G_0}{R_0}\left[\left(\frac{l}{R_0}\right)^2\left(1+\frac{R_0}{l}\right)\frac{e^{-R_0/l}}{r}\sinh\frac{r}{l} + \frac{1}{2}\left(\frac{r}{R_0}\right)^2\right] + const. \tag{7.2.27}$$

with a constant term $const. = -\frac{M_1 G_0}{R_0}\left[\frac{3}{2} + \left(\frac{l}{R_0}\right)^2\right]$. The outer solution ($r \geq R_0$) is

$$\begin{aligned}\Phi_a = &-\frac{M_1 G_0}{r}\left\{1 + \frac{l^2}{R_0^2}\left[\cosh\frac{R_0}{l} - \frac{l}{R_0}\sinh\frac{R_0}{l}\right]e^{-r/l} + \right.\\ &\left. + \frac{l}{R_0}\left(1+\frac{l}{R_0}\right)\left(1 - e^{-R_0/l}\right)\left(\cosh\frac{R_0}{l} - \frac{l}{R_0}\sinh\frac{R_0}{l}\right)\right\}.\end{aligned} \tag{7.2.28}$$

For $l \ll R_0$ and for $l \gg R_0$ (see (7.2.24)), equation (7.2.28) goes through to a $1/r$ potential. For $l \ll R_0$, the newtonian potential (with a small correction from l that should be detectable in a laboratory) with $G_0 = G_N$ is given. On the other hand, for $l \gg R_0$ (and $r \ll l$), there is

$$\Phi_{a\,extreme} = -\frac{M_1 G_0}{r}\left[1 + \frac{1}{3}e^{-r/l}\right] \approx -\frac{M_1 G_0}{r}\left[\frac{4}{3} - \frac{1}{3}\frac{r}{l} + \ldots\right], \quad (l \gg R_0). \tag{7.2.29}$$

If equation (7.2.29) is to give Newton's gravitational potential at small scales, then[3]

$$G_0 = \frac{3}{4}G_N \tag{7.2.30}$$

should be valid up to $\frac{r}{l}$ corrections which pull this value down. Corrections would be expected for large scales only. For them, the gravitational potential may be rewritten as

$$\Phi_a = -\frac{M_1^* G_N}{r}, \tag{7.2.31}$$

being M_1^* an effective mass with

$$M_1^* \equiv M_1(1 - f_1), \quad f_1 = \frac{1}{4}\left(1 - e^{-r/l}\right). \tag{7.2.32}$$

It is valid for vacua such that gravitational sources may be taken as point-like. The effective mass gives the radial dependence for the case $p = 0$. The function f_1 is a correction which for high length scales in relation to the distance has the following form,

$$f_1 \approx \frac{r}{4l}\left(1 + \frac{r}{2l}\right), \quad r \ll l. \tag{7.2.33}$$

Consequently, for larger distances, the effective mass would be measured as smaller than the actual one. The correction term, however, is very small as long as $r \ll l$. For large distances, on the other hand, f_1 tends to a value of 1/4.

The gravitational potential Φ is given by ν. There is still, however, the second potential to analyze in equation (7.2.17). It yields in the case $r \geq R$ (for $\hat{q} = 1$ for a homogeneous sphere of negligible pressure),

$$\begin{aligned}\lambda_a =& \frac{2M_1 G_0}{r}\left\{1 - \frac{l^2}{R_0^2}\left[\cosh\frac{R_0}{l} - \frac{l}{R_0}\sinh\frac{R_0}{l}\right]\left(1 + 2\frac{r}{l}\right)e^{-r/l} + \right.\\ &\left. + \frac{l}{R_0}\left(1 + \frac{l}{R_0}\right)\left(1 - e^{-R_0/l}\right)\left(\cosh\frac{R_0}{l} - \frac{l}{R_0}\sinh\frac{R_0}{l}\right)\right\}.\end{aligned} \tag{7.2.34}$$

For small values of the length scale $l \ll R_0$, equation (7.2.34) leads to

$$\lambda_a = \frac{2M_1 G_0}{r}\left(= \frac{2M_1 G_N}{r}\right), \quad (l \ll R_0), \tag{7.2.35}$$

while, for $l \gg R_0$, there is

$$\lambda_a = \frac{2M_1 G_0}{r}\left[1 - \frac{1}{3}\left(1 + 2\frac{r}{l}\right)e^{-r/l}\right] \longrightarrow \frac{4}{3}\frac{M_1 G_0}{r}\left(= \frac{M_1 G_N}{r}\right), (r \ll l). \tag{7.2.36}$$

Furthermore, for high length scales, λ may be rewritten for G_N and for a Schwarzschild radius $r_S = \frac{2M_1 G_N}{c^2}$ to the form

$$\lambda_a = \frac{r_S}{r}\left\{1 - \frac{1}{4}\left[1 + \left(1 + 2\frac{r}{l}\right)e^{-r/l}\right]\right\}. \tag{7.2.37}$$

[3] Actually, the same re-scaling can be found in [50].

Let us define the following function;

$$f_2 = \frac{1}{4}\left[1 + \left(1 + \frac{r}{l}e^{-r/l}\right)\right]. \tag{7.2.38}$$

For $r \ll l$, it yields

$$f_2 \approx \frac{1}{2}\left(1 - \frac{r}{l} - \frac{5}{4}\frac{r^2}{l^2}\right) \quad \text{for}\, r \ll l. \tag{7.2.39}$$

Using equations (7.2.33) and (7.2.38) as well as the effective mass M_1^* from equation (7.2.32), λ yields then for vacuum,

$$\lambda_a = \frac{2M_1^*}{r}\frac{G_N}{c^2}\frac{1-f_2}{1-f_1}. \tag{7.2.40}$$

Hence, there is the Schwarzschild radius for the effective mass M_1,

$$r_S^* = \frac{2M_1^* G_N}{c^2}. \tag{7.2.41}$$

According to equation (7.2.40), $\lambda_a = -\nu_a$ is not possible as long as there are no pressure terms.

- In short: G_0 is not *a priori* determined to be the newtonian constant of newtonian dynamics. For G_N as the coupling constant as determined by a torsion-balance experiment in the laboratory, if dynamics are to be newtonian, we have
  (a) $G_0 = G_N$ for a distance r which is large in relation to the length scale l in the torsion-balance experiment.
  (b) $G_0 = \frac{3}{4}G_N$ for distances r which are short in relation to length scale l.
  (c) Radial dependence appears for an effective mass which differs from bare, luminous mass. This mass is in general lower than luminous mass.
  (d) For dust, without pressure terms, there is $\nu \neq -\lambda$.

(iii) **Nonvanishing pressures:**
Let us now take pressure terms p which may be relevant for astronomical considerations. Now, with $\kappa_0 = \frac{8\pi G_0}{c^4}$ and after insertion of equation (7.2.4) of the scalar-field excitation in equation (7.2.1) for the gravitational potential Φ, we have the Poisson equation linearized in ν and λ of the following form,

$$\Delta\Phi\,(1+\xi) + \frac{c^2}{2}\left(1 + \frac{3}{2}\xi\right)\Delta\xi = \frac{\kappa_0}{3}\left[3\epsilon - \frac{3}{2}\,(\epsilon - 3p) - \frac{3}{4}\xi\hat{q}\,(\epsilon - 3p)\right]. \tag{7.2.42}$$

Further, let us define a generalized potential

$$\Psi = \Phi + \frac{c^2}{2}\xi \tag{7.2.43}$$

for the joint action of the newtonian-like Φ potential and the scalar field ξ. Now equation (7.2.42) reads

$$\Delta\Psi\,(1+\xi) = \frac{\kappa_0}{2}\left[\epsilon + 3p - \frac{1}{2}\xi\hat{q}\,(\epsilon - 3p)\right] - \frac{c^2}{4}\xi\,\Delta\xi. \tag{7.2.44}$$

In linear approximation, we have the usual Poisson equation for Ψ:

$$\Delta\Psi \;=\; \nabla^2\Psi = \;\frac{\kappa_0}{2}\,(\epsilon+3p)\,. \tag{7.2.45}$$

It is independent on $\hat{q}$. Further, one may easily notice that for a vanishing scalar-field excitation, equation (7.2.45) reduces to that of usual GR. Consequently, the scalar field acts as a further gravitational interaction which at low scales is of newtonian form. Further, it leads to an effective (measured) mass which possesses scalar field contributions *via* pressure p (*cf.* [23, 179]).

Let us now solve the Poisson equation for Ψ, whereas Ψ represents the classical field which fulfils the classical Poisson equation, while Φ represents the actual gravitational potential, dependent on a Yukawa term of the scalar field excitation ξ. Its solution can be written as a term equal to the standard newtonian one plus a term of the pressure p. We have for scales $r \gg R_0$ and general densities,

$$\Psi = -\frac{G_0}{c^2}\int\left(1+\frac{1}{3}e^{|\vec{r}-\vec{r}_s|/l}\right)\frac{\epsilon(\vec{r}_s)}{|\vec{r}-\vec{r}_s|}d\vec{r}_s - \frac{G_0}{c^2}\int\left(3-e^{|\vec{r}-\vec{r}_s|/l}\right)\frac{p(\vec{r}_s)}{|\vec{r}-\vec{r}_s|}d\vec{r}_s. \tag{7.2.46}$$

For a point–particle with barotropic equation-of-state parameter $w = \epsilon/p$, we have

$$\Psi = -\frac{MG_0}{r}\left[1+\frac{1}{3}(1-3w)e^{-r/l}\right], \tag{7.2.47}$$

which possesses newtonian character when the newtonian gravitational constant $G_N = 4G_0/3$ is taken for $R_0 M \ll 1$ (which is valid in torsion experiments of G_N). Obviously, the results above are still valid for $p \neq 0$.

The equations (7.2.16) and (7.2.17) are still valid. Hence, let us give the integration constants valid for $p \neq 0$. Instead of (7.2.23), now we have

$$C_1 = -C_2 - \frac{8\pi G_0}{3\,c^2}l^2\,(R_0+l)\,\epsilon(1-3w)e^{R_0/l}. \tag{7.2.48}$$

Furthermore, there is instead of (7.2.23),

$$A_1 = \frac{2M_1G_0}{c^2}(1-3w)\frac{l^2}{R_0^2}\left[\cosh\left(\frac{R_0}{l}\right)-\frac{l}{R_0}\sinh\left(\frac{R_0}{l}\right)\right], \tag{7.2.49}$$

which, analogously to (7.2.23) reads

$$A_1 = \frac{2M_1G_0}{3\,c^2}(1-3w) \quad \text{for}\, R_0 \ll l. \tag{7.2.50}$$

Further, there is instead of (7.2.26),

$$B_1 = \frac{2M_1G_0}{c^2}(1+3w)\left[1+\frac{l}{R_0}\left(1+\frac{l}{R_0}\right)\left(1-e^{R_0/l}\right)\left(\cosh\left(\frac{R_0}{l}\right)-\frac{l}{R_0}\sinh\left(\frac{R_0}{l}\right)\right)\right]. \tag{7.2.51}$$

For a point–particle of mass M_1 with barotropic equation of state at rest in the origin (equivalently to $R_0 \ll r$), the correction terms (7.2.33) and (7.2.38) of the metric are still valid:

$$f_1 = \frac{1}{4}\left(1-e^{-r/l}\right), \tag{7.2.52}$$

$$f_2 = \frac{1}{4}\left[1+\left(1+2\frac{r}{l}\right)e^{-r/l}\right], \tag{7.2.53}$$

We may now define following correction parameter,

$$h = \frac{1 - f_2 + 2w\left(1 + 2f_2\right)}{1 - f_1 + \frac{3}{2}w\left(1 - \frac{1}{2}f_1\right)}\,, \tag{7.2.54}$$

together with a dynamical mass which now yields

$$M_{dyn} = M_1\left(1 - f_1 + \frac{3}{2}w\left(1 - \frac{1}{2}f_1\right)\right). \tag{7.2.55}$$

For $r/l \gg 1$, (7.2.55) reduces to

$$M_{dyn} = \left(1 + 3w\right)M_1. \tag{7.2.56}$$

For $r/l \ll 1$, on the other hand, it reduces to

$$M_{dyn} = \left(1 + \frac{3}{2}w\right)M_1 \quad \text{for } l \gg r. \tag{7.2.57}$$

Clearly, for nonvanishing pressures, the equation-of-state parameter may lead to higher dynamical mass terms than those of luminous (bare) mass only from densities. A discussion about effective masses in relation to DM may be found in [23,179]. Further, the r/l corrections shall lead to deviations from the standard newtonian potential. In this sense, work as the one in [3] is important.
Now, we define a dynamical (effective) Schwarzschild radius for the dynamical mass M_{dyn}:

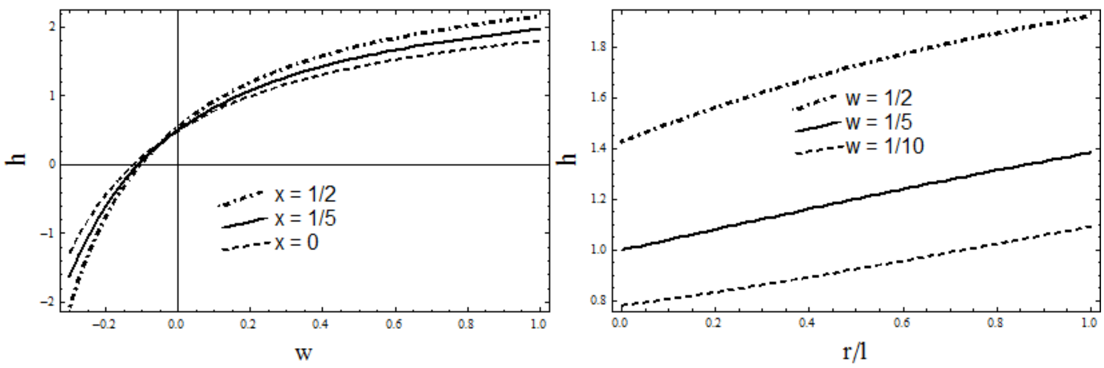

Figure 7.1: Evolution of the parameter h for different equation-of-state (eos) parameters w and distance coefficients $r/l = x$. *Cf.* [23].

$$\tilde{r}_S = \frac{2M_{dyn}G_N}{c^2}. \tag{7.2.58}$$

For the vacuum solutions for $r \gg R_0$ with radii R_0 of the gravitational objects, the potentials in the metric (C.1.1) may now be given as follows,

$$\nu = -\frac{r_{dyn}}{r}, \quad \lambda = h\frac{r_{dyn}}{r}. \tag{7.2.59}$$

Clearly, for weak-field approximation, consistency with a PPN framework is given for $h = 1$ so that $\nu = -\lambda$ is valid.
For $r/l \ll 1$, h simplifies so that one gets

$$h = \frac{1 + 8w}{2 + 3w} \quad \text{for } l \to \infty. \tag{7.2.60}$$

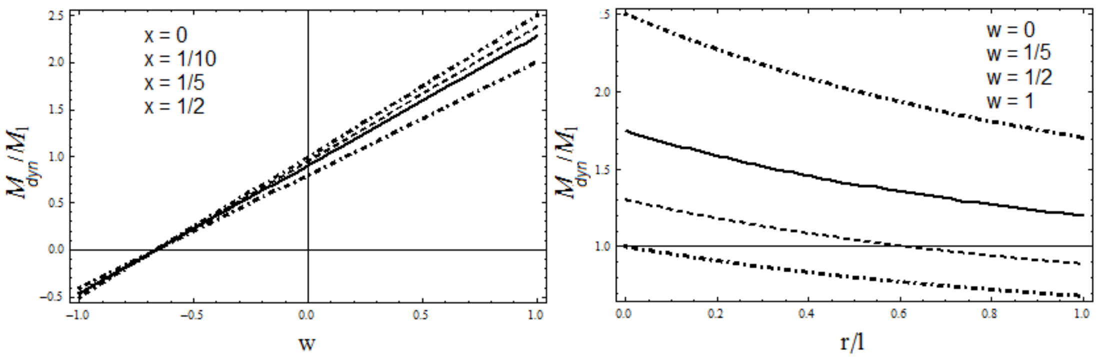

Figure 7.2: Evolution of the dynamical-mass coefficient M_{dyn}/M_1 for different eos parameters w and distance coefficients $r/l = x$. *Cf.* [23].

Terms of $h(w)$ which are actually dependent on distance–length-scale relation are negligible for $r \ll l$, and derivatives of $h(w,r)$ are of the magnitude l^2 (and hence of M^2 parametrized by c and $\hbar$).
Evidently, for $w = 0$, the linear approximation shows $\nu = -2\lambda$ as gotten from equations (7.2.40) and (7.2.31) in the case of vanishing pressure terms. A finite value of the parameter w seems necessary within a PPN framework if $\nu = -\lambda$ is to be valid. Such value, it can be said here, is of $w = 1/5$ within a linear solution (*cf.* [23, 179]). This may be seen as a relevant empirical constraint which is further related to the energy density in Chapter 7.3 and which will be also important in the context of solar-relativistic effects (*cf.* Chapter 7.6) as well as flat rotation curves (*cf.* Chapter 7.8), for instance.

7.3 Energy-density constraints on pressure and mass terms

We know from (7.2.59) that nonvanishing pressure terms are necessary for consistency within a PPN framework. Such pressure terms appear in dynamical (7.2.55) and effective masses (7.5.14) (*v.i.* Chapter 7.5) and should have measurable effects on mass unlike luminous mass from density alone.
The differentiation between bare and effective masses is related to the finiteness of scalar fields (*v.i.* Chapter 7.4, especially the amplitude (7.4.10)). Furthermore, pressure terms are visible in the energy density of gravity (6.4.22) within the energy–stress tensor.
Now, we will analyze the consequences of the scalar field on the energy density for central symmetry by means of comparison with the usual value from GR. Both have to be equal in order not to contradict empirical facts from newtonian dynamics. Hence, it is possible to constraint the values of the equation-of-state parameters for solar-relativistic regimes.

The energy-density of gravitation from equation (6.4.22) of Chapter 6.4 and 1.2 reads

$$s = +\frac{2}{\tilde{\kappa}} u_\mu{}^{;\sigma} u^\mu{}_{;\sigma} + T_{\mu\nu} u^\mu u^\nu - (1 - \hat{q})\, T - \frac{2}{\kappa_0 l^2} \xi + \frac{8}{3} \frac{\Lambda_0}{\kappa_0} \,.$$

For $l \to \infty$ or low scalar-field excitations, together with $\hat{q} = 0$ and $\Lambda_0 = 0$, equation (6.4.22) gives the usual energy density, assuming $\kappa_0 = \kappa_N$. For $\hat{q} = 1$, however, there is a term missing from T and it is clear from earlier works (*viz* [51, 179]) that κ is to be rescaled to κ_N.

If we consider the energy–momentum of an ideal fluid then there is

$$T = T_{\mu\nu}u^{\mu}u^{\nu} - 3p \tag{7.3.1}$$

along with

$$T = \epsilon - 3p, \tag{7.3.2}$$

with energy densities $\epsilon = \varrho c^2$ and pressures p. Now, energy density of gravitation as in (6.4.22) leads to the form below,

$$s = +\frac{2}{\tilde{\kappa}}u_{\mu}{}^{;\sigma}u^{\mu}{}_{;\sigma} + \hat{q}\epsilon + 3(1-\hat{q})p - \frac{2}{\kappa_0 l^2}\xi + \frac{8}{3\kappa_0}\Lambda_0\,. \tag{7.3.3}$$

Let us take cartesian coordinates for central symmetry as follows,

$$ds^2 = (1+e^{\nu})\,dt^2 - \left(1+e^{\lambda}\right)(dx_1^2 + dx_2^2 + dx_3^2). \tag{7.3.4}$$

For the scalar field, there is

$$\frac{1}{c^2}\ddot{\xi}e^{-\nu} - \xi'' e^{-\lambda} - \frac{1}{c^4}\frac{\dot{\nu}-\dot{\lambda}}{2}e^{-\nu}\dot{\xi} - \frac{\nu'-\lambda'}{2}e^{-\lambda}\xi' - \frac{2}{r}e^{-\lambda}\xi' + \frac{1}{l^2}\xi = +\hat{q}\frac{\kappa_0}{3}(\epsilon - 3p). \tag{7.3.5}$$

For an observer which is static to matter, there is a 4-velocity in linear approximation as follows,

$$u_{\mu} = \left(1 - \frac{1}{2}\nu, 0, 0, 0\right). \tag{7.3.6}$$

With $\nu c^2/2 = \Phi$ (the gravitational potential), we have

$$u_{\mu}^{;\sigma}u^{\mu}{}_{;\sigma}c^4 = -(u^0{}_{,1})^2 c^4 = -(\mathrm{grad}\,\Phi)^2. \tag{7.3.7}$$

Therefore, equation (7.3.3) reads in the static case as follows,

$$s = -\frac{2/c^4}{\tilde{\kappa}}(\mathrm{grad}\,\Phi)^2 + \hat{q}\epsilon + 3(1-\hat{q})p - \frac{2}{\kappa_0 l^2}\xi + \frac{8}{3\,\kappa_0}\Lambda_0\,. \tag{7.3.8}$$

For coherent matter and using the pressure-comprising Poisson equation (*viz* equation (7.2.42) in Chapter 7.2), we may write the following for weak fields and nondominant ξ excitations,

$$\mathrm{grad}\,p = -\frac{2}{\kappa_0}\mathrm{grad}\,\Phi\left(\frac{\Delta\Phi}{c^4} - \frac{3}{2}p\,c^2\right). \tag{7.3.9}$$

This problem is analog to the one of GR plus a pressure term. Now, under the assumption $l \to \infty$, using the Gauss theorem several times and taking into account that the pressure p is supposed to vanish at the surface of matter distribution, equation (7.3.9) leads to a relation between the gravitational potential and pressure terms p as below,

$$3c^4\int p dV = \frac{1}{\kappa_0}(1+3w)^{-1}\int(\mathrm{grad}\,\Phi)^2 dV, \tag{7.3.10}$$

using $p = w\epsilon$ with an equation-of-state parameter w, analogously to [65]. This is a relationship between the newtonian gravitational pressure in matter and the gravitational field strength. The equation (7.3.10) is related to the gravitational energy–momentum density by equation (7.3.8). Actually, for the field energy, there is for weak ξ fields with $l \to \infty$ and a vanishing cosmological constant Λ_0,

$$\int s dV = -\frac{2/c^4}{\kappa_0}\int(\mathrm{grad}\,\Phi)^2 dV + \hat{q}\int \epsilon\, dV + 3(1-\hat{q})\int p dV, \tag{7.3.11}$$

with an energy term as defined below,

$$c^4 \int \epsilon dV = \frac{1}{3\kappa_0} \frac{1}{w(1+3w)} \int (\mathrm{grad}\, \Phi)^2 dV. \tag{7.3.12}$$

Inserting equation (7.3.12) in (7.3.11), the field energy yields

$$\int s dV = -\frac{1}{\kappa_0 c^4} \left[2 - \frac{\hat{q}}{3(1+3w)} - \frac{(1-\hat{q})}{(1+3w)} \right] \int (\mathrm{grad}\, \Phi)^2 \, dV. \tag{7.3.13}$$

The solution within GR yields $\int s dV c^4 = -(1/\kappa_N) \int (\mathrm{grad}\, \Phi)^2 dV$. It gives the same value as the potential energy of a body within Newton's gravitational theory, and such is necessary to avoid conflicts with elementary mechanics [64, 65].

The equation (7.3.8) gives GR's solution for $\hat{q} = 0$ and $w = 0$ with $\kappa_0 = \kappa_N$ indeed. For $\hat{q} = 1$, on the other hand, after rescaling with $\kappa_N = 4\kappa_0/3$, equality between the usual gravitational energy of GR and induced-gravity's gravitational energy (as of interpretation given here) leads to a constraint of $w \approx 0.17 \approx 1/6$. This pressure value, necessary for consistency with phenomenology and for nonvanishing scalar fields, should appear in some specific contexts, and variations from it should lead to measurable consequences especially for large-scale dynamics (*cf.* [23]).

Indeed, the constraint value of pressure is near to the value given in Chapter 7.2 for the pressure. Hence, if pressure (which appears by means of the scalar field in relation to A_1) is given such that the theory is consistent with newtonian dynamics. Further, newtonian dynamics is directly given for solar-relativistic effects (further general-relativistic effects are usually obtained as further corrections). However, this has not to be the case for large distances, as pressure acts antiscreening for masses leading to effective masses and thus to phenomenological Dark Matter if understood as the deviation between measured and luminous matter within GR. Newtonian dynamics are no further boundary condition for the gravitational potentials at long ranges. Further, this may respond to nature of Quintessence fields whose fluctuations may behave similar to a relativistic gas (*sc.* [241]).

Given that models of Quintessence usually predict composition-dependent gravity like long-range forces mediated by the fields [242] (*i.e.* breaking of the WEP), measurable consequences should appear for large distances indeed. There, scalar fields may act similarly to a cosmological constant (*cf.* [78]) or as related to the halo mass of galaxies and hence to Dark Matter phenomenology [240]. To analyze such dynamics, we will discuss exact solutions of the model before comparing them in their weak-field limit with the linear results.

7.4 Exact equations and black-hole solutions

Within GR, the exact vacuum solution for central symmetry with a vanishing cosmological constant Λ_0 is the Schwarzschild metric

$$ds^2 = \left(1 - \frac{r_S}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_S}{r}} - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta d\varphi^2, \tag{7.4.1}$$

whereas the constant $r_S = \frac{2M_1 G_0}{c^2} = B$ is the so-called Schwarzschild radius, valid for a constant gravitational coupling G_0 which in GR is $G_0 = G_N$. r_S represents the radius which a body of mass M_1 must have so that its rest-mass $M_1 c^2$ be equal to its internal gravitational potential energy $V_N \simeq G_0 M_1^2 / r_S$ (*cf.* [56]). Within GR, any particle, not even a photon, cannot escape from a region of radius r_S around a body of mass M_1. Hence, the Schwarzschild radius defines the horizon of a black hole, so that for $r = r_S$, there appears an horizon singularity. Then, e^λ diverges.

Here, an analysis of black-hole solutions within this model is fulfilled for vanishing scalar-field masses. In view of the discussion in Chapter 6.3, this is of special relevance indeed.
The limiting case of a vanishing Higgs field mass (6.3.19) of the nonminimally coupled Higgs field as scalar field ($l \to \infty$) can be understood as a double limit $\mu^2 \to 0$ and $\lambda \to 0$, so that $\mu^4/\lambda = 0$ and $v^2 = \mu^2/\lambda = finite$ remain valid throughout. Thus, the ground-state value keeps the degeneracy (remains the one of a Higgs mode and does not go through to one of a Wigner one) even for the massless case of these particles (which is not the case in the standard model), and the symmetry stays broken at low energies. The scalar field still changes the usual dynamics after symmetry breakdown and the excitations are in general nonvanishing. Thus, the field equations do not reduce to the usual ones of GR as long as the excitations ξ do not vanish, and new changes in the dynamics can be acquainted to the scalar field and its gravitational Yukawa interaction.

In the static case and under the assumption of a point–mass at $r = 0$ (or on a distance $r > R_0$ for a radius R_0 of the massive object), the Higgs field equation takes the form (with $l \to \infty$)

$$\xi_a^{''} - \frac{1}{2}(\lambda_a^{'} - \nu_a^{'})\,\xi^{'} + \frac{2}{r}\,\xi_a{}' = 0\,, \tag{7.4.2}$$

where the prime denotes the differentiation with respect to the radial coordinate r as before. The first derivative of the excited scalar field ξ_a from equation (7.4.2) in the case of a point–mass (with internal structure (pressure)) at $r = 0$ then reads

$$\xi_a' = \frac{A}{r^2}\,e^{w/2} = \frac{A}{r^2}\,e^{(\lambda_a-\nu)/2} \quad (l \to \infty)\,, \tag{7.4.3}$$

where (all subscripts a will be let aside afterwards)

$$u := \lambda_a + \nu_a \quad \text{and} \quad w := \lambda_a - \nu_a \tag{7.4.4}$$

are defined. Here, the subscript a means "for vacuum $r > R_0$" with a radius R_0 of the massive object. The integration constant A, which appears in equation (7.4.3), is derived from equation (6.3.21) in the limit of $r \to \infty$ and $l \to \infty$ through

$$\xi^{,\mu}{}_{;\mu} = \frac{1}{\sqrt{-g}}\,(\sqrt{-g}\,\xi^{,\mu})_{,\mu} = \frac{8\pi}{3}\frac{G_0}{c^2}\,T \tag{7.4.5}$$

with $g = \det g_{\mu\nu}$ for the metric $g_{\mu\nu}$. With $g = g_{00}\,k$, after defining $k = \det\, g_{ik}$, equation (7.4.5) can be rewritten as

$$\frac{1}{\sqrt{g_{00}}\,\sqrt{-b}}\,(\sqrt{g_{00}}\,\sqrt{-k}\,\xi^{,\mu})_{,\mu} = \frac{8\pi}{3}\frac{G_0}{c^2}\,T\,, \tag{7.4.6}$$

which is valid for $\hat{q} = 1$ and which, after multiplied with $\sqrt{g_{00}}\,\sqrt{-b}\,d^3x$, leads to the following equation:

$$(\sqrt{g_{00}}\,\sqrt{-k}\,\xi^{,\mu})_{,\mu}d^3x = \frac{8\pi}{3}\frac{G_0}{c^2}T\,\sqrt{-k}d^3x\,\sqrt{g_{00}}\,. \tag{7.4.7}$$

For the limit $r \to \infty$, there is

$$lim_{r\to\infty}\sqrt{g_{00}}\,\xi^{,\mu}\,4\pi r^2 = \frac{8\pi}{3}\frac{G_0}{c^2}\int\sqrt{-g}\,Td^3x\,. \tag{7.4.8}$$

For equation (7.4.8), $lim_{r\to\infty}g_{00} = 1$ is valid according to the boundary conditions of minkowskian space-time. For the signature (+,-,-,-), equation (7.4.8) leads then to

$$-\xi_{,\mu}r^2 = \frac{2}{3}\frac{G_0}{c^2}\int\sqrt{-g}\,T\,d^3x \quad (r \to \infty). \tag{7.4.9}$$

According to equation (7.4.3) for $|e^{w/2}| \approx 1$ approximating to the minkoswkian limit, the integration constant A is then given according to equation (6.3.21) and (7.4.9) by

$$A = -\frac{2}{3}\frac{G_0}{c^2}\int T\sqrt{-g}\,d^3x, \quad (l \to \infty). \tag{7.4.10}$$

The value of A is related to the constant A_1 of linear approximation of Chapter 7.2 for small r/l values for the homogeneous sphere or point–masses. It is related to inner solutions through continuity conditions at the surface $r = R_0$ and, as easily seen, dependent on p. Hence, it may be expected that p do not vanish for vacuum solutions. Otherwise, the scalar-field excitation would vanish. Consequently, the scalar field shall constrain the relation between energy density ϵ and p, as already seen for linear solutions and as will be seen for solar-relativistic effects and for the dynamics of flattening of rotation curves. Such should have implications for measurable mass which is not the integral over density alone but over pressure terms too. Such is seen under the definition of a dynamical effective mass in Chapter 7.2 under equation (7.2.55), and will later be seen in Chapter 7.5. Further, this constraints in relation to gravitational energy may be found in Chapter 7.3.

Now, we define

$$q := \ln(1+\xi) \tag{7.4.11}$$

for the excitation of the scalar field. Inserting it in the non-trivial field equations associated to the Lagrangian (6.1.3) for the metric (7.1.1) (see [23]), and making use of equation (7.4.3), leads in the case of a point–mass in vacuo (or equivalently for the outer region of a massive object) to the following equations for $l \to \infty$ (the subscript a is suppressed) and for the static case [21]:

$$\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} - \frac{\lambda'}{r} = -\frac{1}{1+\xi}\left(\xi'' - \frac{\lambda'}{2}\xi'\right), \tag{7.4.12}$$

$$\left(1 - \frac{r\lambda'}{2} + \frac{r\nu'}{2}\right)e^{-\lambda} - 1 = -\frac{r}{1+\xi}e^{-\lambda}\xi' = -re^{-\lambda}q', \tag{7.4.13}$$

$$\left(\frac{1}{2}\nu'' - \frac{1}{4}\lambda'\nu' + \frac{\nu'}{r} + \frac{\nu'^2}{4}\right)e^{\nu-\lambda} = -\frac{\nu'}{2}q'e^{\nu-\lambda} \tag{7.4.14}$$

Subtraction of equation (7.4.14) from equation (7.4.12), and insertion of w and u, defined in equation (7.4.4), leads to

$$\frac{1}{2}rw' = 1 - e^{(u+w)/2} + r\,q'\,, \tag{7.4.15}$$

$$u'\,(1 + \frac{r}{2}q'\,) \;=\; \frac{r}{2}q'\,(\,w' - \frac{4}{r}\,)\,, \tag{7.4.16}$$

$$\frac{1}{2}\,(u' - w') = \frac{B}{r^2}\,e^{w/2-q} = \frac{B}{A}\,q'\,, \tag{7.4.17}$$

whereas equation (7.4.16) is the substraction of field equations, and equation (7.4.17) is the total integral for ν' with B as an integration constant (these results were mainly published in reference [21]). Especially, it can be seen that

$$\nu' = -\lambda' - q'(r\nu' + 2), \quad u' = -r\frac{B}{A}q'^2 - 2q'. \tag{7.4.18}$$

Using the value of u' given in equation (7.4.16), equation (7.4.17) leads to the following decoupled equation:

$$w' = -2\frac{(A+B)}{r^2}\,e^{w/2-q} - \frac{AB}{r^3}\,e^{w-2q}. \tag{7.4.19}$$

Now, using the equations (7.4.15) and (7.4.19) one immediately deduces

$$e^{u/2+q} = e^{-w/2+q} + \frac{(2A+B)}{r} + \frac{AB}{2r^2}\, e^{w/2-q}\,, \tag{7.4.20}$$

and, therefore, only the differential equation (7.4.19) remains to be solved. These considerations further lead to the solution of the excited Higgs field given by equation (7.4.3) in the following form for $B \neq 0$:

$$\xi = e^q - 1 = -1 + e^{\frac{A}{2B}(u-w)}. \tag{7.4.21}$$

Equation (7.4.21) clearly shows that such excitations of the scalar field are only possible for a nonvanishing value of the integration constant A given by equation (7.4.10).

As boundary condition we postulate the Minkowski metric at spatial infinity. In order to determine the meaning of the integration constant B we consider at first the asymptotic case $r \to \infty$ of the potentials again, *i.e.* $|w| \ll 1$, $|u| \ll 1$. Then, we get from equation (7.4.19):

$$u = 2\frac{A}{r} + \frac{AB}{2r^2}\,, \tag{7.4.22}$$

$$w = \frac{2(A+B)}{r} + \frac{AB}{2r^2}\,. \tag{7.4.23}$$

This results in

$$\nu = \frac{(u-w)}{2} = -\frac{B}{r} \tag{7.4.24}$$

and

$$\lambda = \frac{(u+w)}{2} = \frac{AB}{2r^2} + \frac{(2A+B)}{r}\,, \tag{7.4.25}$$

and consequently,

$$B = \frac{2M_S G_N}{c^2} = r_S \tag{7.4.26}$$

is valid in view of the equation of motion of the line element (7.1.1), where M_S is the asymptotic ($r \to \infty$) visible mass of the particle (and represents the Schwarzschild mass), G_N is the newtonian gravitational coupling constant (since the newtonian character is expected at spatial infinity) and r_S the distance belonging to M_S, belonging to the Schwarzschild radius. Further, the differential equation (7.4.19) is an abelian one and can be solved exactly. With the substitution

$$e^{w/2-q} =: r\,\tilde{g}(r) =: r\,\tilde{g}, \tag{7.4.27}$$

equation (7.4.19) acquires a much simpler form as given below,

$$r\tilde{g}' = \alpha\tilde{g}^3 - K\,\tilde{g}^2 - \tilde{g}\,, \tag{7.4.28}$$

whereas

$$K := 2A + B = 2\left(A + \frac{G_N}{c^2}M_S\right) \quad \text{and} \tag{7.4.29}$$

$$\alpha := -\frac{AB}{2} = -\frac{A}{c^2}M_S G_N. \tag{7.4.30}$$

Equation (7.4.28) can be integrated by using the method of separation of variables, which for $\alpha \neq 0$ reduces to the form given as

$$\left|\frac{\tilde{g}^2}{1 + K\tilde{g} - \alpha\tilde{g}^2}\right| \left|\frac{\sqrt{K^2+4\alpha} + K - 2\alpha\tilde{g}}{\sqrt{K^2+4\alpha} - K + 2\alpha\tilde{g}}\right|^{\frac{K}{\sqrt{K^2+4\alpha}}} = \frac{C}{r^2}\,. \tag{7.4.31}$$

The integration constant C in equation (7.4.31) can be calculated in the minkowskian limit [121] as

$$C = \left(\frac{\sqrt{K^2 + 4\alpha} + K}{\sqrt{K^2 + 4\alpha} - K} \right)^{\frac{K}{\sqrt{K^2+4\alpha}}} . \tag{7.4.32}$$

Thus, the nonminimally coupled massless Higgs field within induced gravity with Higgs potential acts in an analogous way to a massless scalar field within Einstein's theory of gravity in [121]. The integration constants, however, are of different nature to those in [121] since K and the charge parameter α are given by both the parameters of the fields.

In view of the equations (7.4.20), (7.4.21) and (7.4.27), the metric components given by equation (7.1.1) and the scalar field by the equation (7.4.10) for the case $B \neq 0$ may then be expressed in terms of $\tilde{g}$ in the following form:

$$e^{\nu} = \left[\frac{1}{r^2 \tilde{g}^2} \left(1 + K\tilde{g} - \alpha \tilde{g}^2\right) \right]^{\frac{B}{K}} , \tag{7.4.33}$$

$$e^{\lambda} = 1 + K\tilde{g} - \alpha \tilde{g}^2 , \tag{7.4.34}$$

$$\begin{aligned} \xi &= -1 + e^{\frac{A}{K} \ln\left[\frac{1}{r^2\tilde{g}^2}(1+K\tilde{g}-\alpha\tilde{g}^2)\right]} \\ &= -1 + \left[\frac{1}{r^2\tilde{g}^2}(1 + K\tilde{g} - \alpha\tilde{g}^2) \right]^{\frac{A}{K}} . \end{aligned} \tag{7.4.35}$$

The only effective physical parameters remaining in the theory of the present model are the integration constants A and B defined by the equations (7.4.10) and (7.4.26), respectively. Unfortunately, it is quite difficult to solve equation (7.4.31) for $\tilde{g}$ explicitly. However, a transparent discussion of the properties of the solution is feasible in connection to [121]. For the limiting case $A = 0$, *i.e.* for the equation of state $p = \frac{1}{3}\varrho c^2$ (see equation (7.4.10)) and $B \neq 0$ (*i.e.* $\alpha = 0$ and $K = B$), using equations (7.4.20) and (7.4.27), equation (7.4.28) can be exactly solved for $\tilde{g}$ in the following form: [21]

$$\tilde{g} = \frac{1}{r} \left(1 - \frac{B}{r}\right)^{-1} , \tag{7.4.36}$$

and thus for the potentials, using equations (7.4.33) and (7.4.34):

$$\begin{aligned} e^{\lambda} &= \left(1 - \frac{B}{r}\right)^{-1} , \\ e^{\nu} &= \left(1 - \frac{B}{r}\right) . \end{aligned} \tag{7.4.37}$$

The equation (7.4.37) indicates that the metric components of line element given by equation (7.1.1) correspond to the usual Schwarzschild metric (with associated features) which appears in this form only for the limiting case of the vanishing scalar-field excitations (*i.e.* $\xi = 0$) [21]. However, for the general values of $A < 0$, the qualitative results shown in the work of Hardell and Dehnen [121] are valid. It is worth mentioning that the higher values of $|A|$, equation (7.4.33) lead to a decrease in ν through the exponent B/K. In fact, the metric and scalar field are regular everywhere with exception of $r = 0$ as naked singularity, and there exists no Schwarzschild horizon except for the case of vanishing scalar-field excitations. Therefore, Black Holes (in the usual sense) do not appear for the case $A \neq 0$. They may therefore be called Grey Stars. Naked singularities break the Cosmic Censorship Conjecture (CCC) and may be related to brighter, weaker

novae and might be interpreted in terms of some gamma-ray bursts (GRBs) [122, 179, 221].
Herewith, the scalar field leads to a screening of the usual gravitational interaction (*viz* Quintessence and Dark Energy). For higher scalar-field masses, this feature should be also valid, although in a weakened form.
For the derivatives or the potentials, there is

$$\nu' = \frac{B}{K}\left[\frac{2}{r} + \frac{2+K\tilde{g}}{1+K\tilde{g}-\alpha\tilde{g}^2}\frac{\tilde{g}'}{\tilde{g}}\right], \tag{7.4.38}$$

$$\lambda' = \frac{K-2\alpha\tilde{g}}{1+K\tilde{g}-\alpha\tilde{g}^2}\,\tilde{g}', \tag{7.4.39}$$

$$q' = -\frac{A}{K}\left[\frac{2}{r} + \frac{2+K\tilde{g}}{1+K\tilde{g}-\alpha\tilde{g}^2}\,\tilde{g}'\right] = \frac{A}{B}\nu', \tag{7.4.40}$$

where $\tilde{g}$ is given by

$$\tilde{g}' = \frac{1}{r}(\alpha\tilde{g}^3 - K\tilde{g}^2 - \tilde{g}). \tag{7.4.41}$$

Then, there is

$$q'r\nu' = -\,\nu' - \lambda' - 2q' \tag{7.4.42}$$

$$= -\,2\frac{\alpha}{r}\tilde{g}^2 = \frac{AB}{r}\tilde{g}^2. \tag{7.4.43}$$

This term vanishes in a linear approximation. Further, with equation (7.4.28), it is possible to write the derivatives of the field components in terms of $\tilde{g}$ and the constants,

$$\nu' = \frac{B}{r}\tilde{g}, \quad \lambda' = -\frac{K}{r}\tilde{g} + 2\frac{\alpha}{r}\tilde{g}^2, \tag{7.4.44}$$

$$q' = \frac{A}{r}\tilde{g}. \tag{7.4.45}$$

With equation (7.4.40), equation (7.2.36) can further be discussed. An insertion of q' in dependence of ν' leads to

$$\begin{aligned}\nu' &= -\,\lambda' - \frac{A}{B}\nu'(r\nu'+2)\\ &= -\,\lambda' - q'(r\nu'+2).\end{aligned} \tag{7.4.46}$$

Inserting equation (7.4.42) in equation (7.4.46) leads immediately to the λ' equation in (7.4.44), since $K = 2A + B$ is valid. From this point, for weak fields, it follows

$$\nu'\left(1 + 2\frac{A}{B}\right) = -\lambda', \quad \text{(for weak fields)}. \tag{7.4.47}$$

From this equation, it can be seen that $A = 0$ leads to $\nu' = -\lambda'$ of usual GR, as already discussed parting from the linear results.

7.5 The Reissner–Nordström-like solution

The exact solution of equation (7.4.28) as analyzed earlier (*viz* [21]) shows qualitatively, together with [121], that Schwarzschild horizons vanish for nonvanishing excitations $\xi \neq 0$. However, an analytical approach of exact solutions for such fields is very difficult and we have therefore looked for some approximated solutions by using a series-expansion method. This shows basic properties of the solution and of the scalar-field interaction for small but non-vanishing excitations of the scalar field, which is especially valid for long distances to the gravitative source as well as for relatively lowly massive bodies. An exact solution of (7.4.28) for the case $\xi = 0$ for $\tilde{g}$ is discussed in [21], together with the formal solution of the metric.
For the purpose of behavioral analysis, let us consider the series-expansion method as ansatz for $\tilde{g}$ to further simplify the equations (7.2.1)–(7.2.4),

$$\tilde{g} = \sum_{n=1}^{\infty} \frac{C_n}{r^n} = \frac{1}{r}\left[C_1 + \frac{C_2}{r} + \frac{C_3}{r^2} + \frac{C_4}{r^3} + \ldots\right], \tag{7.5.1}$$

Using equation (7.5.1) in (7.4.28) and then comparing the left- and right-hand sides of this equation, the coefficients of r^{-1}, r^{-2} and so on (up to the fifth order in $1/r$) can be obtained with the following simple recursion relations with straightforward calculations as follows (see [179]),

$$\left.\begin{array}{c} C_1 = 1 \\ C_2 = 2A + B \\ C_3 = (2A+B)^2 + \frac{AB}{4} \\ C_4 = (2A+B)^3 + \frac{2AB}{3}(2A+B) \\ C_5 = (2A+B)^4 + \frac{29AB}{24}(2A+B)^2 + \frac{3(AB)^2}{32} \end{array}\right\} . \tag{7.5.2}$$

Clearly, the constants C_i appear as additive and multiplicative terms of A and B, and these are the only two parameters of physical interest of the present model. Consequently, we restructure (7.5.1) as follows,

$$\tilde{g} = \frac{1}{r}\left[1 - \frac{(2A+B)}{r}\right]^{-1} + \frac{AB}{2r} X(A, B; r^{-n}); \quad (n \geq 2), \tag{7.5.3}$$

where $X(A, B; r^{-n})$ is function of r, A and B only, with values up to the fifth order in $1/r$ as

$$X(A, B; r^{-n}) = \frac{1}{2r^2} + \frac{4\,(2A+B)}{3r^3} + \frac{29\,(2A+B)^2}{6r^4} + \frac{3\,AB}{8r^4} + \ldots; \quad (n \geq 2). \tag{7.5.4}$$

There is $n \geq 2$. Accordingly, $X(A, B; r^{-n})$ is negligible for extremely large distances (which is in most of the cases the region of interest). Low potency terms appear as small corrections for smaller distances to the gravitational "source". Such a situation can physically be understood in terms of the weakening of gravity once one moves away from the center of a gravitating mass. And in terms of the scalar-field mass M as discussed in [23] and Chapter 6.3, further corrections appear at large scales.
From the substraction of (7.2.1) from (7.2.3) and using

$$\nu' = \frac{B}{r^2}\frac{1}{1+\xi}\, e^{(\lambda-\nu)/2}, \tag{7.5.5}$$

we get (*cf.* [21, 179])

$$e^{(\lambda+\nu)/2} = (1+\xi)\left[\frac{1}{r\tilde{g}} + \frac{2A+B}{r} + \frac{AB}{2r}\tilde{g}\right]. \tag{7.5.6}$$

Using (7.5.3) and equation (7.5.6) further leads to

$$e^{\lambda} = \left[1 - \left(\frac{2A+B}{r}\right)\right]^{-1} - \frac{AB}{2r^2}\left[1 - \frac{(2A+B)}{r}\right]^{-2} + \\ + \frac{AB}{2r}(2A+B)X(A,B;r^{-n}) - \left(\frac{AB}{2r^2}\right)^2 X_1(A,B;r^{-n}), \tag{7.5.7}$$

where $X_1(A,B;r^{-n})$ is a function of $X(A,B;r^{-n})$ itself.

$$X_1(A,B;r^{-n}) = X(A,B;r^{-n})\left[2\left(1 - \frac{2A+B}{r}\right)^{-1} + \frac{AB}{2}X(A,B;r^{-n})\right] \tag{7.5.8}$$

Equation (7.5.7) may be rewritten onto following form,

$$e^{\lambda} = \left[1 - \frac{(2A+B)}{r} - \frac{AB}{2r^2}\right]\left[1 - \frac{(2A+B)}{r}\right]^{-2} + \\ + \frac{AB}{2r}(2A+B)X(A,B;r^{-n}) - \left(\frac{AB}{2r^2}\right)^2 X_1(A,B;r^{-1}), \tag{7.5.9}$$

which after some calculations leads to

$$e^{\lambda} = \left[1 - \frac{(2A+B)}{r} + \frac{AB}{2r^2}\right]^{-1} + \frac{AB}{2r}(2A+B)X(A,B;r^{-n}) - \frac{(AB)^2}{4r^2}X_2(A,B;r^{-n}), \tag{7.5.10}$$

where $X_2(A,B;r^{-n})$ is a function of $X_1(A,B;r^{-n})$ and further terms related to the first term of (7.5.10). There is

$$X_{2a}(A,B;r^{-n}) = \frac{1}{r^2}\frac{1 - \frac{(2A+B)}{r} - \frac{AB}{2r^2}}{\left(1 - \frac{(2A+B)}{r} + \frac{AB}{2r^2}\right)^2}\left[1 + \frac{(AB)^2}{4r^4}\left(\frac{1 - \frac{(2A+B)}{r} - \frac{AB}{2r^2}}{1 - \frac{(2A+B)}{r} + \frac{AB}{2r^2}}\right)\right]^{-1} \tag{7.5.11}$$

and

$$X_2(A,B;r^{-n}) = \left[X_1(A,B;r^{-n}) + X_{2a}(A,B;r^{-n})\right]. \tag{7.5.12}$$

For the potency n, there is again $n \geq 2$. Finally, up to the second order in $1/r$, the equation (7.5.10) yields

$$e^{\lambda} = \left[1 - \frac{2\tilde{M}G_N}{rc^2} + \frac{\tilde{Q}^2}{r^2}\right]^{-1}, \tag{7.5.13}$$

for which we have defined an effective mass term

$$\frac{\tilde{M}G_N}{c^2} = A + \frac{B}{2} \tag{7.5.14}$$

and a Reißner–Nordström-like (or Reissner–Nordström (RN)-like) charge parameter

$$\tilde{Q}^2 = \frac{AB}{2}. \tag{7.5.15}$$

These parameters are related to the parameters as defined in Chapter 7.4 by reasons of

$$K = 2A + B = \frac{2\tilde{M}G_N}{c^2}, \quad \text{and} \quad \alpha = -\frac{AB}{2} = -\tilde{Q}^2. \tag{7.5.16}$$

The assumption of K taking the place of a generalized mass parameter and of α acting as related to a generalized charge parameter are correct in the sense of a Reissner–Nordström solution as is achieved by means

of series expansion. Hence, the metric component λ acts as the metric component for an electrically charged particle in a gravitational field (*sc.* [206]) with a mass $\tilde{M}$ and a charge $\tilde{Q}$ which is, however, imaginary.
G_N is the newtonian gravitational coupling constant which for theoretical calculations may usually be taken as 1. The effective mass, as a general dynamical mass and in contraposition to the "actual" mass, is dependent on the mass which comes from energy density (ϵ) itself as well as on pressure p which enters a measured mass term through the integral of the trace of the energy–momentum tensor T. Actually, both A and B should contain a pressure factor indeed. Further, the generalized charge parameter $\tilde{Q}$, which appears as a consequence of usual gravitational terms hidden in B coupled to gravitational scalar-field terms in A, may act against usual gravitation of GR in the same way the RN charge does for a charged point–particle in a gravitational field [206]. This quintessential behavior grows for higher "charges" $\sqrt{|Q^2|}$ (*i.e.* for higher masses or field excitations) and smaller distances to the gravitating body. Physical consequences of the solutions are visible, although the exact vanishing of horizons is due to at-low-distance dominant terms which appear for high-order corrections. Yet, it is possible to analytically glance at the nature which ultimately leads to the vanishing of usual Schwarzschild horizons onto Grey Stars which appears in exact solutions (see Chapter 7.4 and [21]). Yet more important, it is possible to interpret the terms which act in dynamics with help of weak-field behavior. Further, the second component of the metric (*cf.* Chapter 7.4 and [21]) results as

$$e^{\nu} = \left[\frac{\left(1-\frac{2\tilde{M}G_N}{rc^2}\right)^2 e^{\lambda}}{\left[1-\left(1-\frac{2\tilde{M}G_N}{rc^2}\right)\tilde{Q}^2 X(A,B;r^{-n})\right]^2}\right]^{\frac{Bc^2}{2\tilde{M}G_N}} \quad ; \quad (n \geq 2). \tag{7.5.17}$$

With some straightforward calculations, the equation (7.5.17) may be rewritten onto the following form,

$$e^{\nu} = \left\{\frac{1-\frac{2\tilde{M}G_N}{rc^2}-\frac{\tilde{Q}^2}{r^2}+\left(1-\frac{2\tilde{M}G_N}{rc^2}\right)^2\left[\frac{\tilde{Q}^2}{r}\frac{2\tilde{M}G_N}{c^2}X(A,B;r^{-n})-\frac{\tilde{Q}^4}{r^2}X_1(A,B;r^{-n})\right]}{\left[1-\left(1-\frac{2\tilde{M}G_N}{rc^2}\right)\tilde{Q}^2X(A,B;r^{-n})\right]^2}\right\}^{\frac{Bc^2}{2\tilde{M}G_N}}, \tag{7.5.18}$$

which may then further be restructured for $n \geq 2$ as follows,

$$\begin{aligned} e^{\nu} = \Bigg\{\Bigg[1-\frac{2\tilde{M}G_N}{rc^2}-\frac{\tilde{Q}^2}{r^2}+\frac{\tilde{Q}^2}{r}\left(1-\frac{2\tilde{M}G_N}{rc^2}\right)^2\Bigg(\frac{2\tilde{M}G_N}{c^2}X(A,B;r^{-n})- \\ -\frac{\tilde{Q}^2}{r}X_1(A,B;r^{-n})\Bigg)\Bigg]\left[1+\frac{\left(1-\frac{2\tilde{M}G_N}{rc^2}\right)\tilde{Q}^4X(A,B;r^{-n})^2}{1-\left(1-\frac{2\tilde{M}G_N}{rc^2}\right)\tilde{Q}^2X(A,B;r^{-n})}\right]^2\Bigg\}^{\frac{Bc^2}{2\tilde{M}G_N}}. \end{aligned} \tag{7.5.19}$$

Unlike in the equation (7.5.13), up to second order, the generalized charge parameter $\tilde{Q}^2$ cancels out in equation (7.5.19), and for the metric component ν thus evolves as given below,

$$e^{\nu} = \left[1-\frac{2\tilde{M}G_N}{rc^2}\right]^{\frac{Bc^2}{2\tilde{M}G_N}}. \tag{7.5.20}$$

The potency term of (7.5.20) may be written as an effective-mass ratio as follows,

$$\frac{r_{dyn}}{\tilde{r}_S} = \frac{Bc^2}{2\tilde{M}G_N}. \tag{7.5.21}$$

The effective Schwarzschild radius $\tilde{r}_S$ and the dynamical radius r_{dyn} of the linear approach may be further related through the following,

$$\tilde{r}_S = 2A + r_{dyn} = \frac{2M_1G_N}{c^2}\left(\frac{1}{2} + 3w\right) \approx h(w)r_{dyn}. \tag{7.5.22}$$

Actually, gravitational energy-density analyses (*viz* Chapter 7.2, [23]) constraint w to *ca.* 1/5 to 1/6, and indeed, for such values both the effective ($\tilde{r}_S$) and the dynamical (r_{dyn}) masses are nearly the same, and for $w = 1/5$ there is $h(w) = 1$, and the metric components valid in linear approximation have the following form,

$$e^{\nu} = 1 - \frac{2M_{dyn}G_N}{rc^2}, \tag{7.5.23}$$

$$e^{\lambda} = 1 + \frac{2M_{dyn}G_N}{rc^2}. \tag{7.5.24}$$

Furthermore, solar-relativistic effects can then be expected to be given as they are measured for all low-energy systems and with advances of perihelion dependent on the system's internal structure (p) (*cf.* Chapter 7.6, [23]). For low gravitating systems, effective masses $\tilde{M}$ and M_{dyn} are approximately the same and the dynamical mass M_{dyn} takes the place of the actual mass M_1.
In linear approximation, from (7.5.20) we have

$$e^{\nu} = 1 - \frac{B}{r}, \tag{7.5.25}$$

with B for the dynamical mass parameter (see Chapter 7.2). In view of equation (7.2.59), B can be written directly in the form given below (*cf.* [179]),

$$B = \frac{2M_1G_N}{c^2}\left(1 + \frac{3}{2}w\right). \tag{7.5.26}$$

Hence, the linear approach is consistent with the series-expansion method as used above.[4]
In linear approximation for $M \to 0$, according to equation (7.4.10) and (7.2.23), A is given as follows,

$$A = -\frac{1}{2}\frac{M_1G_N}{c^2}(1 - 3w). \tag{7.5.27}$$

Hence, for weak-field regimes it is then equal to $-A_1$ as long as $M \to 0$. Further, for point–particles with $Mr \ll 1$, linear approach leads back to

$$\lambda = h(w)\frac{2M_{dyn}G_N}{rc^2}, \tag{7.5.28}$$

with a (lowly Mr-dependent) parameter $h(w)$ well-given by

$$h(w) = \frac{1 + 8w}{2 + 3w}, \tag{7.5.29}$$

for $M \to 0$ and for non-dominant $\tilde{Q}$-charges.

For $A \neq 0$, the power coefficient $Bc^2/(2\tilde{M}G_N)$ may be written as $(1 + 2A/B)^{-1}$, thus showing the deviation from a usual Schwarzschild value of e^{ν} (with $B = r_S$ as Schwarzschild radius), and pointing to different concepts of effective mass for different ranges. Even for weak gravitational fields, a nonvanishing scalar field appears related to the density and pressure terms as a dynamical correction to the bare mass M_1

[4] Further, higher-order corrections are relevant for considerations near to as well as beyond the Schwarzschild and charge radius.

(*cf.* (7.4.10) and Chapter 7.2). For $A/B \ll 1$, for relatively weak field regimes there is clearly a RN-like solution for λ in (7.5.13) with a generalized charge parameter $\tilde{Q}$. For ν in (7.5.20), a quadratic term in r may only appear as consequence of the potency term, *i.e.* from the relation between the amplitude of (7.5.5) and the effective mass parameter (7.5.14). An effective mass appears from an analogy to the Schwarzschild solution. This effective mass, though, depends on scalar-field contributions related to the pressure p.
The excitation of the scalar field for small mass $\tilde{M}$ and charge $\tilde{Q}$ in relation to the distance (*i.e.* beyond high-field regimes) now yields as follows,

$$\xi = \left[1 - \frac{2\tilde{M}G_N}{rc^2}\right]^{\frac{Ac^2}{2\tilde{M}G_N}} - 1\,. \tag{7.5.30}$$

It is exactly vanishing for the Schwarzschild metric ($A \equiv 0$). However, for a RN-like solution ($A \neq 0$), there is $\xi \gtrsim 0$ for the typical value $\tilde{M} > 0$ with $A < 0$.
Clearly, for a vanishing excitation parameter A, the Schwarzschild metric is valid. Negative values of A, on the other hand, lead to a positive field with a singular value at $r = 0$ and the tendency $\xi \to 0$ for spatial infinity.
The metric component (7.5.13) shows a RN-like form. Hence, up to second order e^λ vanishes for

$$r_\pm = \frac{2\tilde{M}G_N \pm \sqrt{(2\tilde{M}G_N)^2 - (2\tilde{Q}c)^2}}{2c^2}\,. \tag{7.5.31}$$

Given the vanishing of horizon for $A < 0$ according to [21], this gives a regime where the validity of approximation clearly breaks. However, it shows a changed behavior from usual quasi-Schwarzschild character for an almost flat metric towards the vanishing of the singularity for the exact solution. Second-order approximation has a Reissner–Nordström character and thus pretends following cases of behavior: (i) - extremal BH when $(\tilde{M}G_N)^2 = \tilde{Q}^2$ (for which the concentric event horizon becomes degenerate), (ii) - a naked singularity with $(\tilde{M}G_N)^2 < \tilde{Q}^2$, and (iii) - a Schwarzschild case for $(\tilde{M}G_N)^2 > \tilde{Q}^2$. The case (iii) also appears when the field excitations ξ vanish completely (for which $\tilde{Q}$ is zero exactly), which is clear from equations (7.5.13), (7.5.20) and (7.5.30).
Clearly, following [21,121] such degeneracy of the horizon as given in (i) is not given exactly. Nevertheless, within RN approximation for $A < 0$ and $B > 0$, only the case (iii) is possible indeed ($\tilde{Q}^2 < 0$), leading to a quasi-Schwarzschild behavior for low-field regimes. Nevertheless, the analogy to RN solutions is an interesting subject which reminds that for a massive object whose charge is not neutralized by further effects, the Schwarzschild radius itself loses its meaning of dominant property of the system. Here, the generalized charge is an intrinsic quality which affects the Schwarzschild radius itself, and the weakening of the latter appears indeed as consequence of the correction terms which already weaken gravitational fields for weak-field regimes (*cf.* figures). Taking this fact into account, it may be possible to establish measurably relevant distinctions of this induced-gravity model to usual dynamics even at long-scale regimes such as those of galactic bulges as well as relevant indications for intermediate regimes towards strong gravitational fields.
It may be established that in all orders, the evolution of gravitational potentials (*i.e.* the metric components) strongly depends on the possible relations between A and B. Such relations are helpful to understand how new physical correction terms act within low gravitational regimes in order to finally break the gravitational collapse onto a Grey Star.
Let us take $B > 0$ throughout for the purpose of our analysis. Further, let us now consider negative values (amplitudes) of A. Accordingly, the charge $\tilde{Q}$ is imaginary and its norm falling for positive scalar-field excitations. We take $B = 2$ for a Schwarzschild radius r_S given by $B = 2M_1G_N$ for the case $A = 0$ (with

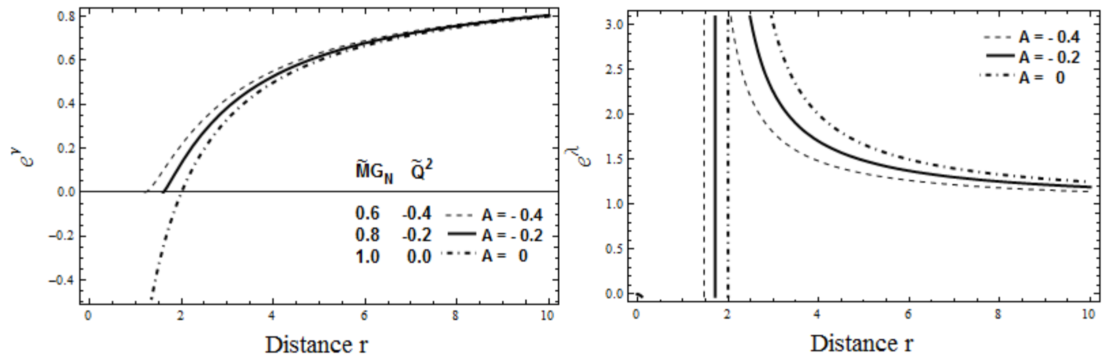

Figure 7.3: Evolution of the metric components in this model ($O(r^{-2})$) for different, (set) negative values of A with $B = 2$ and $M_1G_N/c^2 = 1$. *N.B.: The effective Schwarzschild radius diminishes for decaying values of A.*

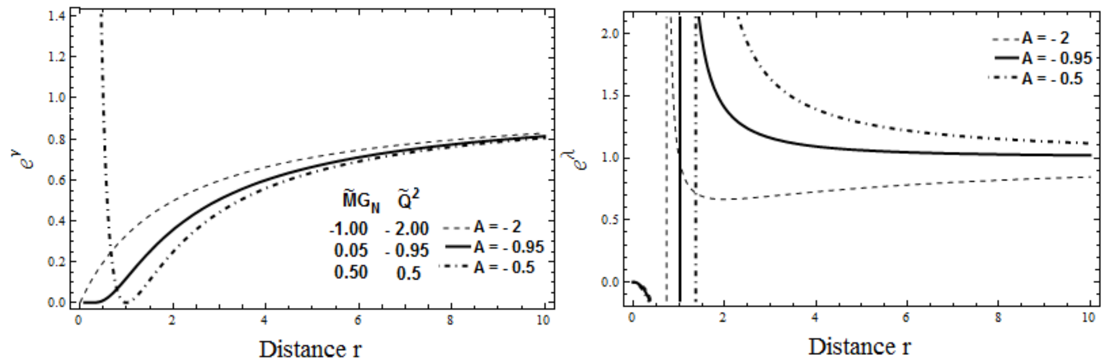

Figure 7.4: Evolution of the metric components in this model for higher negative amplitudes A with $B = 2$ (*v.s.*). *N.B.: There appears a quintessential attraction* (cf. *text) for* e^λ *at higher value of A.*

$M_1G_N = 1$). In Fig.7.3, it can easily be noticed that for very small excitation amplitudes A, Schwarzschild behavior appears almost exactly while for growing values of $-A$, *i.e.* of $|\tilde{Q}^2|$, the Schwarzschild radius diminishes (*i.e.* the singularity distance from the gravitating body center) decreases. A weakening of the horizon is tractable in view of [121] for a low ratio of the scalar-field parameter to the usual Schwarzschild radius ($A/B < 1$). Furthermore, the Schwarzschild radius is now given by the effective mass $\tilde{M}$ which has decreased (hence, we consider an effective radius). Therefore, it is clear that for low gravitational regimes, it is an effective, dynamical mass which is to take the role of bare, luminous mass. Further, for even higher-order terms of $-A$ (Fig.7.4), the curve of e^λ becomes flatter as the effective mass tends to zero. The rise of quintessential terms for the dominance of dynamical behavior is clear as the Schwarzschild mass is not a dominant term anymore. Actually, for mid-field regimes of high $|A|$ with decaying λ field towards the origin, quintessential attraction (*v.i.*) shows a behavior analogue to the exact one in [121]. Such behavior is hence interpretable by means of $\tilde{Q}^2$. For the value $A = -1$ and hence $\tilde{M}G_N = 0$, at large scales, we obtain a flat curve (however non-minkowskian near to $r_Q \equiv \sqrt{|\tilde{Q}^2|}$ where the approximation is broken. Yet, it is r_Q and not r_S itself which marks the singularity which pretends to appear at this order, analogously to RN solutions). Antigravitational properties appear for e^λ, exactly as happens for a RN case with mass and charge as given by the effective parameters as defined in equations (7.5.14) and (7.5.15) (in Fig.7.4 for

$\tilde{M}G_N/c^2 = 1$ and $\tilde{Q}^2 = -2$). This is still of relevance for low gravitational regimes.
Albeit showing a shift of Schwarzschild radius, e^ν does not show gravitationally repulsive behavior for $r > 2\tilde{M}G_N$ nor singular pretension at r_Q (which is not shown in [121] either). However, a weakening of gravitational collapse appearance is also visible, and the patterns of the exact qualitative behavior of e^ν according to [121] are here already noticeable, especially for large excitation parameters A.
In brief, for $A < 0$ the system appears as less massive than the related Schwarzschild system (where dynamics is given strictly by M_1). The case $A \leq -1$, however, is especially interesting in many aspects: It shows best the behavior for mid-strong regimes according to [21, 121]. Further, dynamically speaking, let us call such systems with $M_1 > 0$ and $\tilde{M} < 0$ as "quintessentially attracted". Quintessential because of the antigravitational behavior of e^λ following the negative effective (yet positive actual) mass. Attractive because e^ν still shows attraction of the gravitating body lying at $r = 0$. Its Schwarzschild radius, however, is vanishing. In this analysis, the role of the "charge" radius r_Q is important. A thorough discussion, though, needs of the values of A and B in terms of mass and pressure (*v.i*).

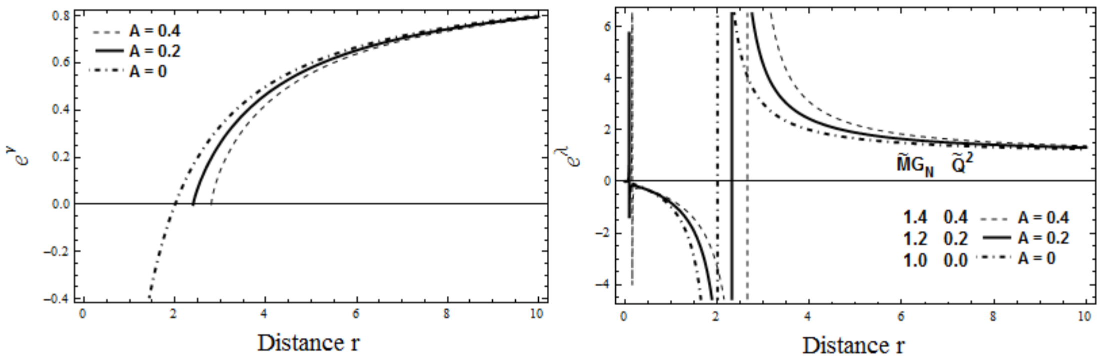

Figure 7.5: Evolution of the metric components for the positive values of A. *N.B.: The effective Schwarzschild radius augments for higher values of A.*

Let us consider positive amplitudes $A > 0$ with $B > 0$. Now, the gravitational field is strengthened and the effective Schwarzschild radius moves to $r > 2M_1G_N/c^2$, and gravitational attraction becomes greater as related to a relatively higher dynamical mass $\tilde{M} > M_1$. Here, the scalar-field excitation leads to a strengthening of the gravitational coupling (*cf.* Fig.7.5) and may thus be of special relevance in terms of Dark Matter.
Using (7.5.26) and (7.5.27), a closer look at e^λ and e^ν in dependence of w may be taken into account. In second approximation, for positive pressures p (Fig.7.6), the effective Schwarzschild radius decreases in respect to the one given by M_1, corresponding to the case $A < 0$ as discussed earlier. For stiff matter $w > 1/3$ ($A > 0$), $e^{-\nu}$ has lower values than e^λ. For negative pressures, on the other hand (see Fig.7.7), $w < -1/6$ leads to quintessential attraction for e^λ. For $w < -2/3$ there is also a gravitational repulsion. $\tilde{Q}^2$ is always smaller than $\tilde{M}$ unless for $w \lesssim -0.7$, for which e^ν is nearly flat.

7.6 Perihelion advance

Solar-relativistic effects need of higher-order corrections of the time-coordinate related to the metric component. Hence, we will consider the solution already derived in Chapter 7.5 (*cf.* [179]) for further analysis.

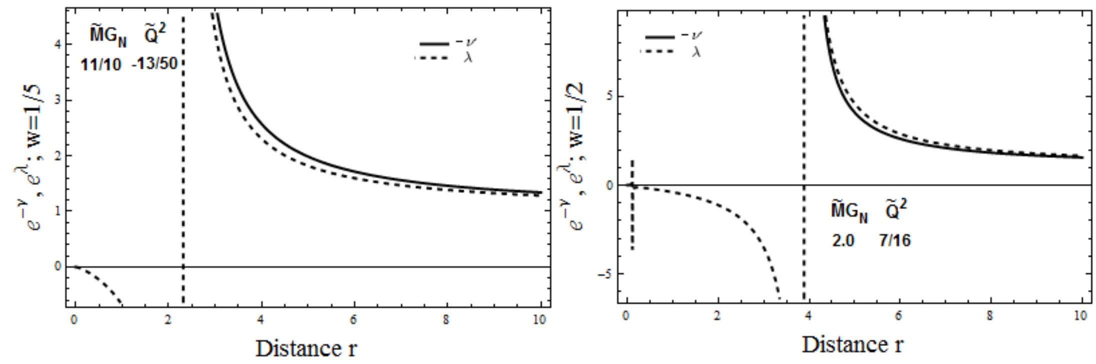

Figure 7.6: Evolution of $e^{-\nu}$ and e^{λ} for $w = 1/5$ and $w = 1/2$ with $M_1 G_N/c^2 = 1$. *Stiff matter* $w > 1/3$ *is related to positive squared charges* $\tilde{Q}^2 > 0$. *For* $w = 1/5$*, the dynamical mass for linear approximation reads* $M_{dyn} = 13/10$.

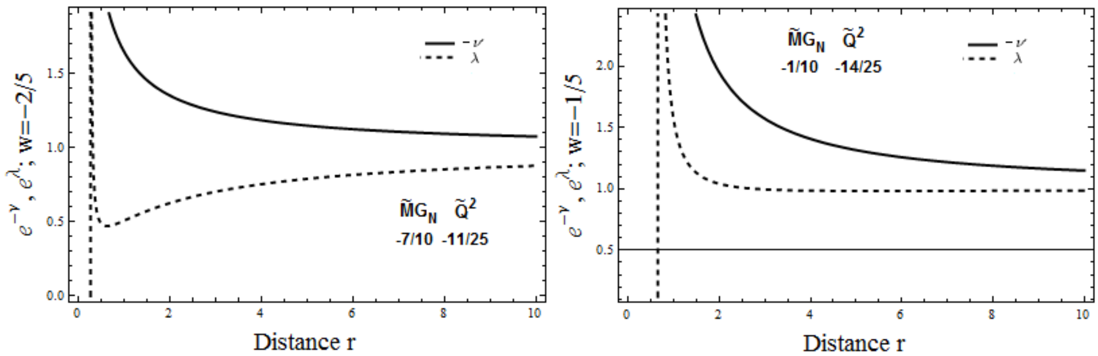

Figure 7.7: Evolution of $e^{-\nu}$ and e^{λ} for $w = -2/5$ and $w = -1/5$ with $M_1 G_N/c^2 = 1$. $w < 0$ *leads to quintessential attraction. The deviation between* $\tilde{M} G_N$ *and* $M_{dyn} G_N$ *is high. For* $w < 2/3$*, where* $M_{dyn} G_N < 0$ *is valid, counter-gravitative behavior appears, together with a naked singularity.*

It may however be compared to Chapter 7.2 in the light of the relation of mass parameters in the context of the mass coefficient $r_{dyn}/\tilde{r}_S$ which is unlike one for $A \neq 0$.
Parting from the case of small Schwarzschild radii in comparison to distance, let us take the result from Chapter 7.5 (*cf.* [179]) which shows a RN-like solution as given below (*cf.* [23]),

$$e^{\nu} = \left[1 - \frac{\tilde{r}_S}{r}\right]^{r_{dyn}/\tilde{r}_S} \quad ; \qquad e^{\lambda} = \left[1 - \frac{\tilde{r}_S}{r} + \frac{r_Q^2}{r^2}\right]^{-1} , \tag{7.6.1}$$

with the dynamical radius r_{dyn} of the linear approach, the effective Schwarzschild radius (with c explicitly)

$$\tilde{r}_S = 2A + r_{dyn} = \frac{2M_1 G_N}{c^2}\left(\frac{1}{2} + 3w\right) \approx h(w) r_{dyn}, \tag{7.6.2}$$

for the bare (luminous) mass M_1 and the squared generalized charge-parameter radius

$$r_Q^2 = |\tilde{Q}^2| = \frac{|A\tilde{r}_S|}{2}. \tag{7.6.3}$$

Geodesics are the applicable trajectories for the theory (*cf.* (6.3.16)). For a well-chosen system in order to get curves along a plane, for $\tilde{r}_S/r \ll 1$, equation (7.6.1) leads to a Lagrange function of geodesic motion of the following form for $\hat{q} = 1$,

$$\int \mathcal{L} d^3x = \frac{m}{2}\left[\left(1-\frac{\tilde{r}_S}{r}\right)^{r_{dyn}/\tilde{r}_S}\left(\frac{dx^0}{d\tau}\right)^2 - \left(1+\frac{\tilde{r}_S}{r}\right)\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\varphi}{d\tau}\right)^2\right], \tag{7.6.4}$$

with the eigentime τ and a cyclic coordinate φ and thus a constant conjugate momentum,

$$L = mr^2\frac{d\varphi}{d\tau} = mC_b^2 = constant, \tag{7.6.5}$$

as well as a cyclic coordinate $x^0 = ct$ so that

$$-m\left(1-\frac{\tilde{r}_S}{r}\right)^{r_{dyn}/\tilde{r}_S}\frac{dct}{d\tau} = mC_a = constant \tag{7.6.6}$$

is valid for a parametrized energy term. Consequently, equation (7.6.4) leads to following relation,

$$\left(1+\frac{\tilde{r}_S}{r}\right)\left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\varphi}{d\tau}\right)^2 - \left(1-\frac{\tilde{r}_S}{r}\right)^{r_{dyn}/\tilde{r}_S}\left(\frac{dct}{d\tau}\right)^2 = -c^2. \tag{7.6.7}$$

Using the definition $u = r^{-1}$ and $' = d/d\varphi$, with the insertion of (7.6.5) and (7.6.6), the equation (7.6.7) reads as follows,

$$-c^2 = (1+\tilde{r}_S u)C_b^2 u'^2 + C_b^2 u^2 - \frac{C_a^2}{(1-\tilde{r}_S u)^{r_{dyn}/\tilde{r}_S}}. \tag{7.6.8}$$

The relation between the effective and the dynamical radii reads

$$\frac{\tilde{r}_S - r_{dyn}}{\tilde{r}_S} = \frac{2A}{B}. \tag{7.6.9}$$

Hence, equation (7.6.8) reads for small Schwarzschild radii,

$$C_b^2 u'^2 + C_b^2 u^2(1-\tilde{r}_S u) - C_a^2(1-\tilde{r}_S u)^{2A/B} = -c^2(1-\tilde{r}_S u). \tag{7.6.10}$$

After a further derivative in φ, and taking small effective Schwarzschild radii, equation (7.6.10) leads to

$$u'' + u\left(1 - \frac{C_a^2}{C_b^2}A\frac{\tilde{r}_S^2}{r_{dyn}}\right) = \frac{3}{2}\tilde{r}_S u^2 + \frac{\tilde{r}_S}{2C_b^2}\bar{X}c^2, \tag{7.6.11}$$

with the parameter X dependent on C_a as follows,

$$\bar{X} = \left[1 - \frac{2A}{r_{dyn}}\frac{C_a^2}{c^2}\right]. \tag{7.6.12}$$

Clearly, for the linear (quasi-newtonian) approximation, (7.6.11) already leads to a trajectory which shows a perihelion shift dependent on the scalar field *via* $C_a^2 A\tilde{r}_S^2/(C_b^2 r_{dyn})$. For **low-energetic** systems, however, the newtonian Kepler orbit appears as first-order solution,

$$u_0 = \frac{\tilde{r}_S}{2C_b^2}c^2(1+\varepsilon\cos\varphi). \tag{7.6.13}$$

In the next-order approximation and only for linear terms in $\varepsilon\varphi$, there is

$$u_1 = \frac{\tilde{r}_S}{2C_b^2}c^2\left[1+\varepsilon\cos\left(1-\frac{3}{4}\frac{\tilde{r}_S c^2}{C_b^2}\right)\varphi\right]. \tag{7.6.14}$$

Equations (7.6.13) and (7.6.14) give the usual value only for $w = 1/6$. The perihelion advance for **low-energetic** systems is then obviously given by

$$\Delta\varphi_P = \frac{6\tilde{M}G_N}{C_b^2}\pi, \tag{7.6.15}$$

which is formally the usual value. It reads as usual for $w = 1/6$ so that $\tilde{M} = M_1$. At about such pressures, there is in $hr_{dyn} \approx \tilde{r}_S \approx r_S$. For higher pressures, effective and dynamical masses are higher than the luminous mass.

7.7 Effect of field excitations on the geodesic motion

We now try to analyze the singularities in view of the completeness of geodesics on the grounds of equation (7.1.1) with the metric components as mentioned in Chapter 7.5 (whereas the RN-like charge parameter basically arises because of the nonvanishing field excitations). Here we have $g_{\mu\nu}g^{\mu\nu} = -\varepsilon$ where $\varepsilon = 0$ and $\varepsilon = -1$ represent the constraints for the null and timelike geodesics respectively. Let us take $c = 1$. The geodesic equations corresponding to the metric (7.1.1) are given as follows,

$$\ddot{t} = -\dot{t}\,\dot{r}\,\nu', \tag{7.7.1}$$

$$\ddot{r} = -\frac{1}{2}e^{-\lambda}\left(-2r\left(\dot{\vartheta}^2 + \sin^2\vartheta\,\dot{\varphi}^2\right) + e^{\lambda}\,\dot{r}^2\,\lambda' + e^{\nu}\,\dot{t}^2\,\nu'\right), \tag{7.7.2}$$

$$\ddot{\vartheta} = -\frac{2}{r}\,\dot{r}\dot{\vartheta} + \cos\vartheta\sin\vartheta\ \ \dot{\varphi}^2, \tag{7.7.3}$$

$$\ddot{\varphi} = -\frac{2}{r}\,\dot{r}\dot{\varphi} - \cot\vartheta\ \dot{\vartheta}\,\dot{\varphi}, \tag{7.7.4}$$

where the dots and primes represent the differentiations with respect to the affine parameter τ and r respectively. Equation (7.7.1) has the solution $\dot{t} = E\,e^{-\nu}$, and using $\vartheta = \pi/2$ (equatorial plane) it leads to $\dot{\varphi} = L/r^2$ where E and L are integration constants. Now, using the constraint for timelike and null geodesics, we obtain

$$v^2 = \left(\frac{dr}{dt}\right)^2 = e^{-(\nu+\lambda)}E^2\left(1 - \frac{L^2\,e^{\nu}}{E^2\,r^2} + \frac{\varepsilon\,e^{\nu}}{E^2}\right). \tag{7.7.5}$$

However, for the tangential velocity, we get from the geodesics equations,

$$\Omega = \frac{d\varphi}{dt} = \frac{1}{r^2}e^{\nu}\left(\frac{L}{E}\right). \tag{7.7.6}$$

Using (7.7.5) and (7.7.6), we can write the angular velocity,

$$\left(\frac{d\varphi}{dr}\right) = \frac{L}{Er^2}\,e^{\frac{(\nu+\lambda)}{2}}\left(1 - \frac{L^2\,e^{\nu}}{E^2\,r^2} + \frac{\varepsilon\,e^{\nu}}{E^2}\right)^{-\frac{1}{2}}, \tag{7.7.7}$$

which counts the radial orbit changes.

Using (7.7.5) and first considering the RN-like charge parameter such that $|\tilde{Q}^2| \ll r^2$, an effective potential may be defined in the following way,

$$V_{eff} = \frac{\varepsilon\tilde{M}G_N}{r} + \frac{L^2}{2r^2} - \frac{\tilde{M}G_N L^2}{r^3}. \tag{7.7.8}$$

The equations (7.7.5) and (7.7.8) satisfy the following energy law,

$$\mathcal{E} = \left(\frac{dr}{dt}\right)^2 + V_{eff} = \frac{1}{2}\left\{\frac{\left(1-\frac{2\tilde{M}G_N}{r}\right)}{\left(1-\frac{2\tilde{M}G_N}{r}\right)^{\frac{B}{2\tilde{M}G_N}}}E^2 + \varepsilon\right\}. \tag{7.7.9}$$

In equation (7.7.9), for $A \ll B$ there is,

$$\frac{\left(1-\frac{2\tilde{M}G_N}{r}\right)}{\left(1-\frac{2\tilde{M}G_N}{r}\right)^{\frac{B}{2\tilde{M}G_N}}} = \left(1-\frac{2\tilde{M}G_N}{r}\right)^{\frac{2A}{B}-\frac{4A^2}{B^2}}. \tag{7.7.10}$$

The effective potential (7.7.8) has newtonian form for $r \to \infty$, and it possesses an extremal value for

$$r = -\frac{L^2}{2\varepsilon\tilde{M}G_N}\left[1 \mp \sqrt{1+\frac{3(2\tilde{M}G_N)^2\varepsilon}{L^2}}\right]. \tag{7.7.11}$$

The innermost stable circular orbit (ISCO) is then given by $r = 6\tilde{M}G_N$, which is related to $L/(\tilde{M}G_N) = \sqrt{12}$. For timelike geodesics the maximal momentum (L_x)–mass relation for the extremum is then given by

$$\frac{L_x^2}{(M_1G_N)^2} = 3(1+6w)\,. \tag{7.7.12}$$

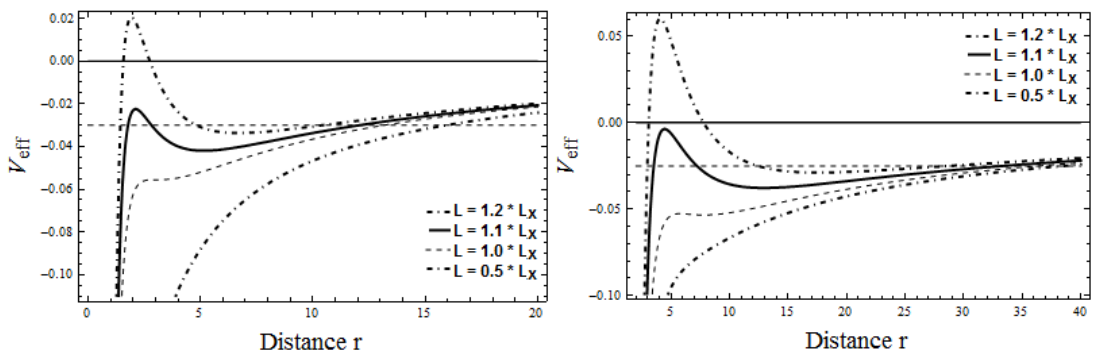

Figure 7.8: Timelike effective potential V_{eff} for $w = 0$ (left) and $w = 0.2$ (right) and different values of L and $M_1G_N = 1$. *N.B.: An orbit for an energy $\mathcal{E}$ equal to the maximum (minimum) is unstable (stable). At an energy given by the dashed horizontal line, for the thick curve there is a bound orbit in which the particle moves between two turning points.*

Bound states appear for high enough stiffness (given by the equation-of-state parameter w) and momentum given by L. For (parameterized) energies $\mathcal{E}$ below the maximum there appear stable bound states. For $\mathcal{E} < 0$ there are orbits which oscillate between two turning points, the perihelion and the aphelion (*cf.* Fig.7.8) as given in usual GR. The difference to usual GR is a dependence on w. Such dependence is in fact related to the difference between luminous and dynamical mass, tangential velocity and angular velocity. In fact, the presence of a scalar field for Quintessence generally changes the singularity of Black Hole solutions [240], and further, models of Quintessence usually predict long-range forces mediated by the fields [242] indeed, leading to different concepts of effective mass, especially as fluctuations of the scalar field which may behave similarly to relativistic gas [241] and/or be associated to halo mass of galaxies [240] (hence related to DM). Tangential and angular velocities are achieved by using equations (7.7.5) and (7.7.7) along with (7.5.20) and (7.5.13) such that ($c = 1$)

$$\left(\frac{dr}{dt}\right)^2 = \left[1 - \frac{2\tilde{M}G_N}{r}\right]^{\frac{B}{2\tilde{M}G_N}} \left[1 - \frac{2\tilde{M}G_N}{r} + \frac{\tilde{Q}^2}{r^2}\right] \left[1 - \frac{\left(1 - \frac{2\tilde{M}G_N}{r}\right)^{\frac{B}{2\tilde{M}G_N}} (L^2 - r^2\varepsilon)}{Er^2}\right], \tag{7.7.13}$$

$$\left(\frac{d\varphi}{dr}\right)^2 = \frac{L^2 \left[\left(1 - \frac{2\tilde{M}G_N}{r}\right)^{\frac{B}{2\tilde{M}G_N}} + \left(1 - \frac{2\tilde{M}G_N}{r} + \frac{\tilde{Q}^2}{r^2}\right)^{-1}\right]}{E^2 r^4 \left[1 - \frac{\left(1 - \frac{2\tilde{M}G_N}{r}\right)^{\frac{B}{2\tilde{M}G_N}}}{E^2 r^2} (L^2 - r^2\varepsilon)\right]}. \tag{7.7.14}$$

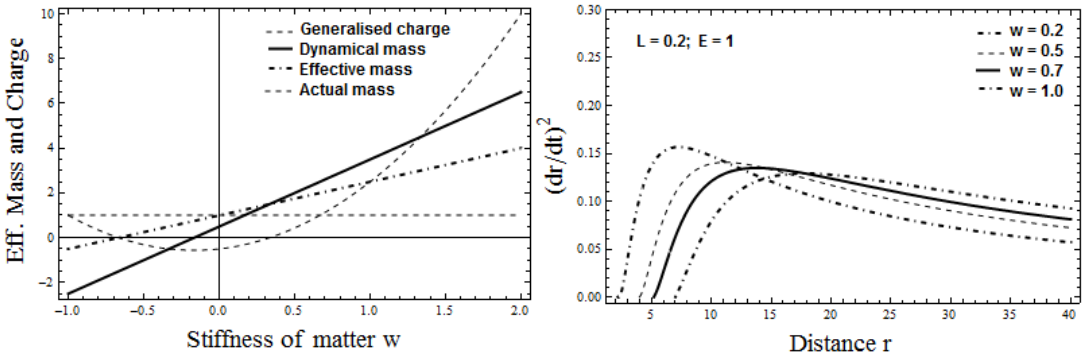

Figure 7.9: Left: Dashed generalized-charge ($\tilde{Q}^2$) plot (dashed curve) and dynamical, effective $\tilde{M}$ (dot-dashed) and M_{dyn} (dashed) and actual (density) mass $M_1 = 1$ (horizontal line) in dependence of stiffness w. $h(w)M_{dyn}$ as thick continuous line. Right: Tangential velocity $(dr/dt)^2$ (with $\varepsilon = -1$) for $M_1 G_N = 1$ and different eos parameters w.

For nonvanishing values of A there is a deviation between M_{dyn} and $\tilde{M}$, which is visible in $B/(2\tilde{M}G_N)$ (*viz* further [23]). As easily seen from the linear analysis and its relation with the general one, for $w = 0$, $\tilde{M}$ gives only half of the dynamical mass. This can be seen in Fig.7.9. $\tilde{M} \approx M_{dyn}$ as measured mass is valid for $w \approx 1/3$ (*i.e.* $A \approx 0$). The deviation between them then grows with stiffness (w) as $\tilde{M}$ grows more rapidly than M_{dyn}. Effective ($\tilde{M}$ and M_{dyn}) masses are then higher than the actual (density, luminous) mass because of pressure terms themselves. For higher stiffness of matter (pressure, inner structure), effective mass $\tilde{M}$ is higher than M_{dyn}. Within linear dynamics, it is the dynamical mass M_{dyn} which dominates, however with nonvanishing values of w according to a PPN framework. The actual effective mass is given for small pressures (w) approximately by

$$\tilde{M} = h(w)M_{dyn}, \tag{7.7.15}$$

with $\tilde{M}$ as actual effective mass for dynamics within an exact solution. At short distance to gravitational sources and for astrophysical considerations, it gives a measured mass which is unlike the bare, luminous mass from density (M_1). For $w = 1$, M_{dyn} is over 3 times higher than density mass M_1 and it gives only a third of the actual dynamical mass which is for $w = 1$ about 10 times higher than luminous (density) mass! Hence, it is reasonable to speculate about a relation to dark-matter phenomenology within this model

of gravity with Higgs potential.
Further, in Fig.7.9 the tangential velocity dr/dt shows a flattening behavior which is greater as greater the stiffness (parting from scalar fields generated purely by hadronic –mainly baryonic– matter and not by themselves, energy-density distribution and scalar field for exactly flat curves is given in [24] which too shows the relevance of inner structure of galaxies in relation to scalar-field excitations and the phenomenology of Dark Matter). Hence, assuming $R_0 M \ll 1$ with R_0 as a galaxy radius, flattened rotation curves can be obtained from (7.5.10) and (7.5.20) even with simple density profiles (*viz* Fig.7.9). Dynamics of galaxies are such as if dynamical mass were higher than the luminous mass. This is in principle the phenomenon of Dark Matter. The potency term $B/(2\tilde{M}G_N)$ in (7.5.20), together with the generalized RN charge $\tilde{Q}^2$ may lead phenomenologically to a halo of non-luminous (effective) matter surrounding a galaxy core. Fourth-order corrections, further, do not change these results much for the relevant values $r > \tilde{r}_S$.

7.8 Flat rotation curves

In [20], the author derived a galactic model for central symmetry of the scalar–tensor theory with Higgs potential in which flat rotation curves appear for a polytropic density of polytropic index 2 in which, parting from the galactic center, space farther than the galactic luminous disc is assumed as vacuum (*i.e.* with a negligible amount of matter in the galactic bars so that each gravitational body may be taken as surrounded by vacuum). Dark Matter phenomenology could be partly reproduced, however with a peak at $r = R_0$ (for a bulge radius R_0) which is not always observationally verified. Yet, a polytropic density distribution for galaxies is useful to achieve a satisfactory agreement between theoretical and empirical data, postulating or not postulating a central massive core for galaxies.
Further, in the latter Chapter (7.7), it was shown that under some circumstances, flat rotation curves are obtained from induced gravity directly, whereas the internal properties of the bulge are relevant in form of the equation-of-state parameter w as a factor of flattening. Actually, in Chapter 7.5, this is shown as a consequence of pressure terms which act as part of effective mass terms changing geodesic motion. For these dynamics, further, the constraint $w \approx 1/5$ for the usual equation-of-state parameter w does not have to hold, since GR dynamics are only to be valid at solar-relativistic ranges.
Flat rotation curves are usually related to the phenomenology of Dark Matter, as mentioned in Chapter 2.3. The cited work [20] leads in this direction. The comparison of the theoretical rotation curves with the rotation curves for several galaxies there indicates that the scalar–tensor theory with the Higgs Mechanism is able to explain and contribute to the flat rotation curves indeed.
Comparable approaches are a Freeman-disk profile as in [52] or a homogeneous mass distribution which then gives the solution for a point–particle when the radius R_0 of the gravitating body is taken as $R_0 \to 0$. Here, we will assume large distances r in relation to the radius R_0 so that the solution for a point–particle with inner structure (*i.e.* pressure which is related to the scalar-field excitation amplitude and which should, thus, not be neglected) will be given. The dark-matter profile for exactly flat rotation curves, for instance, was analyzed in [24], whereas Dark Matter density may be related to a pressure term which is related to the scalar field, the only source of which is usual hadronic matter. Hence, there may be a relation between p and Dark Matter *via* Higgs dynamics. We will go to some details of this in the next pages.
Density profiles for outside galaxy's bulges are usually taken such that they lead to flattening of rotation curves. Thus, they are called DM profiles. Usual profiles are the Navarro–Frenk–White (NFW) [181] and

the (more general) Dehnen or γ profile [72]:

$$\text{NFW: } \hat{\epsilon}(r) \propto \frac{1}{r\left(1+\frac{r}{r_s}\right)^2}, \tag{7.8.1}$$

$$\text{Dehnen: } \hat{\epsilon}(r) \propto \frac{r_s}{r^\gamma (1+r_s)^{4-\gamma}}, \tag{7.8.2}$$

r_s is the profile radius which is a length scale of the spherical system. Clearly, Dehnen's model with $\gamma = 2$ is related to the NFW profile. These models seek for universal halo densities in the context of flat rotation curves. Hence, such are often called universal halo density laws.
Used within a scalar–tensor theory, such densities may be used not as an alternative to Dark Matter in order to solve the missing mass problem but to compute the influence of scalar fields in rotation curves and velocity dispersions with galaxies which possess NFW or Dehnen profiles, for instance (*cf.* [210] for a linear analysis with γ density). Then, universal density laws lead to higher rotation curves than within GR, given mass properties of the nonminimally coupled scalar field. Hence, in that spirit, the present approach does not intend to reproduce rotation and dispersion curves for well-known densities but to derive the necessary density profile for flat rotation curves as an alternate model to CDM indeed but with the scalar field associated with Dark Matter phenomenologically and hence acting as a density contribution the non-newtonian dynamics of which are dominant at galactic ranges.
Let us consider the weak fields for galactic ranges and the tangential velocity of galaxies as given below,

$$v_t = \sqrt{r\frac{d\Phi}{dr}}. \tag{7.8.3}$$

Now, the Poisson equation may be written as follows,

$$\Delta\left(\Phi + \frac{c^2}{2}\xi\right) = \frac{3\pi G_N}{c^2}(\epsilon + 3p). \tag{7.8.4}$$

The scalar-field equation reads

$$\Delta\xi - \frac{1}{l^2}\xi = -\frac{2\pi G_N}{c^4}(\epsilon - 3p). \tag{7.8.5}$$

Phenomenologically, rotation velocity of especially spiral galaxies is nearly constant (problem of flat rotation curves) outside the luminous core as if a spherical halo of nonluminous matter with an extension much greater than the galaxy's visible disc surrounded them (*cf.* [187]). Hence, assume now that the rotation velocity is constant so to analyze the necessary conditions for such case. Then, the gravitational potential to give flat rotation curves is of the following form,

$$\Phi = v_t^2 \ln(r). \tag{7.8.6}$$

The Poisson equation of the model (7.8.4) together with the scalar-field equation (7.8.5) leads to

$$\frac{v_t^2}{r^2} + \frac{c^2}{2l^2}\xi = \frac{4\pi G_N}{c^2}\hat{\epsilon}. \tag{7.8.7}$$

Equation (7.8.7) defines a density profile which is the following,

$$\hat{\epsilon} = \epsilon + \frac{3}{2}p = \frac{(v_t c)^2}{4\pi G_N r^2} + \frac{\xi c^4}{8\pi G_N l^2}. \tag{7.8.8}$$

It possesses, on the one hand, a contribution ϵ of matter density in general and a contribution p of pressure (coming from the inner structure of matter). On the other hand, it possesses a newtonian-type energy density and a scalar-field contribution to density distribution. Hence, we define two energy-density components as follows,

$$\epsilon^* = \frac{(v_t c)^2}{4\pi G_N r^2}, \tag{7.8.9}$$

$$\epsilon_\xi = \frac{\xi c^4}{8\pi G_N l^2}. \tag{7.8.10}$$

Both ϵ^* and ϵ_ξ together give the density profile usually called dark-matter profile ϵ_{DM} (*cf.* [51], [24]). In these terms, the scalar-field contribution (ϵ_ξ) shall act as dark-matter density contribution to the total energy density. The other contribution (ϵ^*) is purely newtonian and represents energy density especially of baryons. Furthermore, the scalar field cannot be its own source, which means that it would have only usual-matter density (ϵ^*) as source. Hence, for equation (7.8.5) there must be

$$\Delta\xi - \frac{1}{l^2}\xi = -\frac{2\pi G_N}{c^4}\epsilon^*. \tag{7.8.11}$$

Accordingly, pressure is given by

$$p = \frac{2}{9}\epsilon_\xi\,. \tag{7.8.12}$$

The pressure is linearly dependent on the scalar-field density and on the scalar field itself. The scalar-field excitation is hence given in such cases as follows,

$$c^2\xi = 36\pi G_N l^2 p\,. \tag{7.8.13}$$

Phenomenologically, there is a relation of about ten to one between hadronic matter (ϵ^*) and Dark Matter. According to the equations (7.8.9) and (7.8.10), Dark Matter is given by the scalar-field contribution of density. Hence, a relation as the one following should be valid:

$$\epsilon_\epsilon \approx 10 \cdot \epsilon^*, \tag{7.8.14}$$

whereas the total energy density $\hat{\epsilon}$ for dark-matter density profile is given according to equation (7.8.8). Interestingly, following the equation (7.8.12), the relation between total energy density $\hat{\epsilon}$ and pressure gives an equation-of-state parameter as follows,

$$\hat{w} = \frac{p}{\hat{\epsilon}} \approx \frac{1}{5}\,. \tag{7.8.15}$$

For large, galactic scales, hence, $\hat{w}$ is given by the Dark Matter contribution which comes from the scalar field. Furthermore, for vanishing contributions of the scalar field, $p/\hat{\epsilon}_\xi$ tends to zero, and for $\epsilon^* = 0$, *i.e.* for a complete dominance of the scalar-field excitation, the total equation-of-state parameter reads exactly 1/4.5. A baryonic density of 1/9 of the scalar-field density leads to $\hat{w} = 1/5$. Astonishingly, these values which are necessary within dark-matter phenomenology of flat rotation curves are comparable to the equation-of-state parameter w within the context of solar-relativistic effects, and especially within the linear approach as well as where density is mainly given by usual matter ϵ^* and newtonian dynamics. Apparently, an equation-of-state parameter of about 1/5 is a weak-field constraint not only for solar-relativistic effects but also within dark-matter phenomenology derived from the present induced-gravity model with a Higgs potential. The behavior of the contributions of pressure, however, differ for both cases. We will now investigate the behavior of density components for galactic dynamics.

After parametrizing distance by a length scale a of the spherical system (a length related to the distance at which galaxies possess flat rotation curves), in the interval between $r = 0$ and $r = r_h$ with r_h as halo radius with $r_h > l$ and $r_h > a$ (see [24], *cf.* [51]), the solution of the scalar field reads

$$\xi = \frac{1}{2r_a}\frac{v_t^2}{c^2}\left[e^{-\frac{r_a}{l_a}}\,\mathrm{Shi}\left(\frac{r_a}{l_a}\right) - \sinh\left(\frac{r_a}{l_a}\right)\mathrm{Ei}\left(-\frac{r_a}{l_a}\right)\right], \tag{7.8.16}$$

whereas $r/a = r_a$ and $l/a = l_a$. Shi(x) is the hyperbolic sine integral function (*i.e.* SinushIntegral(x)) and Ei(x) is the exponential integral function. For the dark-matter profile (total density distribution), there is

$$\hat{\epsilon} = \frac{(v_t c)^2}{4\pi G_N a^2}\left\{\frac{1}{r_a^2} + \frac{1}{4l_a^2 r_a}\left[e^{-\frac{r_a}{l_a}}\,\mathrm{Shi}\left(\frac{r_a}{l_a}\right) - \sinh\left(\frac{r_a}{l_a}\right)\mathrm{Ei}\left(-\frac{r_a}{l_a}\right)\right]\right\}. \tag{7.8.17}$$

It gives the halo structure in a way analogue to NFW or Dehnen profiles with scale radii $r_s = a$ [72, 181].

The scale radius a is of the order of magnitude of a galactic core R_0 (*i.e.* the luminous-disc radius of

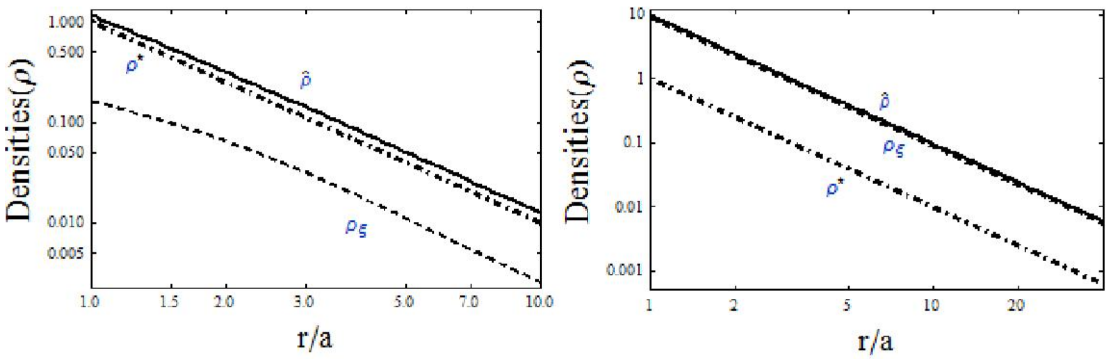

Figure 7.10: Evolution of density distributions for $l_a = 1/5$ (left) and $l_a = 1/35$ (right). *N.B.: Scalar-field (ϵ_ξ) dominance for shorter distances, and baryonic (ϵ^*) dominance for limits of large scales.*

galaxies), and the scalar field is negligible for too high a value of l of the order of magnitude of a. If the length scale l is lower than a, though, from short distances up to some times the scale a, then there is a dominant contribution of the scalar field, as may be seen in Fig.7.10. For longer distances (*viz* Fig.7.10 left panel), usual matter (ϵ^*) dominates the dynamics within the total energy density $\hat{\epsilon}$. Thus, there is dust-matter dominance of the Universe.

Let us now define the ratio of density parameter,

$$\Delta \equiv \hat{\epsilon}/\epsilon^* = 1 + \frac{r_a}{l_a^2}\left[e^{-r_a/l_a}\mathrm{Shi}(r_a/l_a) - \sinh(r_a/l_a)\mathrm{Ei}(-r_a/l_a)\right]. \tag{7.8.18}$$

The density ratio gives non-baryonic behavior ($\Delta - 1 \neq 0$), and it shows three special behavior cases. At lower scales (as shown in the right panel of Fig.7.10), a linearly growing function with relatively high slope, at high scales a constant value, and an intermediate phase with a maximum (*ad loc.* figure 7.11 right). For all length scales l_a, the nonbaryonic behavior $\Delta - 1$ is negligible at shorter ranges, even though scalar-field densities do dominate. Hence, the dominant scalar-field contribution of density acts as a baryonic contribution for shorter distances (even $r_a > 1$). Newtonian behavior dominates at short ranges.

For $l/a \approx 35$ (Fig.7.11 left panel), there is $\Delta \approx 10$ (*i.e.* long-range dynamics are as if there were 10

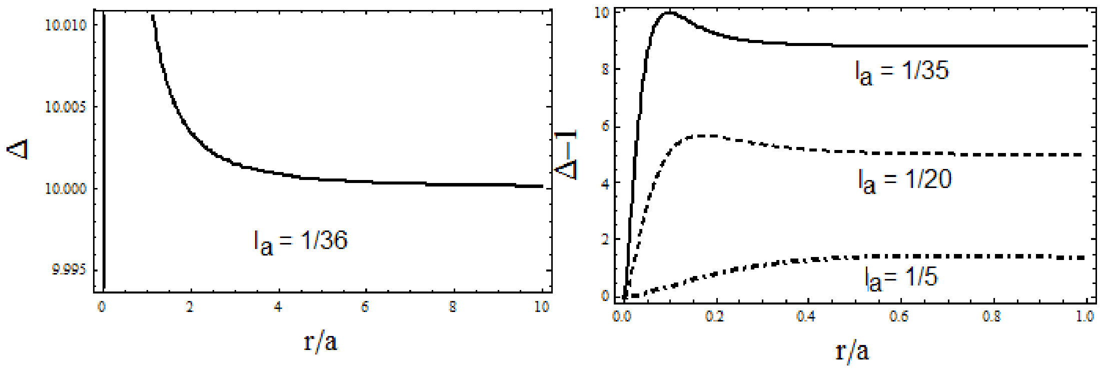

Figure 7.11: Density ratios: *Dark Matter dominance for* $l_a = 1/36$ *(left) and Non-newtonian behavior (right) for* $l_a = 1/5$, $l_a = 20$ *and* $l_a = 35$.

times the baryonic density). There is scalar-field density (ϵ_ξ) dominance at distances of galactic bars, and the relation $\hat{w} = 1/5$ is thus valid and non-newtonian behavior of the scalar field is dominant for flattening dynamics of galaxies. Dark Matter behavior appears at long ranges.

Chapter 8

Friedmann–Robertson–Walker metric

– The model of induced gravity with Higgs potential is analyzed for the Friedmann–Lemaître cosmology with Robertson–Walker symmetry in virtue of Dark Matter and Quintessence from generalized Friedmann equations. Signatures for the primeval Universe and Inflation are also discussed. Introductory aspects may already be found published in [24]. –

8.1 The generalized Friedmann equations and the Hubble parameter

Let us now take a look at the Friedmann–Lemaître–Robertson–Walker (RW) metric, used for general cosmology and cosmic evolution:

$$ds^2 = (cdt)^2 - a(t)^2 \left[d\chi^2 + f(\chi)^2 \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)\right] . \tag{8.1.1}$$

Here, χ is the covariant distance, $a(t)$ is the scale parameter (else, many times found especially as R), $K \in \{1, 0, -1\}$ is the curvature constant and $f \in \{\sin\chi, \chi, \sinh\chi\}$ is a parameter that depends on K. This last symmetry is based on the long-range well-realized assumption that the cosmos is homogeneous and isotropic (the Cosmological Principle).
In the following, let us write down the Einstein and scalar-field equations for this metric. These will be a Higgs-like equation and generalized forms of the Friedmann–Lemaître (or simply Friedmann) equations which have a new term of the scalar-field excitations ξ and derivatives of the same. Further, since these excitations lead to the effective coupling $\tilde{G}$, the whole set of equations can also be written in terms of changings of the gravitational coupling instead of the scalar field. In this way, effects on gravity of the nonminimal coupling with ξ may be clearer.

- **Equations in dependence of ξ:**
  On the grounding of the RW metric, the continuity condition (6.3.16) for the energy density $\epsilon = \varrho c^2$ takes the following form,

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = (1 - \hat{q})\frac{1}{2}\frac{\dot{\xi}}{1 + \xi}(\epsilon - 3p) \ . \tag{8.1.2}$$

  The total energy is conserved and the scalar field produces no entropy process for $\hat{q} = 1$, other than with $\hat{q} = 0$. In the latter case, however, such processes become minimal when the effective

gravitational coupling tends to a constant behavior, *i.e.* for scalar fields with tendency to a constant term.
For a barotropic pressure $p = w\epsilon$, equation (8.1.2) may further be written as

$$\dot{\epsilon} = -3H(1+w)\epsilon + (1-\hat{q})\frac{1}{2}\frac{\dot{\xi}}{1+\xi}(1-3w)\epsilon, \tag{8.1.3}$$

with the Hubble parameter $H = \dot{a}/a$.
The Higgs-like field equation for (8.1.1) reads

$$\ddot{\xi} + 3\frac{\dot{a}}{a}\dot{\xi} + \frac{c^2}{l^2}\xi = \frac{8\pi}{3}\frac{G_0}{c^2}\hat{q}\frac{(\epsilon - 3p)}{(1+\frac{4\pi}{3\breve{\alpha}})} \,. \tag{8.1.4}$$

As expected, its source vanishes for $\hat{q} = 0$, and in such case Higgs particles only interact gravitationally.
With (8.1.4) and (6.3.22), equation (8.1.1) leads to generalized Friedmann equations in forms independent on the source parameter $\hat{q}$. Explicitly, they read

$$\begin{aligned} \frac{\dot{a}^2 + Kc^2}{a^2} &= \frac{1}{1+\xi}\left[\frac{8\pi}{3}\frac{G_0}{c^2}\left(\epsilon + V(\xi)\right) - \frac{\dot{a}}{a}\dot{\xi} + \frac{\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right] \\ &= (1+\xi)^{-1}\left[\frac{8\pi}{c^2}\frac{G_0}{3}\epsilon + \left(\frac{c^2}{4l^2}\xi^2\left(1+\frac{4\pi}{3\breve{\alpha}}\right) - \frac{\dot{a}}{a}\dot{\xi} + \frac{\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right)\right], \end{aligned} \tag{8.1.5}$$

and

$$\begin{aligned} 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + Kc^2}{a^2} &= -\frac{1}{1+\xi}\left[\frac{8\pi G_0}{c^2}\left(p - V(\xi)\right) + \ddot{\xi} + 2\frac{\dot{a}}{a}\dot{\xi} + \frac{\pi}{\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right] \\ &= -8\pi\frac{G_0}{1+\xi}\frac{p}{c^2} - (1+\xi)^{-1}\left[\ddot{\xi} - \frac{3c^2}{4l^2}\xi^2\left(1+\frac{4\pi}{3\breve{\alpha}}\right)\right] - \\ &\quad - 2\frac{\dot{a}}{a}\frac{\dot{\xi}}{1+\xi} + \frac{\pi}{\breve{\alpha}}\left(\frac{\dot{\xi}}{1+\xi}\right)^2, \end{aligned} \tag{8.1.6}$$

with density distribution ϱ, energy-density distribution $\epsilon = \varrho c^2$ and pressure p and the cosmological function $\Lambda(\xi)$. Further, they may be rewritten onto the following form which is helpful for some analyses (see equation (8.2.7)):

$$\frac{\dot{a}^2}{a^2} + \frac{Kc^2}{a^2} = \frac{8\pi}{3}\frac{\tilde{G}}{c^2}\epsilon + f_1 + \frac{\Lambda}{3}c^2 \tag{8.1.7}$$

and (see (8.2.8))

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + Kc^2}{a^2} = -8\pi\tilde{G}\frac{p}{c^2} + \Lambda c^2 + f_2 \,. \tag{8.1.8}$$

Here we find the cosmological function Λ and correction terms f_1 and f_2 to the usual Friedmann equations of usual GR. These correction terms read as follows,

$$f_1(t) \equiv f_1 = -\frac{\dot{a}}{a}\frac{\dot{\xi}}{1+\xi} + \frac{\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2}{(1+\xi)^2} \tag{8.1.9}$$

and

$$f_2(t) \equiv f_2 = -\frac{\ddot{\xi}}{1+\xi} - 2\frac{\dot{a}}{a}\frac{\dot{\xi}}{1+\xi} - \frac{\pi}{\breve{\alpha}}\frac{\dot{\xi}^2}{(1+\xi)^2} \,. \tag{8.1.10}$$

The cosmological function $\Lambda \equiv \Lambda(\xi)$ reads

$$\Lambda = 8\pi \frac{\tilde{G}}{c^4} V = \frac{3}{4l^2} \frac{\xi^2}{1+\xi} \left(1 + \frac{4\pi}{3\breve{\alpha}}\right), \tag{8.1.11}$$

with the scalar-field potential

$$V(\xi) \equiv V = \frac{3\, c^4 \xi^2}{32\pi G_0\, l^2} \left(1 + \frac{4\pi}{3\breve{\alpha}}\right) \tag{8.1.12}$$

and the effective gravitational coupling

$$\tilde{G} \equiv G(\xi) = \frac{G_0}{1+\xi}\,. \tag{8.1.13}$$

In form, Λ takes the place of the cosmological constant Λ_0 of usual GR. Whether it acts as one or not will depend on its dependence on the scale parameter, *i.e.* on time. In case we mean the cosmological constant, Λ will, as throughout, be subscripted as Λ_0. Furthermore, under the assumption of a vanishing scalar field and its derivatives, of course, all corrections and cosmological function vanish. The cosmological function appears as correction for nonvanishing excitations of the scalar field. However, further corrections appear as consequence of its time dependence and hence of the ones of the gravitational coupling itself. The properties of such contributions are analyzed further on in this Chapter.
Let us first take the cosmological function. It is related to the length scale of the scalar field and to the value of the field excitation itself. Thus, the length scale can be given as follows,

$$l^2 = \frac{3}{4\Lambda} \frac{\xi^2}{1+\xi} \left(1 + \frac{4\pi}{3\breve{\alpha}}\right). \tag{8.1.14}$$

For $\breve{\alpha} \gg 1$, high excitations $\xi \gg 1$ are given by

$$\xi = \frac{4}{3} l^2 \Lambda. \tag{8.1.15}$$

For $\Lambda \ll 10^{-50}\mathrm{cm}^{-2}$, in a case as (8.1.15), however, $l^2 \gg \Lambda$ has to be given. There would be

$$l^2 \gg 10^{50}\mathrm{cm}^2 \quad (\text{for } \xi \gg 1). \tag{8.1.16}$$

Hence, we may declare:

– **If Λ gives the dominant term of the measured cosmological constant (*cf.* [198, 208]), this gives a constraint on the length scale** (see density parameters in Chapter 8.4).

However, this constraint is strongly dependent on the value of ξ, since **low excitations constrain l to lower values** in the case of Λ being dominant. For low excitations and $\Lambda = 10^{-50}$, there is $l < 10^{25}$cm (*i.e.* less than 10kpc).
An important cosmological parameter is the so-called deceleration parameter q. It is defined by

$$q = -\frac{\ddot{a}}{aH^2}\,. \tag{8.1.17}$$

It can be found in the Friedman equations, related to $\ddot{a}$. Both equations (8.1.5) and (8.1.6) give, with $V(\xi) \equiv V$,

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi}{c^2} \frac{\tilde{G}}{3} (\epsilon + 3p - 2V) + f \\ &= -\frac{4\pi}{3} \frac{\tilde{G}}{c^2} (\epsilon + 3p) + \frac{1}{3}\Lambda c^2 + f \\ &= -\frac{4\pi}{3} \frac{G_0}{c^2} (\epsilon^* + 3p^*) + \frac{1}{3}\Lambda c^2 + f, \end{aligned} \tag{8.1.18}$$

with effective density and pressure

$$\epsilon^* = \epsilon/(1+\xi) \quad \text{and} \quad p^* = p/(1+\xi) \tag{8.1.19}$$

and

$$f \equiv f(t) = \frac{1}{2}(f_2 - f_1) = -\frac{1}{2}(1+\xi)^{-1}\left[\ddot{\xi} + \frac{\dot{a}}{a}\dot{\xi} + \frac{4\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right]. \tag{8.1.20}$$

Apart from the cosmological function term Λ and the screening of densities and pressures through the effective gravitational coupling, the term f gives the corrections of the theory, and especially those caused by the time-dependence of the scalar field (and hence of Λ) itself.
Further, using the scalar-field equation, (8.1.20) yields

$$f(t) = \frac{1}{2}(1+\xi)^{-1}\left[2\frac{\dot{a}}{a}\dot{\xi} + \frac{c^2}{l^2}\xi - \frac{8\pi}{3}\frac{G_0}{c^2}\hat{q}\frac{\epsilon - 3p}{\left(1+\frac{4\pi}{3\breve{\alpha}}\right)} + \frac{4\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right]. \tag{8.1.21}$$

Furthermore, in the case $\hat{q} = 1$ for high valued scalar-field length scales, it is known that the newtonian gravitational constant is given by a parameterized coupling constant G_0. Hence, for high length scales as are expected here, introduction of Newton's gravitational constant together with a maintenance of a canonical form of the equations may lead to the introduction of

$$\frac{3}{4}\epsilon = \tilde{\epsilon} \quad \text{and} \quad \frac{3}{4}p = \tilde{p}, \quad (l \to \infty, \text{ and } \hat{q} = 1) \tag{8.1.22}$$

so that (*cf.* equation (8.2.8))

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\frac{G_N}{c^2}\left(\tilde{\epsilon}^* + 3\tilde{p}^*\right) + \frac{1}{3}\Lambda c^2 + f \tag{8.1.23}$$

may be written, in canonical form with cosmological term Λ and correction term f. However, ϵ and p are the actual density and pressure.
In the case of a neglection of cosmological corrections, measured terms $\tilde{\epsilon}$ and $\tilde{p}$ would be effective values for which density-like and pressure-like terms coming from Λ and f are taken as part of density and pressure and not as scalar-field terms. In such a case, ξ is partly to be taken as a contribution of matter density. Hence, quintessential cosmological properties coming from it, for instance, would appear as matter-caused (see Chapters 8.3 and 8.6).
Apart from the fact of G and Λ being functional, it is f which makes (8.1.18) formally different to the usual equation in GR, where there is acceleration ($q < 0$) merely for $\Omega_\Lambda > \Omega/2$ (see (8.4)). This new term gives the changes of dynamics caused by the time dependence of the effective coupling constant and (together with the correction to the first Friedmann equation) it can be compared to an analog function derived within Modified Gravity (MOG) by Moffat [168], but here with a functional cosmological term Λ and defining a scalar field which we might write as $x = 1 + \xi$ for a direct comparison with [168].[1]
With the assumption of an equation of state with barotropic equation-of-state parameter,

$$w = \frac{\varrho\, c^2}{p} = \frac{\epsilon}{p}\,, \tag{8.1.24}$$

[1] In [166], there is, for instance, $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\varrho(1+3w) + \varrho_K(1+3w) + \varrho_V(1+w_V) + \varrho_\Lambda(1+3w_\Lambda) + \varrho_G(1+3w_G)\right] + \frac{\ddot{G}}{2G} - \frac{\dot{G}^2}{G^2}$, with usual matter density ϱ, density related to the scalar-field potential, ϱ_V, density from the cosmological function, ϱ_Λ, and density related to $H\dot{G}$ terms, ϱ_G (*cf.* equation 8.1.57 with 8.1.58). Similarly, there is $H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\left[\varrho + \varrho_K + \varrho_V + \varrho_\Lambda + \varrho_G\right]$.

the solution for the density is given by (see [47])

$$\epsilon_w = \frac{M_w\, c^2}{a^{3(1+w)}}\,(1+\xi)^{\frac{1}{2}(1-3w)(1-\hat{q})}\,. \tag{8.1.25}$$

Herewith, M_w is an integration constant which we parameterize explicitly by a subscript w. In this way, it is possible to analyze different matter-type behavior and matter dominance. Consequently, we write a subscript w in ϵ to show it as the energy density related to a specific kind of matter or matter dominance with integration constant M_w.

Matter dominance as given in the hodiernal Universe is gotten with the parameter $w = 0$ (dust-matter).[2] Then there is a dependence as follows,

$$\epsilon \sim a^{-3}, \quad \text{(matter dominance)}, \tag{8.1.26}$$

while radiation-dominated epochs are given by $w = \frac{1}{3}$ such that

$$\epsilon \sim a^{-4}, \quad \text{(radiation dominance)}. \tag{8.1.27}$$

Anti-stiff matter with $w = -1$ represents dark-energy-interacting matter, and it is related to a cosmological constant,

$$\epsilon \sim a^{0}, \quad \text{(dark energy)}. \tag{8.1.28}$$

$w = -1$ is thus related to a density which evolves independently of the scale factor a. This density is constant in time, related to $\Lambda_0 = const.$

For $\hat{q} = 1$, the usual relation of the standard Friedmann models is valid,

$$\epsilon_w a^{3(1+w)} = const. = \epsilon_{0w} a_0^{3(1+w)}. \tag{8.1.29}$$

For $\hat{q} = 0$, an analog, however, changed relation is valid (*cf.* equation (8.1.60)):

$$\epsilon_w\,(1+\xi)^{\frac{1}{2}(1-3w)}\,a^{3(1+w)} = const. = \epsilon_{0w}\,(1+\xi_0)^{\frac{1}{2}(1-3w)}\,a_0^{3(1+w)}\,, \tag{8.1.30}$$

Time $t = t_0$ usually means to be the one of the current Universe (of the observer). a_0 is the observer's current scale factor $a(t_0)$. ϵ_{0w} is the observer's energy density $\epsilon_w(t_0)$ for w-typed matter.[3]

Consequently, the integration constant M_w is related to a_0 in following way for $\hat{q} = 1$ (*cf.* equation (8.1.61)):

$$M_\alpha = a_0^{3(1+w)}\epsilon_{0w}\,, \tag{8.1.31}$$

and for $\hat{q} = 0$ and $w \neq 1/3$,

$$M_w = \frac{a_0^{3(1+w)}}{(1+\xi_0)^{\frac{1}{2}(1-3w)}}\epsilon_{0w}\,. \tag{8.1.32}$$

As commonly known, signs measured by an observer at a time t_0 from astronomical objects such as distant galaxies at a generic time t are redshifted in relation to the originally sent ones. This redshift

$$z = \frac{\lambda_0 - \lambda_{emission}}{\lambda_{emission}} \tag{8.1.33}$$

[2] In general, the hodiernal Universe will be taken as dusty. Quintessential (anti-stiff) properties will be taken as coming not basically from usual matter itself, *i.e.* not from matter density, but from the scalar field leading to a total dark-energy dominance.

[3] The subscript w may usually be let aside and the parametrization be implicitly given.

is related to the scale factor by

$$1 + z = \frac{a_0}{a}. \tag{8.1.34}$$

Therefore, the relation between a density at generic time and the one at the point of observation (usually, the present time) may be expressed by the redshift with

$$\epsilon = \epsilon_0 \left(\frac{\tilde{G}_0}{\tilde{G}} \right)^{\frac{1}{2}(1-3w)(1-\hat{q})} (1+z)^{3(1+w)}. \tag{8.1.35}$$

The redshift parameter z is the redshift as measured at the present time $t = t_0$.
For $t < t_0$, there is $a < a_0$ and a cosmic expansion leads to a redshift of the sent waves.
From (8.1.2) and after definition of the Hubble parameter as a measure of cosmic expansion,

$$H(t) \equiv H := \frac{\dot{a}}{a}\,, \tag{8.1.36}$$

the Hubble parameter may be directly given by

$$c^2 H = -\frac{1}{3}\dot{\epsilon}\,(\epsilon + p)^{-1} + \frac{\dot{\xi}\,c^2}{6}(1-\hat{q})(1+\xi)^{-1}\frac{\epsilon - 3p}{(\epsilon + p)}\,, \tag{8.1.37}$$

It not only depends on the time derivative of density but may also depend on the one of the scalar-field excitation, together with a term from density and pressure themselves. For the last term of the right-hand side of equation (8.1.37), for $p \ll \epsilon$, there is

$$\frac{\varrho + 3\frac{p}{c^2}}{\left(\varrho + \frac{p}{c^2}\right)} \approx 1 + 2\frac{p}{\epsilon} - 2\frac{p^2}{\epsilon^2} + \ldots \tag{8.1.38}$$

Equation-of-state terms may then be neglected for the last term of the right-hand side of equation (8.1.37) for $p \ll \epsilon$.
Insertion of equation (8.1.24) into (8.1.37) leads to the latter in the following form (with w explicitly),

$$c^2 H_w = -\frac{1}{3}\frac{\dot{\epsilon}}{\epsilon}(1+w)^{-1} + \frac{(1-\hat{q})}{6(1+w)}\frac{\dot{\xi}}{(1+\xi)}\,(1-3w)\;. \tag{8.1.39}$$

It is easily noticed that an increase (decrease) in the density is related to a contraction ($\dot{a} < 0$) (expansion ($\dot{a} > 0$)) of the cosmos and that for $\hat{q} = 0$ the time variation of the scalar-field excitation plays a role in cosmic expansion, too: higher derivatives reduce the value of H.
Explicitly, for the hodiernal, matter-dominated Universe ($w = 0$), equation (8.1.39) gives directly

$$c^2 H_0 = -\frac{1}{3}\frac{a_0^3}{M_0 c^2}\dot{\epsilon}_0(1+\xi_0)^{-\frac{1}{2}(1-\hat{q})} + (1-\hat{q})\,\frac{1}{6}\,\dot{\xi}_0 c^2\,(1+\xi_0)^{-1} \tag{8.1.40}$$

with the integration constant $M_0 \equiv M_{w=0}$ and $\xi_0 \equiv \xi_{w=0}$ and $\epsilon_0 \equiv \epsilon_{w=0}$. For radiation (rad, $w = 1/3$), there is

$$c^2 H_{1/3} = -\frac{1}{4}\frac{a_{rad}^4}{M_{1/3}c^2}\dot{\epsilon}_{rad}\,, \tag{8.1.41}$$

and for stiff matter (SM),

$$c^2 H_1 = -\frac{1}{6}\frac{a_{SM}^6}{M_1 c^2}\dot{\epsilon}_{SM}(1+\xi_{SM})^{(1-\hat{q})} - (1-\hat{q})\,\frac{c^2}{6}\,\dot{\xi}_{SM}\,(1+\xi_{SM})^{-1}\,. \tag{8.1.42}$$

For $w > 1/3$, and especially for stiff matter, a negative changing of the scalar-field excitation leads to a higher Hubble parameter if $\hat{q} = 0$.
The present Universe should be given by $w = 0$. Since the Hubble function at the present time is quasi constant, then time-changes of density shall not be too high.

- **Equations in dependence of $\tilde{G}$:**
Since the effective gravitational coupling is related to the scalar field *via*

$$G(\xi) \equiv \tilde{G} = \frac{G_0}{1+\xi}\,, \tag{8.1.43}$$

it possesses a time-dependence coefficient as follows,

$$\frac{\dot{G}_{eff}}{G_{eff}} = -\frac{\dot{\xi}}{1+\xi}\,. \tag{8.1.44}$$

The gravitational coupling is directly dependent on the scalar field and its behavior in time. With it, equation (8.1.2) may as well be written as follows,

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}\left(\epsilon + p\right) = -(1-\hat{q})\frac{1}{2}\frac{\dot{G}(\xi)}{G(\xi)}\left(\epsilon - 3p\right), \tag{8.1.45}$$

so that the dependence on coupling deviations is explicitly given. Small time deviations of the effective coupling, thus, means for $\hat{q} = 0$ a small-valued source for continuity condition. There appear entropy-production processes which, however, become minimal when the effective gravitational coupling tends to constant behavior. For scalar fields with tendency to a constant term, entropy production vanishes.
Further, with a barotropic equation-of-state parameter, equation (8.1.45) may be written as follows,

$$\dot{\epsilon} = -3H\left[1 + w - \frac{s}{6H}\frac{\dot{G}(\xi)}{G(\xi)}\right]\epsilon, \tag{8.1.46}$$

whereas we have defined

$$s \equiv (1-\hat{q})(3w-1). \tag{8.1.47}$$

Hence, for $\hat{q} = 0$, an effective equation-of-state parameter

$$w_{eff} = w - \frac{s}{6H}\frac{\dot{G}(\xi)}{G(\xi)} \tag{8.1.48}$$

may be defined. It is related to the changed density distribution in equation (8.1.25) in the case of $\hat{q} = 0$. As a cosmological parameter, w_{eff} must depend on the dynamical scale H^{-1}, as does also $G(\xi)$. The value of w_{eff} may then differ from w and lead to Quintessence within a model of only gravitationally coupled Higgs bosons ($\hat{q} = 0$) for $w = 0$ and $s = -1$. This may be related to [248, 249] in which ΛCDM and Chaplygin gas profiles are derived without a cosmological constant.

Further, the second derivative of the scalar-field excitation may also be rewritten so that following is valid,

$$\frac{\ddot{\xi}}{1+\xi} = \frac{1}{G(\xi)^2}\left[2\,\dot{G}(\xi)^2 - \ddot{G}(\xi)\,G(\xi)\right]. \tag{8.1.49}$$

This means that the second derivative of the gravitational coupling is related to the ones of scalar-field excitations as

$$\frac{\ddot{G}(\xi)}{G(\xi)} = -\frac{\ddot{\xi}}{1+\xi} + 2\left(\frac{\dot{\xi}}{1+\xi}\right)^2 . \tag{8.1.50}$$

With equation (8.1.49), the scalar-field equation (8.1.4) may be rewritten onto the following form,

$$\frac{1}{G_{eff}^2}\left(\ddot{G}(\xi)\,G(\xi) - 2\,\dot{G}(\xi)^2\right) + 3\frac{\dot{a}}{a}\frac{\dot{G}(\xi)}{G(\xi)} + \frac{c^2}{l^2}\left(\frac{G(\xi)}{G(v)} - 1\right) = -\frac{8\pi}{c^2}\frac{G(\xi)}{3}\hat{q}\frac{(\epsilon - 3p)}{(1+\frac{4\pi}{3\breve{\alpha}})}. \tag{8.1.51}$$

The dynamics of the cosmological scalar-field excitation are given by the time dependence of the effective gravitational coupling $\tilde{G}$. For vanishing deviations of the effective gravitational coupling from the ground-state coupling constant, it is directly seen in (8.1.51) that the length-scale term vanishes. Furthermore, the Friedmann equations (8.1.5) and (8.1.6) in the forms (8.1.7) and (8.1.8) may be written, too, in terms of corrections coming from time derivatives of the gravitational coupling (*cf.* equation (8.1.9)):

$$f_1(G) \equiv f_1 = \frac{\dot{G}(\xi)}{G(\xi)}\left[\frac{\dot{a}}{a} + \frac{\pi}{3\breve{\alpha}}\frac{\dot{G}(\xi)}{G(\xi)}\right], \tag{8.1.52}$$

and (*cf.* equation (8.1.10))

$$f_2(G) \equiv f_2 = 2\frac{\dot{a}}{a}\frac{\dot{G}(\xi)}{G(\xi)} - \frac{\dot{G}(\xi)^2}{G(\xi)^2}\left(2 + \frac{\pi}{\breve{\alpha}}\right) + \frac{\ddot{G}(\xi)}{G(\xi)}\,. \tag{8.1.53}$$

Explicitly, the Friedmann equations (8.1.5) and (8.1.6) read now

$$\frac{\dot{a}^2 + Kc^2}{a^2} = \frac{8\pi}{3}\frac{G_{eff}}{c^2}\epsilon + \frac{1}{3}\Lambda c^2 + \frac{\dot{a}}{a}\frac{\dot{G}(\xi)}{G(\xi)} + \frac{\pi}{3\breve{\alpha}}\frac{\dot{G}(\xi)^2}{G(\xi)^2} \tag{8.1.54}$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + Kc^2}{a^2} = -8\pi G_{eff}\frac{p}{c^2} + \Lambda c^2 - (1+\xi)^{-1}\ddot{\xi} + 2\frac{\dot{a}}{a}\frac{\dot{G}(\xi)}{G)(\xi)} + \frac{\pi}{\breve{\alpha}}\frac{\dot{G}(\xi)^2}{G(\xi)^2}\,. \tag{8.1.55}$$

The cosmological function (8.1.11) now reads as follows,

$$\Lambda \equiv \Lambda(\tilde{G}) = \frac{3}{4l^2}\left(2 - \frac{G_0^2 + \tilde{G}^2}{G_0\tilde{G}}\right)\left(1 + \frac{4\pi}{3\breve{\alpha}}\right)\,. \tag{8.1.56}$$

Again, (8.1.7) and (8.1.8) give together equation (8.1.18),

$$\begin{aligned}\frac{\ddot{a}}{a} &= -\frac{4\pi}{c^2}\frac{\tilde{G}}{3}(\epsilon + 3p - 2V) + f \\ &= -\frac{4\pi}{3}\frac{\tilde{G}}{c^2}(\epsilon + 3p) + \frac{1}{3}\Lambda c^2 + f \\ &= -\frac{4\pi}{3}\frac{G_0}{c^2}(\epsilon^* + 3p^*) + \frac{1}{3}\Lambda c^2 + f,\end{aligned} \tag{8.1.57}$$

again with effective density and pressure

$$\epsilon^* = \epsilon/(1+\xi) \quad \text{and} \quad p^* = p/(1+\xi)\,.$$

For explicit gravitational-coupling derivatives, there is now equation (8.1.20) in the form

$$f \equiv f(G(\xi)) = \frac{1}{2}(f_2 - f_1) = \frac{1}{2}\left[\frac{1}{G(\xi)}\left(\ddot{G}(\xi) + \frac{\dot{a}}{a}\dot{G}(\xi)\right) - 2\frac{\dot{G}(\xi)^2}{G(\xi)^2}\left(1 + \frac{2\pi}{3\breve{\alpha}}\right)\right]. \qquad (8.1.58)$$

In the same way, energy density may be written in terms of the gravitational coupling,

$$\epsilon_w = \frac{M_w\, c^2}{a^{3(1+w)}}\left(\frac{G(v)}{G(\xi)}\right)^{\frac{1}{2}(1-3w)(1-\hat{q})}. \qquad (8.1.59)$$

For $\hat{q} = 0$ and $w \neq 1/3$, high effective gravitational couplings (*i.e.* $\xi < 0$) in relation to the ground-state coupling $G(v) = G_0$ lead to smaller (as measured) values of the energy density, with

$$\tilde{G}\epsilon > G_0\epsilon\,.$$

Analogously, small effective couplings ($\xi > 0$) in relation to G_0 lead to higher densities than within the standard Friedmann models, assuming $w < 1/3$. The evolution of ξ, thus, is of special relevance. Furthermore, there is (*cf.* [24] and (8.1.30))

$$\epsilon_w\left(\frac{G_0}{\tilde{G}}\right)^{\frac{1}{2}(1-3w)(1-\hat{q})} a^{3(1+w)} = const. = \epsilon_{0w}\left(\frac{G_0}{\tilde{G}_0}\right)^{\frac{1}{2}(1-3w)(1-\hat{q})} a_0^{3(1+w)}\,, \qquad (8.1.60)$$

with $\tilde{G}_0$ as the effective coupling for the time $t = t_0$, usually meant to be the one of the current Universe (of the observer). a_0 is the observer's current scale factor $a(t_0)$. ϵ_{0w} is the observer's energy density $\epsilon_w(t_0)$ for w-typed matter.[4]
Consequently, the integration constant M_w is related to a_0 in following way for $\hat{q} = 1$:

$$M_w = \left(\frac{\tilde{G}_0}{G_0}\right)^{\frac{1}{2}(1-3w)(1-\hat{q})} a^{3(1+w)}\epsilon_{0w}\,. \qquad (8.1.61)$$

The Hubble parameter reads

$$c^2 H_w = -\frac{1}{3}\frac{\dot{\epsilon}}{\epsilon}(1+w)^{-1} - \frac{(1-\hat{q})}{6(1+w)}\frac{\dot{G}(\xi)}{G(\xi)}\,(1-3w)\,. \qquad (8.1.62)$$

Further, equation (8.1.25) leads to (8.1.39) as

$$c^2 H_w = -\frac{1}{3}\dot{\epsilon}\frac{a^{3(1+w)}}{M_w c^2}\left(\frac{G(v)}{G(\xi)}\right)^{-\frac{1}{2}(1-3w)(1-\hat{q})} - \frac{(1-\hat{q})}{6(1+w)}\frac{\dot{G}(\xi)}{G(\xi)}c^2\,(1-3\alpha)\,. \qquad (8.1.63)$$

The variation of the coupling constant leads to a screening of the density parameter ϵ in (8.1.59) in relation to the case where the G-coefficient is negligible. The value of the density for $\hat{q} = 1$ is smaller if $\xi < 0$, *i.e.* $G(\xi) > G(v)$ (anti-screening of the gravitational constant).
For $w > 1/3$, and especially for stiff matter, a positive derivative of the coupling constant leads to a higher Hubble parameter if $\hat{q} = 0$.

[4]The subscript w may usually be let aside and the parametrization be implicitly given.

8.2 Equation-of-state parameters of the scalar field

Let us define a scalar-field component of the total density in terms of the scalar-field excitations,

$$\epsilon_\Lambda = V - \frac{3c^2}{8\pi G_0}\frac{\dot a}{a}\dot\xi + \frac{v^2}{8}\frac{\dot\xi^2}{1+\xi}. \tag{8.2.1}$$

This density is dependent on the scalar-field potential, on a term of the scalar-field derivative and on the Hubble parameter.
In terms of gravitational-coupling changings, equation (8.2.1) yields (related to the parameter Ω^*_Λ in (8.4.2) with (8.4.3)),

$$\epsilon_\Lambda = V + \frac{3c^2}{8\pi G(\xi)}\left(H\frac{\dot G(\xi)}{G(\xi)} + \frac{1}{3\breve\alpha}\frac{\dot G(\xi)^2}{G(\xi)^2}\right). \tag{8.2.2}$$

Further, the total density read

$$\epsilon_T = \epsilon + \epsilon_\Lambda. \tag{8.2.3}$$

Analogously, the screened density

$$\epsilon^*_T = \frac{\epsilon_T}{1+\xi} = \frac{\tilde G}{G_0}\epsilon_T \tag{8.2.4}$$

may be defined.
Furthermore, we define a scalar-field term of the pressure for the second Friedmann equation (8.1.6),

$$p_\Lambda = -V + \frac{c^2}{8\pi G_0}\left(\ddot\xi + 2\frac{\dot a}{a}\dot\xi - \frac{\pi}{3\,\breve\alpha}\frac{\dot\xi^2}{1+\xi}\right), \tag{8.2.5}$$

or equivalently in dependence of the gravitational coupling,

$$p_\Lambda = -V - \frac{c^2}{8\pi G(\xi)}\left[\frac{\ddot G(\xi)}{G(\xi)} + 2H\frac{\dot G(\xi)}{G(\xi)} - 2\frac{\dot G(\xi)^2}{G(\xi)^2}\left(1+\frac{\pi}{6\breve\alpha}\right)\right]. \tag{8.2.6}$$

So, equations (8.1.5) and (8.1.18) yield

$$\begin{aligned} \frac{\dot a^2 + Kc^2}{a^2} &= \frac{8\pi}{c^2}\frac{\tilde G}{3}\epsilon_T \qquad (8.2.7)\\ &= \frac{8\pi}{c^2}\frac{G_0}{3}\epsilon^*_T\,,\\ \frac{\ddot a}{a} &= -\frac{4\pi}{c^2}\frac{\tilde G}{3}\left(\epsilon_T + 3p_T\right) \qquad (8.2.8)\\ &= -\frac{4\pi}{c^2}\frac{G_0}{3}\left(\epsilon^*_T + 3p^*_T\right). \end{aligned}$$

All changes on dynamics are now written in terms of screened densities and pressures or effective densities and pressures with screened gravitational coupling. Furthermore, there is an equation of state for scalar-field dominance in the following way:

$$\epsilon_\Lambda + 3p_\Lambda = -2V - \frac{3\,c^2}{8\pi G(\xi)}\left[H\frac{\dot G(\xi)}{G(\xi)} + \frac{\ddot G(\xi)}{G(\xi)} - 2\frac{\dot G(\xi)^2}{G(\xi)^2}\left(1+\frac{2\pi}{3\breve\alpha}\right)\right]. \tag{8.2.9}$$

Introducing the scalar-field equation, it is seen that the latter is equal to

$$\epsilon_\Lambda + 3p_\Lambda = -2V + \hat{q}\frac{(\epsilon - 3p)}{1+\frac{4\pi}{3\breve{\alpha}}} - \frac{3\,c^2}{8\pi G_0}\left[\frac{c^2}{l^2}\xi + 2H\dot{\xi}\right]. \tag{8.2.10}$$

The total equation of state then reads

$$\epsilon_T + 3p_T = \left(1 + \frac{\hat{q}}{1+\frac{4\pi}{3\breve{\alpha}}}\right)\epsilon + 3\left(1 - \frac{\hat{q}}{1+\frac{4\pi}{3\breve{\alpha}}}\right)p - 2V - \frac{3c^2}{8\pi\tilde{G}}\left[2H\frac{\dot{\xi}}{1+\xi} + \frac{c^2}{l^2}\xi\right]. \tag{8.2.11}$$

Hence, as the first term of the right-hand side of (8.2.8) vanishes for $\breve{\alpha} \gg 1$, in terms of equation (8.2.8), pressure hardly contributes to acceleration in case of $\hat{q} = 1$. Acceleration is mainly given by excitations ξ in V or by positive time derivatives of the excitations themselves, which in terms of the effective gravitational coupling is analogous to in-time diminishing effective couplings $\tilde{G}$. For $\hat{q} = 1$, densities, which we assume always positive ($\epsilon \geq 0$), act against acceleration and naturally contribute to deceleration as within standard dynamics with gravitation as an attractive elementary interaction (for the primeval Universe, consequences of this may be found in Chapter 8.8).

The scalar-field equation of state comprises the correction term f to usual Friedmann equations together with the cosmological function. The relation of equation (8.2.9) to the parameter f in (8.1.58) is

$$-\frac{8\pi}{3}\frac{\tilde{G}}{c^2}(\epsilon_\Lambda + p_\Lambda) = f_2 - f_1 + \frac{6\pi}{3}\frac{\tilde{G}}{c^2}V \tag{8.2.12}$$

$$= 2\left(f + \frac{1}{3}\Lambda\right), \tag{8.2.13}$$

and its value can clearly be negative for a dominance of ξ over its derivatives or for a dominance of its second derivative, for instance. Thus, in a scalar-field dominated universe, acceleration terms of cosmos may dominate. Such terms, which come from the corrections, are herewith related to density- and pressure-acting terms (*cf.* (8.2.7) and (8.2.8)).

Consequently to equation (8.2.13), there is

$$\Lambda = -\frac{3}{2}f - \frac{4\pi\tilde{G}}{c^2}\left(\epsilon_\Lambda + p_\Lambda\right). \tag{8.2.14}$$

The cosmological function may be split onto the correction f and the equation-of-state contribution of the scalar field. Doing so, non-explicitly time dependent contributions of the scalar field to density can be treated as density contributions, with no other cosmological term other than f.

The equation-of-state parameter of the ξ-related density and pressure reads

$$\frac{p_\Lambda}{\epsilon_\Lambda} = w_\Lambda = -\frac{\frac{2}{G(\xi)^2}\left[\frac{1}{2}\ddot{G}(\xi)G(\xi) + H\dot{G}(\xi)G(\xi) - \dot{G}(\xi)^2\left(1+\frac{\pi}{6\breve{\alpha}}\right) + \frac{4\pi G(\xi)}{3c^2}V\right]}{H\frac{\dot{G}(\xi)}{G(\xi)} + \frac{1}{3\breve{\alpha}}\frac{\dot{G}(\xi)^2}{G(\xi)^2} + \frac{8\pi}{3}\frac{G(\xi)}{c^2}V}. \tag{8.2.15}$$

With

$$\frac{8\pi}{3}\frac{\tilde{G}}{c^2}V = \frac{\xi^2c^2}{4l^2}(1+\xi)^{-1}\left(1+\frac{4\pi}{3\breve{\alpha}}\right) \tag{8.2.16}$$

$$= \frac{c^2}{4l^2}\frac{\tilde{G}^2 - 2G_0\tilde{G} + G_0^2}{\tilde{G}G_0}\left(1+\frac{4\pi}{3\breve{\alpha}}\right), \tag{8.2.17}$$

equation (8.2.15) can further be fully rewritten in terms of the gravitational coupling:

$$w_\Lambda = -\frac{\ddot{G}(\xi) + 2H\dot{G}(\xi) - 2\frac{\dot{G}(\xi)^2}{G(\xi)}\left(1+\frac{\pi}{6\breve{\alpha}}\right) + \frac{3c^2}{4l^2}\frac{G(\xi)^2 - 2G_0G(\xi) + G_0^2}{G_0}\left(1+\frac{4\pi}{3\breve{\alpha}}\right)}{H\dot{G}(\xi) + \frac{1}{3\breve{\alpha}}\frac{\dot{G}(\xi)^2}{G(\xi)} + \frac{c^2}{4l^2}\frac{G(\xi)^2 - 2G_0G(\xi) + G_0^2}{G_0}\left(1+\frac{4\pi}{3\breve{\alpha}}\right)}. \tag{8.2.18}$$

This is the equation-of-state parameter for a scalar-field dominated universe. For $\breve{\alpha} \gg 1$, it simplifies as follows,

$$w_{\Lambda\,\text{sf-dom}} = -\frac{-H\dot{G}(\xi) + \frac{3c^2}{4l^2}\frac{G(\xi)^2 - 2G(\xi)G_0 + G_0^2 - 4G(\xi) + 4G_0}{G_0}}{H\dot{G}(\xi) + \frac{3c^2}{4l^2}\frac{G(\xi)^2 - 2G_0G(\xi) + G_0^2}{G_0}}. \tag{8.2.19}$$

For $\dot{G}(\xi) = 0$ (statical field, subscribed 'c'), there is further

$$w_{\Lambda\,\text{sf-dom,c}} = -3\frac{G(\xi)^2 - 2G(\xi)G_0 + G_0^2 - 4G(\xi) + 4G_0}{G(\xi)2 - 2G_0G(\xi) + G_0^2}. \tag{8.2.20}$$

Vanishing values of the scalar-field excitation for vacuum, *i.e.* $\tilde{G} = G_0$ with $\epsilon = p = 0$ would lead to a divergence denoting the absence of all fields. However, **for the scalar-field pressure p_Λ (*cf.* 8.2.5), it is easily noticed that for absolute scalar-field dominance, $p_\Lambda = w_\Lambda \epsilon_\Lambda$ is quintessential, *i.e.* it is accompanied by a negative equation-of-state parameter w_Λ so that** $w_T < w$ (see equation (8.3.20)). The scalar field possesses antigravitational properties and it may act as anti-stiff matter. Furthermore, a deSitter state, which is a quintessential matter-vacuum state, is important to get an inflationary epoch after $t = 0$ and so to solve problems as the one of horizon through a highly accelerated expansion of the universe. Furthermore, the appearance of negative effective pressures is important for the appearance of cosmic accelerations in the present Universe as well as for the nature of the initial state of the same (see Chapter 8.8).

8.3 Deceleration parameter and Dark Energy

- **The deceleration parameter and its importance within measurements:**
  Let us take the deceleration parameter as defined in equation (8.1.17). In general, for values $q \geq 1/2$, the deceleration parameter gives a cosmic deceleration in accordance with a gravitational character of densities and pressures leading to attraction. For values smaller than 1/2, it gives an acceleration.
  Positive pressures p lead to higher values of q and thus strengthen deceleration (pressure acts gravitationally). A cosmic fluid with $p < \frac{1}{3}\epsilon$ acts antigravitationally and thus strengthens acceleration (gravitational repulsion).
  The current Universe with only matter and negligible radiation should fulfill a decelerated expansion as consequence of the Big Bang and then gravitational attraction. Furthermore, it is mostly accepted that geometry is given by $K = 0$, wherein this value is deduced from observations of the cosmic microwave background (CMB) radiation (however, this depends on the exact value of total density in the Universe). Observations of Super Novae of type Ia, however, indicate a negative q_0 for the current cosmos (hence the subscript 0), which then would mean acceleration of cosmic expansion. The mechanism of a cosmic medium which should be cause of such antigravitational interaction is usually called Dark Energy or Quintessence (see Chapter 2.4). Quintessence, however, is usually used for a kind of Dark Energy which has as source a scalar field which almost does not evolve in time.

Equation (8.2.18) already showed that the scalar-field terms may lead to negative values for the total equation-of-state parameter. Given such, the scalar field may lead to quintessential properties in form of cosmic acceleration.
The total pressure term which carries scalar-field terms is dependent on $\tilde{G}$ and its derivatives. One can see that high effective values of the scalar-field excitation as well as especially positively valued (second) derivatives of the gravitational coupling $\tilde{G}$, may lead to negative total pressures. Such strengthen a cosmological acceleration, whereas $\dot{G}(\xi)^2$ may act as deceleration factor if it is especially high. The latter term, however, may be expected as negligible under normal circumstances, and relevant only for the primordial Universe, possibly in relation with primeval Inflation. The concept of primeval, cosmic Inflation, first proposed by Alan Guth in 1981 [118], based on ideas of Starobinsky [224], and later improved by Albrecht, Steinhardt [1] and Linde [154], assumes a phase of very highly accelerated expansion in the early Universe to explain horizon and flatness problems of cosmology. Often, an hypothetical scalar field, namely the inflaton field, is proposed in this context. "New" and "Chaotic" Inflation differ from the original one (called "Old"), by means of the initial conditions of this scalar field.

The relevance of the deceleration parameter can be seen within the relation between a generic time-scale factor and the current one for $t = t_0$. There is

$$a(t) = a_0 \left[1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots\right], \tag{8.3.1}$$

being H_0, a_0 and q_0 the hodiernal Hubble parameter, scale factor and deceleration parameter, and $a(t)$ be the scale factor for a time t which can be given in terms of the redshift of a luminous source such as a distant galaxy (*cf.* [56]),

$$z = H_0(t_0 - t) + \left(1 + \frac{1}{2} q_0\right) H_0^2 (t_0 - t)^2 + \ldots , \tag{8.3.2}$$

invertible to

$$t_0 - t = \frac{1}{H_0} \left[z - \left(1 + \frac{1}{2} q_0\right) z^2 + \ldots\right]. \tag{8.3.3}$$

Clearly, for high redshift values, the exact value of q_0 plays an important role in the evolution of $a(t)$. With equations (8.1.34) and (8.3.3), one can take

$$\int_t^{t_0} \frac{c dt}{a} = \int_0^r \frac{dr}{\sqrt{1 - Kr^2}} \tag{8.3.4}$$

and convert it into

$$\frac{c}{a_0} \int_t^{t_0} \left[1 + H_0(t_0 - t) + \left(1 + \frac{1}{2} q_0\right) H_0^2 (t_0 - t)^2 + \ldots\right] dt = r + O(r^3), \tag{8.3.5}$$

using

$$q_0 = -\frac{\ddot{a}(t_0) a_0}{\dot{a}(t_0)^2} . \tag{8.3.6}$$

Therefore, there is (*cf.* [56])

$$r = \frac{c}{a_0} \left[(t_0 - t) + \frac{1}{2} H_0 (t_0 - t)^2 + \ldots\right], \tag{8.3.7}$$

which using equation (8.3.2) yields

$$r = \frac{c}{a_0 H_0}\left[z - \frac{1}{2}(1-q_0)z^2 + \ldots\right]. \tag{8.3.8}$$

Thus, the radial coordinate may be expressed by the redshift in terms of acceleration with the acceleration parameter q_0. Deceleration however depends on the total pressure p_T and energy density ϵ_T, as well as on curvature K (all at the time $t = t_0$). A nonvanishing, positive term of the curvature K leads to smaller, effective, values of the deceleration compared with the actual one.

Proper distances to astronomical objects cannot be measured in any direct way. Astronomical objects are observed through the electromagnetic radiation they emit. Radiation, on the other hand, takes time to travel from the emission to the observation point (usually P_0 at t_0). It is thus only possible to make measurements along the set of paths traveling to us from the past, *i.e.* part of our light cone. Everything outside of it, of course, is not causally related with the present.
One of the measurable distances is the luminosity distance d_L. This distance is defined in a way to preserve the euclidean inverse-square law for the diminution of light with distance from a point–source.
L denote the power emitted by a source at a point P at a coordinate distance r at time t. l be the power received per unit area (the flux) at t_0 by an observer P_0. Then, the luminosity distance be defined by

$$d_L = \sqrt{\frac{L}{2\pi l}}\,. \tag{8.3.9}$$

The area of spherical surface centered on P and passing through P_0 at time t_0 is $4\pi a_0^2 r^2$. The photons emitted by the source arrive at this surface having been redshifted by the expansion of the Universe by a factor a/a_0. There is

$$l = \frac{L}{4\pi a_0^2 r^2}\left(\frac{a}{a_0}\right)^2, \tag{8.3.10}$$

and thus with equation (8.3.7),

$$d_L = a_0^2\frac{r}{a} = \frac{c}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2 + \ldots\right]. \tag{8.3.11}$$

From d_L, there is (*cf.* [56])

$$l = \frac{L}{4\pi d_L^2} = \frac{LH_0}{4\pi c^2 z^2}\left[1 + (1-q_0)z + \ldots\right] \tag{8.3.12}$$

for apparent (l) and absolute (L) luminosity. In astronomy, however, it is custom to use magnitudes instead; the absolute (M) and the apparent one (m). They are defined in a logarithmical scale by taking a factor 100 in received flux to be a difference of 5 magnitudes. Per convention, Polaris (α UMi) is given an apparent magnitude of 2.12 in visible light. The absolute magnitude is defined to be the apparent magnitude the source would have if it were placed at a distance of 10 parsec.[5] The following relation is given,

$$d_L = 10^{1+(m+M)/5}\text{pc}, \tag{8.3.13}$$

[5] As commonly known, parallax of one arcsecond or parsec (pc) is the length of an adjacent side of an imaginary right triangle in space. The two dimensions that form this triangle are the parallax angle (defined as 1 arcsecond) and the opposite side (which is defined as 1 astronomical unit (AU), the distance from the Earth to the Sun). Given these two measurements, the length of the adjacent side (the parsec) can be found. It is of about 3.26 light-years length.

where the distance modulus

$$DM = m - M = -5 + 5\log d_L(\mathrm{pc}) \tag{8.3.14}$$

may be defined. Using equation (8.3.12), the latter is given by

$$m - M \approx 25 - 5\log_{10} H_0 + 5\log cz + 1.086(1-q_0)z + \dots \tag{8.3.15}$$

The apparent magnitude is dependent on z and on q_0. The q_0-dependence, however, is relevant only for redshifts $z > 0.1$. Therefore, cosmic acceleration does not play a dominant role in low-z-analysis, and measured distances indirectly gotten through observations may be accurately given independently on the exact Friedmann model. However then, high-z-analyses have to be made to constraint dark-energetic behavior. Furthermore, given other factors intervening in the analysis, magnitudes can give little accurate information about the deceleration parameter. The regime of accuracy of m is $z < z_{max} \approx 0.2$ where the distance modulus however confirms the Hubble law and, therefore, the cosmological principle.

The relation between luminosity in form of the distance modulus $DM = m - M$ of magnitudes and redshift z is called Hubble diagram. It can be used to prove the value of q_0 directly. The problem: it needs of objects of known intrinsic luminosities which are therefore called standard candles.
The use of SNe as standard candles in cosmology was discussed by Sandage [214] because of their rather homogeneous and extremely luminous peak but it was not only until the realization, though, that SNe are actually subdivided in underclasses, that they lead to the current progress starting with Hamuy *et al.*'s [119] and Riess *et al.*'s [207] work in 1995.
First observations [197] had suggested a positive deceleration parameter $q_0 > 0$. Then it became apparent that high-redshift supernovae might be fainter than they should when compatible with $q_0 > 0$ under the assumption of usual Friedmann models. The works of Riess *et al.* [208] and Perlmutter *et al.* [198] in 1998 concluded an acceleration of the Universe together with a dominant cosmological constant within standard Friedmann models. Indeed, within our model, the dominance of scalar-field (thus cosmological, exotic) components of density and pressure seem dominant for the current, else dust-dominated Universe.

- **The deceleration parameter and the equation of state:**
  Division between both equations (8.2.7) and (8.2.8) leads to

$$\begin{aligned}\frac{\ddot{a}a}{\dot{a}^2 + Kc^2} &= -\frac{1}{2}\left(1 + 3\frac{p_T^*}{\epsilon_T^*}\right) \\ &= -\frac{1}{2}\left(1 + 3\frac{p_T}{\epsilon_T}\right),\end{aligned} \tag{8.3.16}$$

with equations (8.2.1), (8.2.3) and (8.2.5). Equation (8.3.16) may be used as redefinition of an effective deceleration parameter $\tilde{q}$ which shall contain a curvature term with

$$2\tilde{q} = 1 + 3\frac{p_T}{\epsilon_T} \tag{8.3.17}$$

$$= \frac{2q}{1 + \frac{Kc^2}{\dot{a}^2}}. \tag{8.3.18}$$

Thus, the proper deceleration parameter q reads

$$q = \frac{1}{2}\left(1+3w_T\right)\left(1+\frac{Kc^2}{\dot{a}^2}\right), \tag{8.3.19}$$

with $w_T = p_T/\epsilon_T$ as total equation-of-state parameter. q is an effective parameter which is negative for quintessence-dominance, even though the equation-of-state parameter of matter *per se* should be zero or positive. Hence, w_T takes exotic contributions to matter (which come from ξ) into account.

The total equation-of-state parameter reads using the scalar-field equation,

$$w_T = \frac{p+p_\Lambda}{\epsilon+\epsilon_\Lambda} = \frac{\left[1-\frac{\hat{q}}{\left(1+\frac{4\pi}{3\breve{\alpha}}\right)}\right]p + \frac{\hat{q}}{3\left(1+\frac{4\pi}{3\breve{\alpha}}\right)}\epsilon - \frac{V}{3} - \frac{c^2}{8\pi G_0}\left(H\dot{\xi}+\frac{c^2}{l^2}\xi - \frac{\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right)}{\epsilon + V - \frac{3c^2}{8\pi G_0}\left(H\dot{\xi} - \frac{1}{3\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right)}\,. \tag{8.3.20}$$

In terms of the equation-of-state parameters ϵ and p and the scalar-field terms, the latter may be written for $\breve{\alpha} \gg 1$ as follows,

$$3w_T = \frac{3(1-\hat{q})p + \hat{q}\epsilon - \frac{3c^2}{8\pi G_0}\left[\frac{c^2}{4l^2}\xi\left(4+\xi\right)+H\dot{\xi}\right]}{\epsilon + \frac{3c^2}{8\pi G_0 l^2}\left[\frac{c^2}{4l^2}\xi^2 - H\dot{\xi}\right]}. \tag{8.3.21}$$

For $\breve{\alpha} \gg 1$ and $\hat{q} = 1$, equation (8.3.20) leads to

$$w_T = \frac{1}{3}\left[\frac{\epsilon - V - \frac{3c^2}{8\pi G_0}\left(H\dot{\xi}+\frac{c^2}{l^2}\xi\right)}{\epsilon + V - \frac{3c^2}{8\pi G_0}H\dot{\xi}}\right]. \tag{8.3.22}$$

$\xi > 0$ as well as $\dot{\xi} > 0$ lead to anti-stiff behavior.
For $\hat{q} = 0$, there is

$$w_T = \frac{1}{3}\left[\frac{3p - V - \frac{3c^2}{8\pi G_0}\left(H\dot{\xi}-\frac{c^2}{l^2}\xi\right)}{\epsilon + V - \frac{3}{8\pi G_0}H\dot{\xi}}\right]. \tag{8.3.23}$$

As directly seen, for the limiting case of constant scalar-field excitations $\xi = const.$ (thus $V = V_c$), the equation-of-state parameter reads

$$w_{T\,\xi=const.} = \frac{1}{3}\left[\frac{\epsilon - V_c - \frac{3c^4}{8\pi G_0 l^2}\xi}{\epsilon + V_c}\right] \quad \text{for} \quad \hat{q}=1, \tag{8.3.24}$$

$$= \frac{1}{3}\left[\frac{3p - V_c - \frac{3c^4}{8\pi G_0 l^2}\xi}{\epsilon + V_c}\right] \quad \text{for} \quad \hat{q}=0\,. \tag{8.3.25}$$

Since $V(\xi) \geq 0$, for high excitations there is for equation (8.3.24), $w_{T\,\xi=const.} \leq 1/3$. For small excitations also, as long as $\xi > 0$. For equation (8.3.25), in the same cases, there is $w_{T\,\xi=const.} \leq w/3$ if $p = w\epsilon$ is valid. In the latter case $\hat{q} = 0$, further, vanishing excitations lead to the usual equation-of-state parameter $w = p/\epsilon$.

The Friedmann equations may be given in dependence of the effective deceleration in the following form,

$$\frac{p_T}{\epsilon_T} = \frac{1}{3}(2\tilde{q}-1) \tag{8.3.26}$$

$$= \frac{\dot{a}^2}{3}\left(\frac{2q-1-\frac{Kc^2}{\dot{a}^2}}{\dot{a}^2+Kc^2}\right) = w_T . \tag{8.3.27}$$

For a flat Universe, *i.e.* $K = 0$, the equation above gives the usually given Friedmann equation containing the deceleration parameter, with Dark Matter terms added to density and pressure, and with the scalar-field terms as dark constituents which are here still to define in terms of their properties. Further, in terms of the effective gravitational coupling, there is

$$2\tilde{q} = 1 + 3\left\{\frac{p - V - \frac{c^2}{8\pi G(\xi)}\left[\frac{\ddot{G}(\xi)}{G(\xi)} + 2H\frac{\dot{G}(\xi)}{G(\xi)} - 2\frac{\dot{G}(\xi)^2}{G(\xi)^2}\left(1+\frac{\pi}{2\breve{\alpha}}\right)\right]}{\epsilon + V + \frac{3c^2}{8\pi G(\xi)}\left(H\frac{\dot{G}(\xi)}{G(\xi)} + \frac{1}{3\breve{\alpha}}\frac{\dot{G}(\xi)^2}{G(\xi)^2}\right)}\right\} . \tag{8.3.28}$$

Obviously, for vanishing derivatives (subscript *sfd=0*) of ξ ($V = const. \equiv V_c$), the effective deceleration parameter is given by

$$2\tilde{q}_{sfd=0} = 1 + 3\left(\frac{p - V_c}{\epsilon + V_c}\right) , \tag{8.3.29}$$

for an effective pressure $p_{eff} = p - V_c$ and effective density $\epsilon_{eff} = \epsilon + V_c$ of a constant potential V_c: $\epsilon_{eff} + 3p_{eff} = \epsilon + 3p - 2V_c$. Following equation (8.3.28),

$$\tilde{q} = \frac{1}{2}(1 + 3w_T) ,$$

the deceleration parameter $\hat{q}$ reads for scalar-field excitations (according to equation (8.3.20)),

$$2\tilde{q} = 1 + \frac{3\left[1 - \frac{\hat{q}}{\left(1+\frac{4\pi}{3\breve{\alpha}}\right)}\right]p + \frac{\hat{q}\epsilon}{\left(1+\frac{\pi}{3\breve{\alpha}}\right)} - 3V - \frac{3\,c^2}{8\pi G_0}\left(H\dot{\xi} + \frac{c^2}{l^2}\xi - \frac{4\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right)}{\epsilon - V - \frac{3c^2}{8\pi G_0}\left(H\dot{\xi} - \frac{1}{3\breve{\alpha}}\frac{\xi^2}{1+\xi}\right)} . \tag{8.3.30}$$

Inserting V, there is further

$$2\tilde{q} = 1 + \frac{3\left[1 - \frac{\hat{q}}{1+\frac{4\pi}{3\breve{\alpha}}}\right]p + \frac{\hat{q}}{1+\frac{4\pi}{3\breve{\alpha}}}\epsilon - \frac{3\,c^2}{8\pi G_0}\left[\frac{c^2}{4l^2}\xi\left(4+\xi\left(1+\frac{4\pi}{3\breve{\alpha}}\right)\right) + H\dot{\xi} - \frac{\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right]}{\epsilon + \frac{3\,c^2}{8\pi G_0}\left[\frac{c^2}{4l^2}\xi^2\left(1+\frac{4\pi}{3\breve{\alpha}}\right) + H\dot{\xi} - \frac{1}{3\breve{\alpha}}\frac{\dot{\xi}^2}{1+\xi}\right]} . \tag{8.3.31}$$

For $\breve{\alpha} \gg 1$ and $\hat{q} = 1$, as directly seen from equation (8.3.24), the latter simplifies to

$$2\tilde{q} = 1 + \frac{\epsilon - 3V - \frac{3c^2}{8\pi G_0}\left(H\dot{\xi} + \frac{c^2}{l^2}\xi\right)}{\epsilon + V - \frac{3c^2}{8\pi G_0}H\dot{\xi}} . \tag{8.3.32}$$

On the other hand, for $\breve{\alpha} \gg 1$ and $\hat{q} = 0$, there is, as directly seen from equation (8.3.23),

$$2\tilde{q} = 1 + \frac{3p - V - \frac{3c^2}{8\pi G_0}\left(H\dot{\xi} + \frac{c^2}{l^2}\xi\right)}{\epsilon + V - \frac{3c^2}{8\pi G_0}H\dot{\xi}} . \tag{8.3.33}$$

However, the $c^2\xi/l^2$ term for nonvanishing scalar-field excitations pushes down the equation-of-state-parameter value for $\xi > 0$.
Further, there is

$$-V_c - \frac{3c^4}{8\pi G_0 l^2}\xi = -\frac{3c^4}{32\pi G_0 l^2}\xi(4+\xi) . \tag{8.3.34}$$

A neglection of the scalar-field potential is equivalent to linear approximation of the scalar-field excitations (*cf.* $\xi^2 = 0$). Hence, low-excitation analyses may be fulfilled for $V \approx 0$. In such case, the dominant term is the third term of equations (8.3.24) and (8.3.25) which for $\xi > 0$ is quintessential (*i.e.* it diminishes w_T), however high valued for lower length scales l. Given the smallness of p and ϵ in cosmological terms, a small value of the excitation ξ (in relation to 1) may still have a dominant character in terms of the total equation-of-state parameter and cosmological acceleration, which may still be given for $0 < \xi \ll 1$.

For equation (8.3.32), there is the value $w_T = 1/3$ (radiation) for $\dot{\xi} = \xi = 0$. The value of the equation-of-state parameter is $w_{T\,1c} < 1$ for $-1/4 < \xi$ and $\xi > 0$.

Let us write down the total equation-of-state parameter for $\hat{q} = 1$ according to equation (8.3.33) with $p = w\epsilon$. There is

$$w_T = \frac{1}{3}\left[\frac{w - V/\epsilon - \frac{3c^2}{8\pi G_0 \epsilon}\left(H\dot{\xi} + \frac{c^2}{l^2}\xi\right)}{1 - V/\epsilon - \frac{3c^2}{8\pi G_0 \epsilon}H\dot{\xi}}\right]. \tag{8.3.35}$$

For static, vanishing excitations, the usual equation-of-state parameter is recovered, $w_T = w$.

Before further analyses, the density parameters should be introduced. This may relate the deceleration parameters with other measured quantities This is fulfilled in the Chapter 8.4.

8.4 The density parameters

Let us take the first of the generalized Friedmann equation (8.1.5). It may be rewritten so that following be valid:

$$a^2 = \frac{\dot{a}^2 + Kc^2}{\frac{8\pi\tilde{G}}{3}\left(\varrho + 3\frac{p}{c^2}\right) + \frac{1}{3}\Lambda c^2 + \frac{1}{3}\Lambda_I c^2}\,. \tag{8.4.1}$$

Herewith, $\Lambda_I \equiv \Lambda_I(\xi)$.

In this way, the density parameters Ω_i may be defined (at this point with the functional Hubble rate H). Be

$$\Omega = \frac{8\pi G(\xi)}{3\,H^2}\varrho\,(1 + 3w)\,, \quad \Omega^*_\Lambda = \frac{c^2}{3}\,(\Lambda + \Lambda_I)\;H^{-2} \equiv \Omega_\Lambda + \Omega_I\,, \tag{8.4.2}$$

with a further cosmological-function term

$$\Lambda_I(\xi) \equiv \Lambda_I := -\,\frac{3\,H}{c^2}\,\frac{\dot{\xi}}{1+\xi} + \frac{\pi}{c^2\,\breve{\alpha}}\,\frac{\dot{\xi}^2}{(1+\xi)^2} \tag{8.4.3}$$

$$= \frac{8\pi G(\xi)}{c^2\,H^2}\,(\epsilon_\Lambda - V)\,, \tag{8.4.4}$$

related to equation (8.2.1) of the scalar-field density. The density parameter Ω^*_Λ is hence the density parameter of the scalar-field density of Chapter 8.3. This strengthens the interpretation of the equation-of-state parameters of the scalar field as components of the total equation of state and part of an equation-of-state parameter w_Λ for scalar-field dominance in equation (8.2.15). The same parameter possesses quintessential properties.

Density parameters Ω_i are dimensionless parameters for density contributions. Ω_Λ and Ω_I are density-parameter components of a total density parameter Ω^*_Λ which entail the whole scalar-field density distribution ϵ_Λ. In this sense, Ω^*_Λ is the density parameter of a scalar dark sector with energy density ϵ_Λ, here separated into its cosmological function and time-derivative parts.

Density parameters are in principle defined by means of the density of a certain density distribution $\epsilon_i = \varrho_i c^2$ and the critical density

$$\varrho_c = \frac{3H^2}{8\pi\tilde{G}}\,, \tag{8.4.5}$$

usually for time $t = t_0$.
ϱ_c is defined as the density which is necessary for a flat Universe $K = 0$. Furthermore, there is

$$\Omega_i = \frac{\epsilon_i}{\epsilon_c} = \frac{\varrho_i}{\varrho_c}\,. \tag{8.4.6}$$

Ω is the density parameter of matter ($i =$ matter): usually baryonic matter or baryonic matter plus additional relevant, yet unknown matter. It may in principle be defined in a more general way to contain further terms like neutrinos and other kinds of dark matter.
Radiation is here also given within Ω, especially within w,

$$\Omega = \frac{8\pi G_{eff}}{3H^2}\varrho + \frac{8\pi G_{eff}}{3H^2c^2}p \tag{8.4.7}$$

$$= \Omega_\epsilon + \Omega_p = \Omega_M + \Omega_R + \Omega_p + \ldots \tag{8.4.8}$$

Herein,

$$\Omega_\epsilon = \Omega_{Baryon} + \Omega_{Meson} + \Omega_\nu + \Omega_X + \Omega_R\,. \tag{8.4.9}$$

Herein, Ω_R is the radiation contribution in the Universe. Further $\Omega_{Baryon} \equiv \Omega_B$, Ω_{Meson}, Ω_{Lepton} and Ω_ν are the baryonic, mesonic and leptonic contributions. Ω_X stays for further contributions that might be Cold Dark Matter (CDM), for instance. Ω_ν, part of Ω_{Lepton} for that instance, may be called Hot Dark Matter (HDM) contribution (see Chapter 2.3).
Ω_p is the pressure contribution which is negligible for dust-dominance ($w \approx 0$).
Apart from possible CDM terms (which seem dominant), the only matter terms which are relevant in dynamics are the baryonic ones. Furthermore, in the current Universe where matter dominates, Ω_R and Ω_P of radiation are negligible, too.
However, there is Dark Energy as measured in high-z-measurements using super novae of type Ia. It acts antigravitationally and in terms of matter, as a negative pressure. Should it be considered as matter-typed, then an anti-stiff pressure term $w \approx -1$ would dominate. Such term is, however, often taken separately within a dark-energy density parameter Ω_Λ. In this model, it is related to the scalar-field excitation ξ. Additionally, there is Ω_I from equation (8.4.2) which is also related to ξ. The question is in which way do density parameters of the scalar field, together with $\tilde{G}$, relate to dark sectors of density in sense of usual models. All these parameters here may differ from the standard ones. For instance, Ω_i in the standard approach represent observed quantities based on a screened value of the gravitational constant (or of density), so that

$$\Omega_i = \left(\tilde{G}/G_0\right)\Omega_i^{std} \tag{8.4.10}$$

is given, whereas the geometry of the Universe is determined by the constant's "bare" value. For $\xi > 0$, there is $\Omega_i < \Omega_i^{std}$.

Let us write down the densities and density parameters in some of their forms for $\breve{\alpha} \gg 1$. There are the critical density

$$\epsilon_c = \frac{3H^2}{8\pi G(\xi)} \tag{8.4.11}$$

and the matter-density parameters

$$\begin{aligned}\Omega &= \Omega_\epsilon + \Omega_p \\ &= \frac{8\pi G(\xi)}{3H^2c^2}(\epsilon + 3p) = \frac{\epsilon + 3p}{\epsilon_c}\end{aligned} \tag{8.4.12}$$

and

$$\Omega_\epsilon = \Omega_M + \Omega_R, \tag{8.4.13}$$

wherein Ω_ϵ entails Ω_M of usual matter, Ω_R of radiation and further dark-matter sectors as defined in equations (8.4.8) and (8.4.9). Ω_M give the measured matter-density term as long as Ω_I does not contribute to that sector.

The scalar-field related parameters with the scalar-field excitation or its first derivative are

$$\begin{aligned}\Omega_\Lambda &= \frac{8\pi G(\xi)}{3H^2c^2}V = \frac{1\Lambda}{3H^2} \\ &= \frac{c^2}{4l^2H^2}\left(2 - \frac{G_0^2 + G(\xi)^2}{G_0G(\xi)}\right),\end{aligned} \tag{8.4.14}$$

$$\begin{aligned}\Omega_I &= \frac{8\pi G(\xi)}{3H^2c^2}(\epsilon_\Lambda - V) = \frac{\epsilon_I}{\epsilon_c} \\ &= \frac{c^2}{3H^2}\Lambda_I = -\frac{\dot{\xi}}{H(1+\xi)}.\end{aligned} \tag{8.4.15}$$

Here, the following energy densities and pressures are used,

$$\epsilon_I = \epsilon_\Lambda - V, \tag{8.4.16}$$

$$\begin{aligned}\epsilon_\Lambda &= V + \frac{3c^2}{8\pi G(\xi)^2}H\dot{G}(\xi) \\ &= V - \frac{3c^2}{8\pi G_0}H\dot{\xi},\end{aligned} \tag{8.4.17}$$

$$p_\Lambda = V + \frac{c^2}{8\pi G_0}\left(\ddot{\xi} + 2\frac{\dot{a}}{a}\dot{\xi}\right). \tag{8.4.18}$$

These equation-of-state components have already been discussed for a scalar-field dominated Universe in Chapter 8.2.

Finally, from equation (8.1.6), the density parameter related to second derivatives of the scalar-field excitations reads

$$\begin{aligned}\Omega_{II} &= -\frac{1}{3H^2}\left(\frac{\ddot{G}(\xi)}{G(\xi)} - 2\frac{\dot{G}(\xi)^2}{G(\xi)^2}\right) \\ &= \frac{8\pi G(\xi)}{3H^2c^2}(p_\Lambda + V) - \frac{2}{3}\frac{\dot{G}(\xi)}{G(\xi)H} \\ &= \frac{8\pi G(\xi)}{3H^2c^2}(p_\Lambda + V) - \frac{2}{3}\Omega_I\,.\end{aligned} \tag{8.4.19}$$

Ω_{II} is a term related to p_Λ, other than Ω_I, which is related to ϵ_Λ. In equation (8.4.2), $\Omega_\Lambda + \Omega_I$ equals the density parameter for the energy density ϵ_Λ. Hence,

$$\Omega_\Lambda^* = \frac{8\pi\tilde{G}}{3\,H^2c^2}\epsilon_\Lambda. \tag{8.4.20}$$

For Ω_{II} using the scalar-field equation, there is

$$\Omega_{II} = \frac{1}{H}\frac{\dot{G}(\xi)}{G(\xi)} + \frac{c^2}{3H^2l^2}\left(\frac{G(\xi)}{G_0} - 1\right) + \frac{8\pi G(\xi)}{9H^2}\hat{q}\,(\epsilon - 3p)\,. \tag{8.4.21}$$

This leads to

$$\Omega_{II} = \Omega_I + \frac{\hat{q}}{3}\,(\Omega_\epsilon - \Omega_p) + \frac{c^2}{3H^2l^2}\left(\frac{G(\xi)}{G(v)} - 1\right). \tag{8.4.22}$$

Its form depends on the coupling $\hat{q}$ to matter. For $\hat{q} = 0$, *i.e.* for Higgs particles which lead to the appearance of mass and decouple themselves from all particles, the density parameter Ω_{II} equals Ω_I.
Further, on the meaning of density parameters, the denominator in equation (8.4.1) may be written as $\bar{\Omega}H$. Let us define the following,

$$a^2 = \frac{\dot{a}^2 + Kc^2}{\bar{\Omega}H^2} =: \frac{Kc^2}{bH^2}\,. \tag{8.4.23}$$

Let us assume

$$b = \bar{\Omega} - 1. \tag{8.4.24}$$

This is valid, since

$$\frac{Kc^2}{bH^2} = a^2 = \frac{a^2}{\bar{\Omega}} + \frac{Kc^2}{\bar{\Omega}H^2} \iff a^2\bar{\Omega} = a^2 + \frac{Kc^2}{H^2} \tag{8.4.25}$$

$$\iff a^2 = \frac{Kc^2}{\left(\bar{\Omega} - 1\right)H^2} = \frac{Kc^2}{bH^2}. \tag{8.4.26}$$

(8.4.23) may be rewritten to get directly

$$\dot{a}^2 + Kc^2 = \frac{Kc^2\bar{\Omega}}{b} = \frac{Kc^2\bar{\Omega}}{\bar{\Omega} - 1} \tag{8.4.27}$$

and hence

$$b = \frac{Kc^2\bar{\Omega}}{\dot{a}^2 + Kc^2}\,. \tag{8.4.28}$$

Thus, there is

$$\dot{a}^2\left(\bar{\Omega} - 1\right) = Kc^2\,. \tag{8.4.29}$$

$\bar{\Omega} = \Omega_{total}$ and a value $\Omega_{total} \equiv 1$ entails $K = 0$ and a flat geometry of the Universe (see later), while smaller values entail $K = -1$ and an hyperbolic Universe.[6] $\Omega_{total} > 1$ means $K = 1$ and a closed Universe. Equivalently, $b = 0$ entails $K = 0$, $b > 0$ entails $K > 0$ and $b < 0$ entails $K < 1$. From observations of the cosmic microwave background (CMB), it is deduced that the dominant contribution of energy density comes from $\tilde{\Omega}_\Lambda$ of the cosmological constant or a form of dark energy with $\tilde{\Omega}_\Lambda \approx 0.7$ (for which we use a tilde to point out that it does not have to be the same, not even in nature, as here). Then there is Cold Dark Matter with $\Omega_{CDM} \approx 0.3$, in standard approaches as part of Ω_ϵ. Within standard Friedmann models, energy density of usual, baryonic matter is only about 1/10 the value of that of dark matter, hence almost negligible in cosmological terms (see Chapter 2.3). Furthermore, parting from the two-year-results of WMAP, the total density parameter Ω_{total} possesses a value near to unity. The experimental uncertainty is, however, too high to conclude $K = 0$.

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[6]The name Ω_{total}, may let one assume that the parameter Ω_{II} is part of it and this is not the case! $\bar{\Omega}$, however, is indeed the parameter of the total density according to usual approaches.

8.5 Deceleration and the equation-of-state parameters

Let us first take (*pro tem.*) $\dot{\xi} = 0$. The total equation-of-state parameter for $\dot{\xi} = 0$ yields for $\breve{\alpha} \gg 1$,

$$w_T = \frac{1}{3}\left[\frac{\hat{q} + 3(1-\hat{q})w - \frac{3c^2}{8\pi\tilde{G}\epsilon}\left(\frac{\Lambda}{3} + \frac{\xi}{l^2(1+\xi)}\right)}{1 + \frac{c^2}{8\pi\tilde{G}\epsilon}\Lambda}\right]. \tag{8.5.1}$$

With the critical density

$$\epsilon_c = \frac{3H^2}{8\pi\tilde{G}},$$

it yields

$$w_T = \frac{1}{3}\left[\frac{\hat{q} + 3(1-\hat{q})w - \frac{c^2}{3H^2}\frac{\epsilon_c}{\epsilon}\left(\frac{\Lambda}{3} + \frac{\xi}{l^2(1+\xi)}\right)}{1 + \frac{c^2}{3H^2}\frac{\epsilon_c}{\epsilon}\Lambda}\right]. \tag{8.5.2}$$

Further, using

$$\epsilon_c = \frac{\epsilon}{\Omega_\epsilon}$$

and

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H^2},$$

the equation-of-state parameter may be written in terms of density parameters. So, using

$$l^2 = \frac{c^2}{4H^2\Omega_\Lambda}\frac{\xi^2}{1+\xi}, \tag{8.5.3}$$

there is

$$w_T = \frac{1}{3}\left[\frac{\hat{q} + 3(1-\hat{q})\frac{\Omega_p}{\Omega_\epsilon} - \frac{\Omega_\Lambda}{\Omega_\epsilon}\left(1 + \frac{4}{\xi}\right)}{1 + \frac{\Omega_\Lambda}{\Omega_\epsilon}}\right] \quad \text{for} \quad \dot{\xi} = 0\,. \tag{8.5.4}$$

The evolution of the equation-of-state parameter w_T may be seen in figures 8.1 and 8.2.

According to equation (8.5.4), a negative equation-of-state parameter appears for

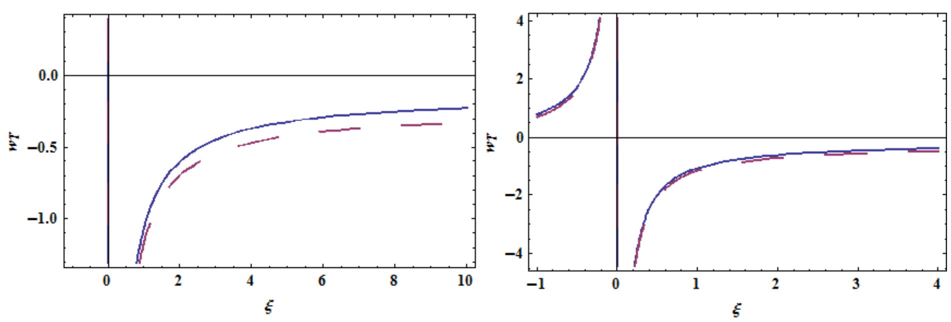

Figure 8.1: Total eos parameter w_T. *Pro tem.*: $\dot{\xi} = 0$, $\Omega_\epsilon = 0.3$, $\Omega_\Lambda = 0.7$). *N.B.: The continuous line stays for* $\hat{q} = 1$. *The dashed one for* $\hat{q} = 0$.

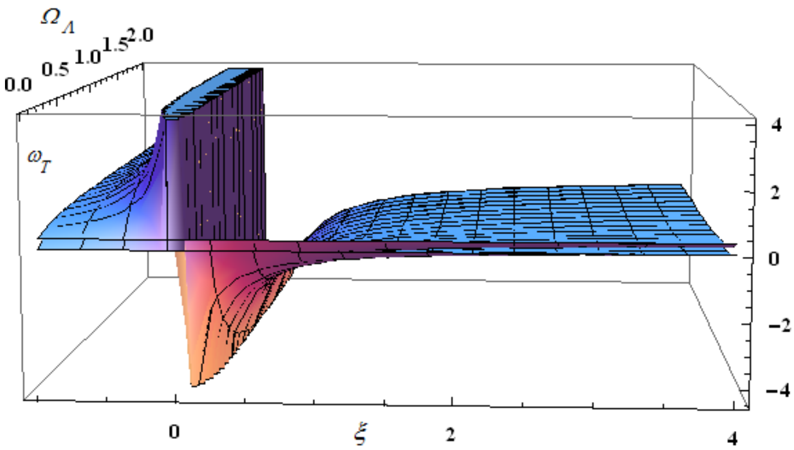

Figure 8.2: Total eos parameter w_T. *Pro tem.*: $\dot{\xi} = 0$ and $\Omega_\epsilon = 0.3$.

$$\hat{q}\Omega_\epsilon + 3(1-\hat{q})\Omega_p < \Omega_\Lambda \left(1 + \frac{4}{\xi}\right). \tag{8.5.5}$$

This can be visualized in figure 8.3 by means of the minimal values of matter and pressure densities in relation to Ω_Λ terms for positive total equation-of-state parameters.

Further, in the nonstatical case, the equation-of-state parameter may be easily generalized. There is

$$w_T = \frac{1}{3}\left[\frac{\hat{q}\Omega_\epsilon + 3(1-\hat{q})\Omega_p + \Omega_I - \Omega_\Lambda\left(1+\frac{4}{\xi}\right)}{\Omega_\epsilon + \Omega_I + \Omega_\Lambda}\right]. \tag{8.5.6}$$

Analogously to equation (8.5.5), there is $w_T < 0$ for

$$\hat{q}\Omega_\epsilon + \Omega_I + 3(1-\hat{q})\Omega_p < \Omega_\Lambda \left(1 + \frac{4}{\xi}\right). \tag{8.5.7}$$

This may be visualized in figures 8.4 and 8.5.
Matter and pressure terms Ω_ϵ and Ω_p, but also a positive density parameter Ω_I strengthen a deceleration behavior according to gravitation as an attractive interaction. They act against Quintessence as do usually sectors of matter. Consequently, it may be assumed that Ω_I acts as a dark sector of some kind of matter indeed. In other words, assume that there is

$$\Omega_M = \Omega_\epsilon + \Omega_I. \tag{8.5.8}$$

Furthermore, be Ω_ϵ roughly given by baryonic matter (subscript B). Hence, there should be a relation of the following form,[7]

$$\Omega_\epsilon \propto \Omega_B. \tag{8.5.9}$$

[7] Later, in equation (8.5.15), we will introduce baryonic matter as an antiscreened term of Ω_ϵ. Ω_M will then be given as in equation (8.5.8).

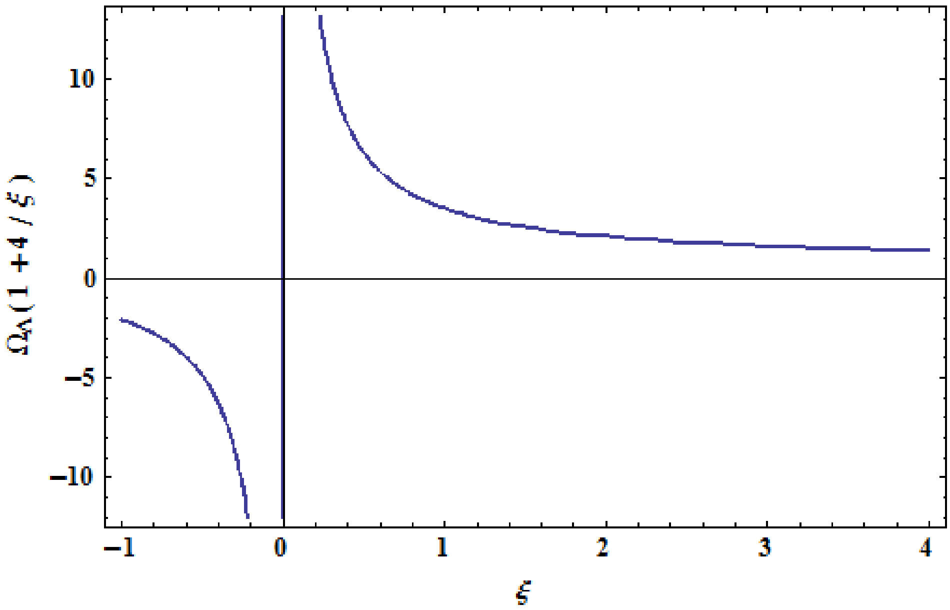

Figure 8.3: Minimal value of $\hat{q}\,\Omega_\epsilon + 3(1-\hat{q})\Omega_p$ for $w_T > 0$ with $\dot{\xi} = 0$ according to equation (8.5.5). Set: $\Omega_\Lambda = 0.7$.

Hence, Ω_I give a dark sector of phenomenological Dark Matter. Then, there is

$$\dot{\xi} < 0! \implies \dot{G}(\xi) < 0. \tag{8.5.10}$$

So, following equations (8.4.15) and (8.1.44) there is

$$\Omega_I > 0. \tag{8.5.11}$$

This gives a positive energy-density distribution of a dark-sector component. Such is related to in-time diminishing scalar-field excitations which are themselves related to diminishing changes of the coupling constant $G(\xi)$.
Take equation (8.5.6). Now, according to the latter interpretations, be the following set given:

$$\Omega_\epsilon = 0.03, \quad \Omega_p = 0, \quad \Omega_I = 0.27, \quad \Omega_\Lambda = 0.7. \tag{8.5.12}$$

Ad. val., there is

$$w_T \approx 0.33\left(-0.40 - \frac{2.8}{\xi}\right) \quad \text{for} \quad \hat{q} = 1, \tag{8.5.13}$$

$$\approx 0.33\left(-0.43 - \frac{2.8}{\xi}\right) \quad \text{for} \quad \hat{q} = 0\,. \tag{8.5.14}$$

This may be seen in figures 8.6 and 8.7. w_T is negative for $\xi > 0$. For $\xi = 1$, there is $w_T \approx -1.1$. For $\xi = 0.1$, there is $w_T \approx -9.5$. For $\xi = 10$, $w_T \approx -0.2$. There is $w_T = -1$ for $\xi = 1.077$ for $\hat{q} = 1$ and for $\xi = 1.089$ for $\hat{q} = 0$. For $0 < \xi < 4$, there is $\tilde{q} < 0$.

Another term to take into account is $\tilde{G}$ which too should lead to a difference between Ω_B and Ω_ϵ given by

$$\Omega_\epsilon \approx \frac{\Omega_B}{1+\xi}. \tag{8.5.15}$$

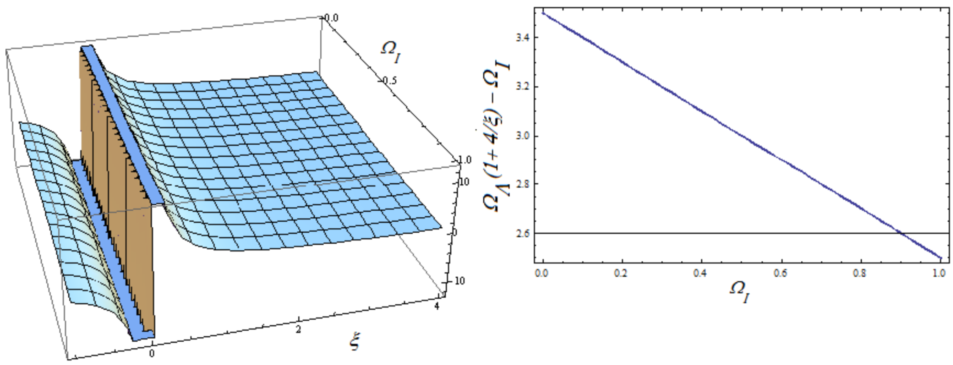

Figure 8.4: Minimal value of $\hat{q}\,\Omega_\epsilon + 3(1-\hat{q})\Omega_p$ for $w_T > 0$. Left: *pro tem.*: $0 \le \Omega_I \le 1$ and $-1 \le \xi \le 4$, according to equation (8.5.7). Set: $\Omega_\Lambda = 0.7$. Right: *pro tem.* $\xi = 1$.

For positive values of ξ, this means an anti-screening of matter density with $\Omega_\epsilon \lesssim \Omega_B$.
Let us now take the effective gravitational coupling into account. Assume $\Omega_B = 0.03$. Then, for $\xi = 1$, for instance, there is $\Omega_\epsilon = 0.015$, together with $\Omega_I = 0.285$ (for $\Omega_\epsilon + \Omega_I = \Omega_M$). If, on the other hand, there is $\xi = 0.1$, then there is $\Omega_\epsilon = 0.027$. Equivalently, for $\xi = 4$, there is $\Omega_\epsilon = 0.006$.
For $\Omega_M = \Omega_\epsilon + \Omega_I$ as the measured term of matter density, we have:

- Under neglection of further terms, the relation to baryonic matter reads $\Omega_\epsilon = \Omega_B/(1+\xi)$.
- Matter density may be given by Ω_ϵ as screened baryonic density plus further dark terms Ω_I.
- For $\dot{\xi} < 0$, there is $\Omega_I > 0$.
- For $\xi > 0$ ($\xi < 0$), for $\Omega_\epsilon + \Omega_I = \Omega_M$, there is $\Omega_I > \Omega_{DM}^{SM}$ ($\Omega_I < \Omega_{DM}^{SM}$).

There are the following values of w_{T0} and w_{T1} for $\Omega_\Lambda = 0.7$ and $\hat{q} = 0$ and $\hat{q} = 1$ respectively:

ξ	Ω_ϵ	Ω_I	w_{T0}	w_{T1}
0.01	0.030	0.270	-93.477	-93.467
0.10	0.027	0.273	-9.476	-9.467
0.50	0.020	0.28	-2.007	-2.000
1.00	0.015	0.285	-2.007	-1.067
1.50	0.012	0.288	-0.756	-0.756
2.00	0.010	0.290	-0.603	-0.603
4.00	0.006	0.294	-0.369	-0.367

Under these assumptions (with $\Omega_M = \Omega_\epsilon + \Omega_I$), there is for $\hat{q} = 1$,

$$w_T = \frac{1}{3}\left[\frac{\Omega_M - \Omega_\Lambda\left(1 + \frac{4}{\xi}\right)}{\Omega_M + \Omega_\Lambda}\right]. \tag{8.5.16}$$

For it, w_T is exactly -1 for $\xi = 1.07692$. Such a value would mean an anti-screening of Ω_ϵ by roughly one half.[8]

[8] *N.B.*: An analysis of the behavior of Ω_I is necessary for different epochs of the Universe. Such an analysis needs of a better comprehension of the relation between bare and effective densities and density parameters as well as of the time-dependence of the effective parameters. Hence, as grounding of exact analyses of the nature of Ω_I and its relation to dark sectors, the work in Chapter 8.6 is of special relevance.

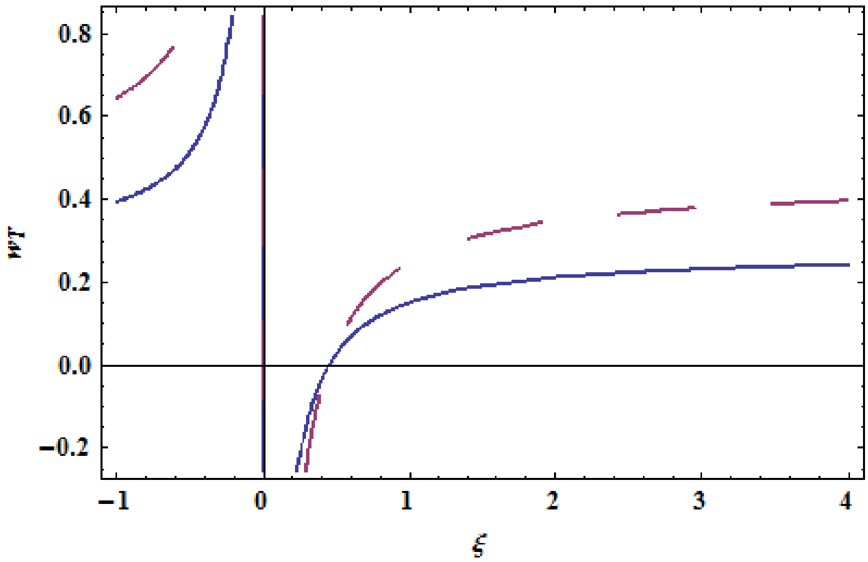

Figure 8.5: Example of a positive eos parameter in a highly matter-dominated, closed universe for $\Omega_\epsilon = 3$, $\Omega_I = 4, \Omega_\Lambda = 0.7$ for different excitations ξ. The dashed curve stays for $\hat{q} = 0$ and the continuous one for $\hat{q} = 1$. *N.B.: Low excitations would need of higher matter-dominance for total dust or stiffness of w_T to be acquired. This may be translated in higher length scales l.*

8.6 Effective and bare density parameters

Let us now rewrite the Friedmann equations by dividing the first generalized Friedmann equation by the present scale factor a_0. This leads to

$$\frac{\dot{a}^2}{a_0^2} + \frac{Kc^2}{a_0^2} = \frac{8\pi}{3}\frac{G_{eff}}{c^2}\epsilon\left(\frac{a}{a_0}\right)^2 + \frac{1}{3}\Lambda(\xi)c^2\left(\frac{a}{a_0}\right)^2 + \frac{\dot{a}a}{a_0^2}\frac{\dot{G}_{eff}}{G_{eff}} + \left(\frac{a}{a_0}\right)^2\frac{\pi}{3\breve{\alpha}}\frac{\dot{G}_{eff}^2}{G_{eff}^2}. \tag{8.6.1}$$

Equivalently, writing the matter term explicitly, there is

$$\begin{aligned}\frac{\dot{a}^2}{a_0^2} + \frac{Kc^2}{a_0^2} =& \frac{8\pi}{3}\frac{G_0}{c^2}(1+\xi)^{-1}\epsilon\left(\frac{a}{a_0}\right)^2 + \frac{1}{3}\Lambda(\xi)c^2\left(\frac{a}{a_0}\right)^2 + \\ &+ \frac{\dot{a}a}{a_0^2}\frac{\dot{G}_{eff}}{G_{eff}} + \left(\frac{a}{a_0}\right)^2\frac{\pi}{3\breve{\alpha}}\frac{\dot{G}_{eff}^2}{G_{eff}^2}.\end{aligned} \tag{8.6.2}$$

So, ξ affects the matter term.

We have for a generic-time (t) density and for the present one ($t = t_0$) the following relation,

$$\epsilon = \epsilon_0\left(\frac{a_0}{a}\right)^{3(1+w)}\left(\frac{\tilde{G}_0}{\tilde{G}}\right)^{\frac{1}{2}(1-3w)(1-\hat{q})}, \tag{8.6.3}$$

and therefore for $a = a_0$ (or equivalently $z = 0$),

$$\Omega_{0\epsilon} = \frac{\epsilon_0}{\epsilon_{0c}}. \tag{8.6.4}$$

For generic times there are two parts to be taken into account for time dependence:

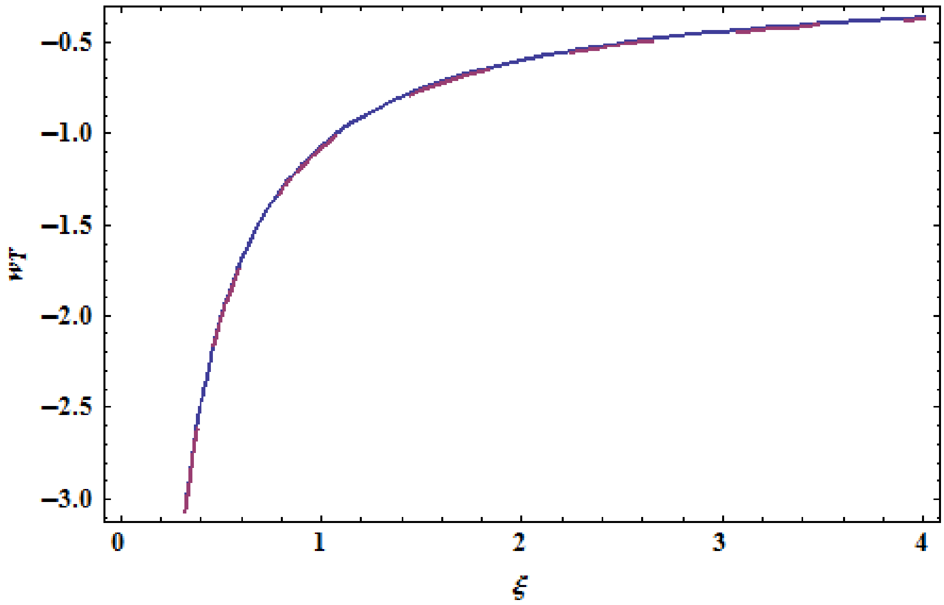

Figure 8.6: Total eos parameter. *Pro tem.*: $\Omega_\epsilon = 0.03$, $\Omega_I = 0.27$, Ω_Λ. ξ (length scale, *cf.* (8.5.3)) variable. *N.B.: For negative excitations ξ, w_T is positive.*

(i) The time dependence of ϵ which is given by $a^{-3(1+w)}$. This is the usual dependence of standard Friedmann models.

(ii) The time dependence of $\tilde{G}^{-1}$ in ϵ in a term nonvanishing for $\hat{q} = 0$.

(iii) The time dependence of ϵ_c, where $\tilde{G}^{-1}$ is found. For $\hat{q} = 1$ and $w = 1/3$, however, the $\tilde{G}$ terms cancel from ϵ. For $\hat{q} = 1$, higher scalar-field excitations belonging to a higher-valued critical density ϵ_c, a given value of ϵ_0 would mean lower values of Ω_ϵ. If the density parameter Ω_ϵ is, on the other hand, set, then ϵ has to possess a larger value (anti-screening) within standard formalism.
We may write the terms coming from the time-changing of the critical density by some functions y or y_i respectively (which depend on w). They depend on ξ in terms of $\tilde{G}$.

There is

$$\epsilon_c = \frac{3H^2}{8\pi\tilde{G}} = \frac{3H_0^2}{8\pi G_0}(1+\xi). \tag{8.6.5}$$

Thus, we define

$$\epsilon^* = \frac{3H^2}{8\pi G_0} = \frac{\epsilon}{1+\xi} \tag{8.6.6}$$

as the screened density which is, especially in the case $\hat{q} = 1$, independent on ξ. Equivalently for density parameters Ω_i. In the same way, we define an "anti-screened" quantity

$$x^= = x(1+\xi). \tag{8.6.7}$$

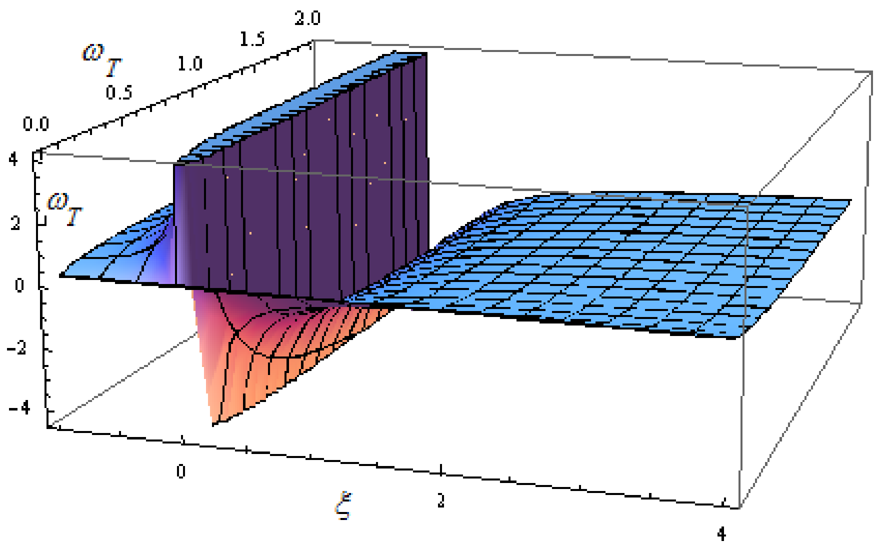

Figure 8.7: Total eos parameter. *Pro tem.*: $\Omega_\epsilon = 0.03$, $\Omega_I = 0.27$, $\Omega_\epsilon + \Omega_I = \Omega_M$. *N.B.: The curve is the same as for the case* $\dot{\xi} = 0$ *and* $\Omega_\epsilon = 0.3$.

There is

$$\Omega_\epsilon = (1+\xi)^{-1}\Omega^{=}_{0\epsilon}\left(\frac{\tilde{G}_0}{\tilde{G}}\right)^{\frac{1}{2}(1-3w)(1-\hat{q})}\left(\frac{a_0}{a}\right)^{3(1+w)} \tag{8.6.8}$$

$$= \tilde{\Omega}^*_{0\epsilon}\left(\frac{a_0}{a}\right)^{3(1+w)}, \tag{8.6.9}$$

with an effective parameter as follows,

$$\tilde{\Omega}^*_{0\epsilon} = \Omega^{=}_{0\epsilon}\left(\frac{\tilde{G}_0}{\tilde{G}}\right)^{\frac{1}{2}(1-3w)(1-\hat{q})}. \tag{8.6.10}$$

This parameter contains the screening effect for $\hat{q} = 0$ and the screening from ϵ_c shown explicitly in equation (8.6.5). This should be a measured value within standard formalism, entailing not only matter-density terms. Taking time dependence for the density parameter with $\epsilon(a)$, there is

$$\Omega^{=}_\epsilon = \Omega_\epsilon \cdot (1+\xi)\,. \tag{8.6.11}$$

ξ is a function of time and hence of H^{-1}. Its value is of some form

$$\xi \propto \xi_0\left(\frac{a_0}{a}\right)^{m_w}, \tag{8.6.12}$$

with an amplitude ξ_0 and a time-dependence term m_w which is high-valued and negative for negligible time dependence of the scalar field. High amplitudes of the excitation, however, screen the density parameter of matter to be smaller than the actual density would lead to assume. The one parameter for unscreened terms is

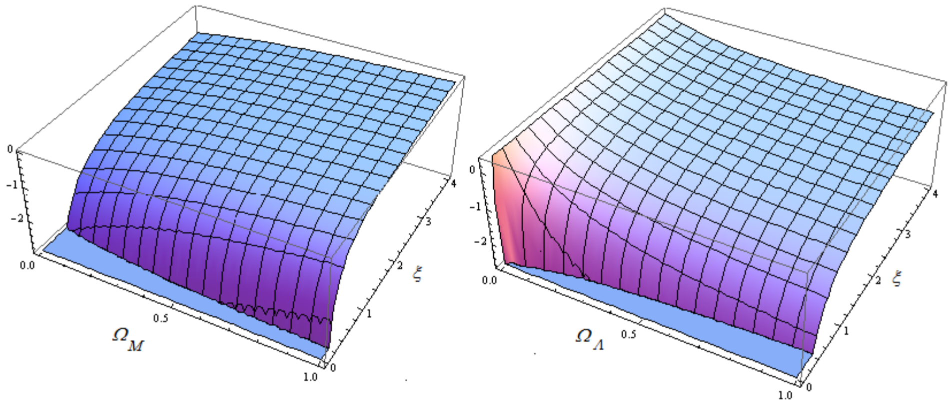

Figure 8.8: Total eos parameter w_T. *N.B.*: $\Omega_\epsilon + \Omega_I = \Omega_M$, Ω_ϵ: anti-screened baryonic parameter. Left: $\Omega_\Lambda = 0.7$, $\Omega_B = 0.03$. Right: $\Omega_B = 0.03$, $\Omega_M = 0.3$.

$\Omega^{=}_{0\epsilon}$. It should give the actual value of density while $\Omega_{0\epsilon}$ is the screened (measured) parameter, analogously to (bare, luminous) mass and effective (measured) mass (see Chapters 7.2 and 7.5). Hence, for small time variations of ξ, high amplitudes of the scalar-field excitations would lead to the phenomenological appearance of Dark Matter in terms of a screening effect of scalar fields on density.

Now, we rewrite equation (8.6.1) as follows,

$$\begin{aligned}\frac{\dot{a}^2}{a_0^2} + \frac{Kc^2}{a_0^2} =&\Omega^{=}_{0\epsilon}\left(\frac{\tilde{G}_0}{\tilde{G}}\right)^{\frac{1}{2}(1-3w)(1-\hat{q})}\left(\frac{a_0}{a}\right)^{1+3w}(1+\xi)^{-1} + \frac{1}{3}\Lambda(\xi)c^2\left(\frac{a}{a_0}\right)^2 + \\ &+ \frac{\dot{a}a}{a_0^2}\frac{\dot{G}(\xi)}{G(\xi)} + \left(\frac{a}{a_0}\right)^2\frac{\pi}{3\breve{\alpha}}\frac{\dot{G}(\xi)^2}{G(\xi)^2}.\end{aligned} \tag{8.6.13}$$

Using equation (8.6.3), the Friedmann equation then reads for "usual" matter M with $w = 0$, radiation ($w = 1/3$) and dark sectors,

$$\begin{aligned}\left(\frac{\dot{a}}{a_0}\right)^2 =&H_0^2\left[\Omega^{=}_{0M}(1+\xi)^{-1}\left(\frac{\tilde{G}_0}{\tilde{G}}\right)^{\frac{1}{2}(1-\hat{q})}\left(\frac{a_0}{a}\right) + (1+\xi)^{-1}\Omega^{=}_{0R}\left(\frac{a_0}{a}\right)^2 + \right.\\ &+(1+\xi)^{-1}(\Omega^{=}_{\Lambda} + \Omega^{=}_{I})\left(\frac{a_0}{a}\right)^{-2} + \\ &\left.+(1-(1+\xi_0)^{-1}(\Omega^{=}_{0M} + \Omega^{=}_{0R} + \Omega^{=}_{0\Lambda} + \Omega^{=}_{0I}))\right],\end{aligned} \tag{8.6.14}$$

$\Omega^{=}_i$ are the "anti-effective" terms $\Omega_i \cdot (1+\xi)$, thus constants not explicitly dependent on ξ. $(1+\xi_0)^{-1}\Omega^{=}_{0i} = \Omega_{0i}$ is the measured density parameter.
Alternatively, there is

$$\begin{aligned}H^2(t) =&H_0^2\left(\left(\frac{a_0}{a}\right)^2\left[\left(\Omega^*_{0M}\left(\frac{a_0}{a}\right) + \Omega^*_{0R}\left(\frac{a_0}{a}\right)^2 + (\Omega^*_\Lambda + \Omega^*_I)\left(\frac{a_0}{a}\right)^{-2}\right) + \right.\right.\\ &\left.\left.+(1 - \Omega^*_{0M} - \Omega^*_{0R} - \Omega^*_{0\Lambda} - \Omega^*_{0I}))\right]\right).\end{aligned} \tag{8.6.15}$$

Here, we define $\Omega^*_R = \Omega_R$, $\Omega^*_\Lambda = \Omega_\Lambda$ and $\Omega^*_I = \Omega_I$. Furthermore, density parameters $\Omega_{0\epsilon} + \Omega_{0\Lambda} + \Omega_{0I} = 1$ lead to $K = 0$.

- **Constant and quasi constant excitations $\xi = const.$:**
There is the cosmological function Λ which gives the density parameter Ω_Λ by means of equation (8.4.14) for $H = H_0$. In case of negligible time dependence of the scalar field, this term is exactly as within standard dynamics with dark energy $\Lambda = \Lambda_0$. There is

$$\Omega_{0\Lambda} = \frac{c^2}{3}\frac{\Lambda}{H_0^2} \quad \text{for } \xi = const. \tag{8.6.16}$$

In the case of equation (8.6.16), then the parameters Ω_I and Ω_{II} given by equations (8.4.15) and (8.4.19) vanish. However, if equation (8.6.16) is only nearly given in the current Universe, these parameters might play an important role in primeval dynamics. A short discussion of the, however rather standard, cases of $\xi = const.$, especially for $\xi = 0$ (*i.e.* Einstein–deSitter and ΛCDM cosmology) is shown in Appendix C.4.
For $\xi = const.$, it can be stated here for a three-fluid system with cosmological constant,

$$\left(\frac{\dot{a}}{a_0}\right)^2 = H_0^2\left[\Omega_{0M}^*\left(\frac{a_0}{a}\right) + \Omega_{0R}^*\left(\frac{a_0}{a}\right)^2 + \Omega_{0\Lambda}^*\left(\frac{a_0}{a}\right)^{-2} + (1 - \Omega_{0M}^* - \Omega_{0R}^* - \Omega_{0\Lambda}^*)\right]. \tag{8.6.17}$$

Here, we have used the effective density parameters as defined in equation (8.6.10). For $\hat{q} = 1$, $\Omega_{0\epsilon}^*$ equals $\Omega_{0\epsilon}$ which is also a screened value. For high values of the amplitude ξ_0, these screened values are smaller than the actual bare parameter $\Omega_{0\epsilon}^{=}$. For $\xi < 0$, this leads to phenomena like the ones of Dark Matter. $\xi_0 = -0.9$ would lead, for instance, to $\Omega_i = 10\Omega_i^{=}$, with Ω_i as dynamical measured value.
In case of a constant scalar field, let there be (*pro tem.*[9])

$$\Omega_\epsilon^{=} \approx \Omega_M(1 + \xi_0)^{-1}, \quad \Omega_M^{=} = \Omega_B = 0.03\,. \tag{8.6.18}$$

Further, let there be

$$\Omega_\epsilon \approx \Omega_M = \Omega_{dyn} = 0.3\,. \tag{8.6.19}$$

Then, there would be an amplitude

$$\xi_0 = -0.9\,. \tag{8.6.20}$$

On the other hand, for $\Omega_{0\Lambda}$, a cosmological constant is given here by $\xi = const$. This may account for the phenomenon of Dark Energy, depending on the value of the amplitude of the scalar-field excitations and its length scale ($\Omega_\Lambda \sim l^{-2}\xi_0^2$). There is

$$\Lambda = \frac{3}{4l^2}\frac{\xi^2}{1+\xi}. \tag{8.6.21}$$

The general form of Ω_Λ reads

$$\Omega_\Lambda = \frac{1}{H_0^2}\frac{c^2}{4l^2}\xi^2(1+\xi)^{-1}\,. \tag{8.6.22}$$

[9] This is in contraposition with the analysis of Chapter 8.6. However, this is an analysis for $\xi \approx const.$ while in the latter Chapter, the analysis focuses on Ω_I. Such a different approach is important given still unknown matters of the evolution of effective density parameters. See the next point of discussion within this Chapter.

For $\xi = -0.9 = const.$, there is

$$\Omega_\Lambda = 20.25 \frac{c^2}{l^2 H_0^2} \,. \tag{8.6.23}$$

The Hubble rate reads approximately

$$H_0 = 73\text{km/s/Mpc} \approx 2.3 \cdot 10^{-18}/\text{s} \,. \tag{8.6.24}$$

Thus, there would be

$$\Omega_\Lambda \approx 3.4 \cdot 10^{56} \frac{\text{cm}^2}{l^2} \,. \tag{8.6.25}$$

Should this parameter possess a value of about 0.7, then, the length scale would be

$$l \approx 2.2 \cdot 10^{28}\text{cm} \approx 7.2\text{Gpc}, . \tag{8.6.26}$$

A parameter of about 0.7 with length scales in the range of kpc would entail excitations ξ nearer to zero. $\xi = -0.1$ would mean length scales in the range of Mpc. For ranges of some kpc, ξ would have to lie below $\pm 10^{-5}$, see equation (8.1.4).
For $\xi = const.$, the interpretation depends on ξ_0, *i.e.* on the value of the constant scalar field. Unlike ΛCDM, it may be expected not only as main contribution to the cosmological constant Λ_0 and thus of the Dark Energy parameter, but it may be expected within matter distribution as well. For constant fields, a contribution both as Dark Matter and as Dark Energy is only possible for high-scaled nearly vanishing scalar fields $\xi \approx -1$ (*i.e.* $\phi \approx 0$) which screen matter density. Nearly vanishing constant excitations may lead only to Dark Energy. However, according to Chapter 7.8, the length scale for Dark Matter phenomenology should be of the order of magnitude of galaxy bulges. The Milky Way (as a usual galaxy) has a bulge of 5 kpc, which is of the order of magnitude of 10^{20}m. Hence, the length scale for Dark Energy in the case $\xi = const.$ is too high. On the other hand, a length scale of the order of magnitude of galaxy bulges leads to far too high values of Ω_Λ. Scalar-field time dependence shall indeed play a role within dark sectors of density.

- **Derivatives and possible dark sectors:**
Scalar-field excitations cannot be exactly statical and further terms are to be added in terms of Ω_I and Ω_{II}. In that case, Dark Matter may be given as discussed in Chapter 8.5 with Ω_I as part of a dark sector of matter. Another option may be seen from (8.4.15), which may be written as (for $\breve{\alpha} \gg 1$)

$$\Omega_I = \frac{1}{3H_0^2} \Lambda_I = \frac{\dot{G}(\xi)}{H_0^2 G(\xi)} \frac{\dot{a}a}{a_0} . \tag{8.6.27}$$

Hence, it may be expected that for the measured density parameter of Dark Energy, there be

$$\Omega_\Lambda^{std} = \Omega_\Lambda + \Omega_I . \tag{8.6.28}$$

However, if we assume that Ω_Λ is nearly constant, given low dynamical behavior of the scalar field, then Ω_I is to possess negative values. The effective coupling $\tilde{G}$ is to diminish with time. In that case, there would be $\xi \to -1$. Nonetheless, it is however possible that more complex dynamical behavior of the scalar field leads to other situations. For instance, Ω_I might possess the scale-factor dependence of a matter density and hence act as a dark sector of matter (as discussed in Chapter 8.5). If this were so and Dark Energy were fully given by Ω_Λ, a density value of Dark Energy of 0.7 with a length scale as given by flat rotation curves (say $35l = 5$kpc) would mean a field excitation of

$$\xi \approx \pm 2 \cdot 10^{-23}. \tag{8.6.29}$$

In this case, a dark sector of matter Ω_I had to possess the following property,

$$\frac{\dot{a}a}{a_0^2}\frac{\dot{G}(\xi)}{G(\xi)} = k\frac{8\pi G(\xi)}{3\,c^2}\epsilon\left(\frac{a}{a_0}\right)^2, \tag{8.6.30}$$

with a constant $k \approx 9$. Further, for $t = t_0$, there were

$$\dot{G}(\xi)_0 \approx k\epsilon 1.7 \cdot 10^{-18}\mathrm{m}^4\mathrm{kg}^{-2}s^1. \tag{8.6.31}$$

Taking only baryons and with $\Omega_B \approx 0.03$, the baryonic energy density for $k = 9$ reads

$$\epsilon \propto 10^{-13}\mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{-2}. \tag{8.6.32}$$

Hence, there would be

$$\dot{G}(\xi)_0 \approx 10^{-4}\mathrm{m}^2 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^3. \tag{8.6.33}$$

Further, given time derivatives of $\tilde{G}$, and the appearance of $\tilde{G}$ in Ω_i, the redshift dependence of Ω_i as a screened term may possess a different behavior in dependence of time. Such analysis is to be fulfilled in further developments.

8.7 Breaking of energy conditions and conditions of a Bounce

For perfect fluids, there is the weak energy condition which may be written as follows,

$$\epsilon \geq 0. \tag{8.7.1}$$

Furthermore, there is the strong energy condition, which we write as below,

$$\epsilon + 3p \geq 0. \tag{8.7.2}$$

Already from equation (8.1.18) it is clear that for vanishing scalar-field excitations and derivatives of the same (without antigravitative matter terms), if the energy conditions (8.7.1) and (8.7.2) are valid, there can be no accelerations $\ddot{a} > 0$ [123, 194]. The conditions (8.7.1) and (8.7.2) together are known as Penrose–Hawking condition. Further, given the concaveness of $a(t)$ for all times under equations (8.7.1) and (8.7.2), $a(t)$ must be equal to zero at some time in the past (which we usually label $t = 0$). Since $a(0) = 0$ at this point, the density diverges, as does the Hubble expansion parameter. There appears a singularity, usually known as the Big Bang, and because $a(t)$ is concave, the time between the singularity and the epoch t must always be less than the Hubble time $\tau_H = 1/H$.
In all homogeneous and isotropic models for which the Zel'dovich interval of equation-of-state parameters is valid ($0 \geq w \geq 1$), and even for $-1/3 < w < 0$, a Big Bang singularity is unavoidable. The Big Bang can, however, be avoided in models with dominant negative pressure with $p \leq -\epsilon/3$ or in those containing a nonvanishing cosmological constant or with some values of the cosmological function, *i.e.* of scalar-field

excitations or of its derivative (concaveness of $a(t)$ is then no longer valid throughout). In other words, it can be avoided for a dominance of Ω_Λ or Ω_I in the dynamics. These terms then contribute as pressure terms which violate the Penrose–Hawking condition. They lead to dark-energy behavior.
Within standard cosmology, there are problems as horizon and reheating which are usually solved by means of Inflation. Inflation may be explained as a mechanism by which the Universe expands very rapidly, and in usual models, exponentially. The Universe passes a deSitter epoch in which a cosmological constant or a related negative pressure dominates (*cf.* Chapters 2.4 and 8.3). The way this pressure (or cosmological function) actually evolves is determinant for determining the initial state of the Universe. Furthermore, how the scalar field (as inflaton field) evolves is crucial for the dynamics of Inflation and for the values taken by effective pressures, given the discussion of Chapter 2.4.
An inflationary universe with induced gravitation can be derived within the context of induced gravity with Higgs potential (*cf.* [47–49]). This model can lead to primeval *New* or *Chaotic* Inflation indeed (*op. cit.*). As a matter of fact, the Penrose–Hawking energy condition $3p+\varrho c^2 \geq 0$ [123, 194] may be broken for Chaotic Inflation. For this kind of Inflation, a Big Bounce would be expected (that means no initial singularity before Inflation). This case can be compared with the case of the works in [67], according to which Yukawa interactions of the magnitude of the nuclear density can lead to negative pressures that might play an important role in early stages of the Universe so that the Penrose–Hawking condition may not be satisfied. This Yukawa interaction in the primordial Universe would be related to a pressure as p_Λ (coming from the potential $V(\xi)$ and the scalar-field derivatives, translated as the variable gravitational coupling), possibly contributing to the mechanism of Inflation and Dark Energy as part of the cosmological term Λ.
Without further introduction, the Friedmann equations with the scalar field equation lead to

$$2\left[\frac{\ddot{a}}{a}+\frac{\dot{a}^2+Kc^2}{a^2}\right]=\frac{8\pi\tilde{G}}{3c^2}(1-\hat{q})(\epsilon-3p)+\frac{c^2}{l^2}\xi. \tag{8.7.3}$$

Let us take a general time $t=t_q$ which shall have the following properties:

(i) $a(t_q)\neq 0$,

(ii) $\dot{a}(t_q)=0$,

(iii) $\varrho(t_q)=0$.

t_q shall be identified with $t\approx 0$. This shall be a statical universe without initial singularity. For it, in general, there is from equation (8.1.5),

$$\begin{aligned} f_1(t_q) =& \frac{\pi}{3\breve{\alpha}}\frac{\dot{G}(\xi)(t_q)^2}{G(\xi)(t_q)^2} \\ =& \frac{\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2(t_q)}{(1+\xi(t_q))^2}. \qquad (8.7.4)\\ =& \frac{\pi}{\breve{\alpha}}\frac{\dot{\xi}^2(t_q)}{\xi(t_q)^2}. \qquad (8.7.5)\end{aligned}$$

Further, it is easily seen that for $\breve{\alpha}\gg 1$, this correction vanishes with

$$f_1(t_q)=0\,. \tag{8.7.6}$$

The cosmological function for the time $t=t_q$ reads

$$\Lambda(\xi(t_q))\equiv\Lambda_q=\frac{3}{4l^2}\frac{\xi^2(t_q)}{1+\xi(t_q)}\left(1+\frac{4\pi}{3\breve{\alpha}}\right). \tag{8.7.7}$$

It is positive for $\xi(t_q) > -1$.
The first Friedmann equation (8.1.5) then yields

$$\frac{Kc^2}{a^2(t_q)} = \frac{\pi}{3\breve{\alpha}} \frac{\dot{\xi}^2(t_q)}{(1+\xi(t_q))^2} + \frac{c^2}{4l^2} \frac{\xi^2(t_q)}{(1+\xi(t_q))^2} \left(1 + \frac{4\pi}{3\breve{\alpha}}\right). \tag{8.7.8}$$

Given the properties of ξ and a, the Universe has to be closed or flat, *i.e.*

$$K \geq 0. \tag{8.7.9}$$

$K = 0$, however, is given only if both $\dot{\xi}(t_q) = 0$ and $\xi(t_q) = 0$ are valid.
Let us further write

$$\xi(t_q) \equiv \xi_q \quad \text{and} \quad a(t_q) \equiv a_q \tag{8.7.10}$$

and equivalently with all other quantities.
With help of the definition of the effective gravitational coupling $\tilde{G}$, there is

$$\frac{\xi^2}{1+\xi} = \frac{\tilde{G}}{G_0}\xi^2. \tag{8.7.11}$$

Further, following the first Friedmann equation (8.1.5),

$$\frac{Kc^2}{a_q^2} = \frac{\pi}{3\breve{\alpha}} \frac{\dot{\xi}_q}{(1+\xi_q^2)} + \frac{c^2}{4l^2} \frac{\xi_q^2}{(1+\xi_q)} \left(1 + \frac{4\pi}{3\breve{\alpha}}\right), \tag{8.7.12}$$

there is for $K = 1$,

$$\frac{\xi_q^2}{1+\xi_q} = \frac{4l^2}{a_q^2} \left(1 + \frac{4\pi}{3\breve{\alpha}}\right)^{-1} - \frac{\pi}{3\breve{\alpha}} \frac{4l^2 \dot{\xi}_q^2}{(1+\xi_q)^2}. \tag{8.7.13}$$

For $\breve{\alpha} \gg 1$, equation (8.7.13) further reads as follows,

$$\frac{\xi_q^2}{1+\xi_q} = \frac{4l^2}{a_q^2}. \tag{8.7.14}$$

Λ_q is thus basically given by l^2/a_q^2. Furthermore, for $\breve{\alpha} \gg 1$, the scalar-field excitation for $t = t_q$ reads

$$\xi_q = \frac{2l^2}{a_q^2} \left(1 \mp \sqrt{1 + \frac{a_q^2}{l^2}}\right), \tag{8.7.15}$$

and for $\xi_q \gg 1$, consequently,

$$\xi_q = \frac{4l^2}{a_q^2}. \tag{8.7.16}$$

Equation (8.7.15) gives only negative values for a negative sign before the square-root. We have two cases to analyze. Let us define:

- **The upper-sign case (minus) in equation (8.7.15) we will call (-)**.

- **The lower-case (plus) sign in equation (8.7.15) will be (+)**.

For $a_q/l \to 0$ in **(-)**, there is $\xi_q \to -1$. Let us see this as follows: first, take for the case **(-)**,

$$\delta \equiv (a(t_q)/l)^2 \quad \text{and} \quad \delta \ll 1\,. \tag{8.7.17}$$

Then, there is

$$\sqrt{1+\delta} \approx 1 + \frac{1}{2}\delta, \tag{8.7.18}$$

and hence for **(-)**,

$$\begin{aligned}\xi_q =& \frac{2l^2}{a_q^2}\left(1 - \sqrt{1+\delta}\right) \\ =& -\frac{l^2}{a_q^2}\delta. \end{aligned}\tag{8.7.19}$$

(8.7.20)

Hence, we have for the case (-),

$$\xi_q \approx -1\,. \tag{8.7.21}$$

This represents a vanishing of the scalar field $\phi \cong v\sqrt{1+\xi}$.

The continuity condition reads for $t = t_q$,

$$\dot{\varrho}_q = -(1-\hat{q})\frac{3}{2}\frac{\dot{\xi}_q}{1+\xi_q}p_q. \tag{8.7.22}$$

$\dot{\varrho}_q$ is to be zero for sign changing to be given at $t = t_q$. For $\hat{q} = 1$, this does not have to be forced as a condition (*cf.* equation (8.7.22)), as it is directly given. For $\hat{q} = 0$, there must be

$$0 = -\frac{6l^2}{a_q^2}p_q\frac{\dot{\xi}_q}{\xi_q^2} \tag{8.7.23}$$

$$= -\frac{3}{2}\frac{a_q^2}{l^2}p_q\dot{\xi}_q\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right)^{-2}. \tag{8.7.24}$$

Therefore:

- **For $\hat{q} = 0$, there is either $\dot{\xi}(t_q) = 0$ or $p(t_q) = 0$, for both cases (+) and (-)!**

Take the scalar-field equation (8.1.4). For $t = t_q$, it is

$$\ddot{\xi}_q = -\frac{c^2}{l^2}\xi_q - \kappa_0\hat{q}p_q c^2. \tag{8.7.25}$$

With equation (8.7.15), we have

$$\ddot{\xi}_q = -\frac{2}{a_q^2}\left[1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right] - \hat{q}\kappa_0 p_q. \tag{8.7.26}$$

For $\hat{q} = 0$ or $p(t_q) = 0$, $\ddot{\xi}(t_q)$ is positive for **(-)** and negative for **(+)**. In the case **(-)**, $|\ddot{\xi}(t_q)|$ is very small if $a_q \ll l$. In the same case for **(+)**, there is with both general $\hat{q}$ and p_q,

$$\ddot{\xi}_q \approx -\frac{4}{a_q^2} - \hat{q}\kappa_0 p_q c^2, \quad \text{for} \quad a_q \ll l, \quad \text{and } (+). \tag{8.7.27}$$

Furthermore, there is for high initial scales,

$$\ddot{\xi}(t_q) \approx \pm\frac{2}{a(t_q)l} - \hat{q}\kappa_0 p(t_q)c^2, \quad \text{for} \quad a_q \gg l. \tag{8.7.28}$$

Here, the upper sign belongs to **(-)**, and the lower one to **(+)**.
With either $\hat{q}=0$ or $p_q=0$, **there is** $\xi_q=0$ **for**

$$1 \mp \sqrt{1+\frac{a^2(t_q)}{l^2}} = 0. \tag{8.7.29}$$

Hence, it is especially relevant that, although small-valued, δ be not zero. Else, there is $\xi_q=0$ in **(-)**. Such would further mean $K=0$.

The second Friedmann equation (8.1.6) leads to the following,

$$2\frac{\ddot{a}_q}{a_q} + \frac{Kc^2}{a_q^2} = -\frac{\kappa_0}{1+\xi_q}\frac{p_q}{c^2} + \frac{3}{4l^2}\frac{\xi_q^2}{1+\xi_q} - \frac{\ddot{\xi}_q}{1+\xi_q}, \tag{8.7.30}$$

which with equation (8.7.14) directly leads to the equation below (take $K=1$),

$$\ddot{a}_q a_q - c^2 = -\,2l^2c^2\left(\kappa_0\frac{p_q}{\xi_q^2} + \frac{\ddot{\xi}_q}{\xi_q^2}\right) \tag{8.7.31}$$

$$= -\frac{a_q^4}{2\,l^2}\left(1 \mp \sqrt{1+\frac{a_q^2}{2l^2}}\right)^{-2}\left[\kappa_0(1-\hat{q})p_q - \frac{2}{a_q^2}\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right)\right]. \tag{8.7.32}$$

Let us take two cases:

(i) For $p_q=0$, equation (8.7.32) yields

$$\frac{\ddot{a}_q a_q}{c^2} = 1 + \frac{a_q^2}{l^2\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right)}. \tag{8.7.33}$$

It can be easily seen that for $l \gg a_q$, the latter yields

$$\ddot{a}_q a_q/c^2 \longrightarrow 1, \quad (l \gg a_q), \tag{8.7.34}$$

which is the usual relation of Friedmann models.
For $a_q \gg l$ and $p_q=0$, there is

$$\frac{\ddot{a}_q a_q}{c^2} = 1 + \frac{a_q}{l} \quad (a_q \gg l \quad \text{and} \quad (+)). \tag{8.7.35}$$

- Hence, there is accelerated expansion without the necessity of initial values of pressure p_q. Acceleration comes from curvature K and from the scalar-field excitations. For (-), on the other hand, expansion is decelerated.

(ii) For $p_q \neq 0$, there is

$$\frac{\ddot{a}_q a_q}{c^2} = 1 + \frac{a_q^2}{l^2}\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right)^{-1} - \frac{a_q^4}{l^2c^4}\frac{1}{2}(1-\hat{q})\kappa_0 p_q\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right)^{-2}, \tag{8.7.36}$$

with the following extremal cases:

$$\frac{\ddot{a}_q a_q}{c^2} = 1 + \frac{a_q^2}{2\left(1+\frac{1}{4}\delta\right)} - \frac{a_q^4}{4l^2c^4}\frac{1}{2}(1-\hat{q})\kappa_0 p_q \quad \text{for} \quad (+) \text{ and } a_q \ll l, \tag{8.7.37}$$

$$= \mp\frac{a_q}{l} - \kappa_0 p_q \frac{a_q^2}{l^2c^4} \quad \text{for } a_q \gg 1 \quad (+,-). \tag{8.7.38}$$

$p_q > 0$ acts against acceleration for **(+)** and $\hat{q} = 0$ while the low δ term acts accelerating. For **(-)**, however, the following is valid. For $a_q \ll l$ in **(-)**,

$$\frac{\ddot{a}_q a_q}{c^2} = 1 - 2\frac{\delta}{\delta} - \frac{2a_q^4}{l^2c^4}(1-\hat{q})\kappa_0 p_q \quad (a_q \ll l), \quad (-)\,, \tag{8.7.39}$$

For $\xi \neq 0$, the δ contribution acts decelerating as does p_q also for $\hat{q} = 0$.
For $\ddot{a}_q a_q > 0$, there is the following condition,

$$\frac{1}{2}(1-\hat{q})p_q < \frac{1}{\kappa_0 a_q^2}\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right) + \frac{l^2c^4}{\kappa_0 a_q^4}\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right)^2. \tag{8.7.40}$$

We have:

- For high length scales $l \gg a_q$, <u>acceleration</u> is given for **(+)** <u>unless there are very high pressures $p(t_q)$</u>.
- For **(+)** and $\hat{q} = 0$, small pressures p_q may lead again to deceleration in case of high initial length scales.

For **(-)**, there are no accelerated initial states for positive initial pressures.
Basically:

- **There can be no initial acceleration for (-), unless there is $\hat{q} = 0$ and $p(t_q) < 0$.**

Furthermore, there is for equation (8.7.32),

$$\frac{\ddot{a}_q a_q}{c^2} = 1 + \frac{2}{\xi_q^2}\left(\xi_q - l^2(1-\hat{q})\kappa_0 p_q\right). \tag{8.7.41}$$

Here, the different accelerating (K and positive ξ_q) and decelerating terms ($p_q > 0$ for $\hat{q} = 0$) can be seen. Positive values of ξ_q (*i.e.* (+)) lead to acceleration terms. Pressure acts decelerating (gravitationally attractive) for $\hat{q} = 0$.

Take now the time derivative of the first Friedmann equation. Then, there is

$$2\frac{\dot{a}}{a}\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{Kc^2}{a^2}\right) = (1+\xi)^{-1}\left[\frac{\kappa_0}{3}\left(\dot{\varrho} - \frac{\varrho\dot{\xi}}{1+\xi}\right) + \frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}\dot{\xi} - \ddot{\xi} + \frac{\dot{\xi}^2}{1+\xi}\right) + \frac{\ddot{a}}{a}\dot{\xi}\right] + \dot{\Lambda}. \tag{8.7.42}$$

For $t = t_q$, there is then

$$0 = \frac{\ddot{a}(t_q)}{a(t_q)}\frac{\dot{\xi}(t_q)}{1+\xi(t_q)} + \dot{\Lambda}(t_q). \tag{8.7.43}$$

This means for the derivative of the cosmological function, using (8.7.41),

$$\dot{\Lambda}(t_q) = -\frac{c^2}{a_q^2}\left[1 + \frac{2}{\xi_q^2}\left(\xi_q - l^2(1-\hat{q})\kappa_0 p_q\right)\right]\frac{\dot{\xi}_q}{1+\xi_q} \tag{8.7.44}$$

$$= -\frac{1}{a_q^2}\left[1 + \frac{2}{\xi_q^2}\left(\xi_q - l^2(1-\hat{q})\kappa_0 p_q\right)\right]\frac{4l^2}{a_q^2}\frac{\dot{\xi}_q}{\xi_q^2}. \tag{8.7.45}$$

For $\dot{\xi}_q = 0$, the latter means $\dot{\Lambda}_q = 0$.
Further, on the one hand, $\dot{\xi}_q > 0$ leads to a negative derivative of the cosmological function as long as $0 < \xi_q > l^2(1-\hat{q})\kappa_0 p_q$. According to equation (8.7.45), acceleration is related to $\dot{\Lambda}_q$ when $\dot{\xi}_q \neq 0$.
On the other hand, the definition of Λ leads to

$$\dot{\Lambda} = \frac{3\dot{\xi}}{4l^2}(1+\xi)^{-1}\xi\left(2 - \frac{\xi}{1+\xi}\right). \tag{8.7.46}$$

Specifically for $t = t_q$, we then have the following,

$$\dot{\Lambda}_q = \frac{3}{a_q^2}\dot{\xi}_q\xi_q^{-1}\left(2 - \frac{4l^2}{a_q^2}\xi_q^{-1}\right). \tag{8.7.47}$$

Here, $\dot{\xi} > 0$ means $\dot{\Lambda}_q > 0$.
For matters of consistency, if $\ddot{a}_q$ is to be positive (+)/negative (-), the first derivative of the scalar-field excitation at $t = t_q$ is to be vanishing! If, however, the sign of $\ddot{a}(t_q)$ were changed by means of $p(t_q) < 0$, the same would be valid since we already have the constraint $p(t_q) = 0$ or $\dot{\xi}(t_q) = 0$. Hence,

- **The derivative of the scalar-field excitation is vanishing at $t = t_q$, *i.e.***

$$\dot{\xi}(t_q) \equiv \dot{\xi}_q \equiv 0. \tag{8.7.48}$$

Hence, the cosmological function is constant at and the scalar field is static at $t = t_q$.

Take again the Friedmann equations. The second one can be rewritten to take the following form,

$$\frac{\ddot{a}}{a} = -\frac{\kappa_0}{6}(\varrho + 3p) + k(t) + \frac{\Lambda}{3}. \tag{8.7.49}$$

Here, we have

$$k(t) := -(1+\xi)^{-1}\frac{1}{2}\left[\ddot{\xi} + \frac{\dot{a}}{a}\dot{\xi}\right] \tag{8.7.50}$$

$$= -\frac{1}{2}(1+\xi)^{-1}\ddot{\Theta}(t), \tag{8.7.51}$$

which gives new dynamics. For $t = t_q$, k reads

$$k(t_q) \equiv -\frac{1}{2}(1+\xi)^{-1}\ddot{\Theta}(t_q), \tag{8.7.52}$$

with

$$\ddot{\Theta}(t_q) = \ddot{\xi}(t_q) \tag{8.7.53}$$

$$= -\hat{q}\kappa_0 p(t_q) - \xi(t_q)l^{-2} \tag{8.7.54}$$

$$= -\hat{q}\kappa_0 p_q - \frac{2}{a_q^2}\left(1 \mp \sqrt{1 + \frac{a_q^2}{l^2}}\right). \tag{8.7.55}$$

Furthermore, $\ddot{\Theta}$ gives a pressure term

$$p_G \equiv \frac{1}{8\pi G_0}\ddot{\Theta} \tag{8.7.56}$$

which equals $\epsilon_\Lambda + p_\Lambda$ without taking the terms from the cosmological function/Higgs potential. So, there is in terms of density, pressure and the cosmological function,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_0}{(1+\xi)}\left(\frac{1}{3}\epsilon + p + p_G\right) + \frac{\Lambda}{3}c^2. \tag{8.7.57}$$

For $t = t_q$, we have

$$\frac{\ddot{a}(t_q)}{a(t_q)} = -\frac{4\pi G_0}{1+\xi}\left(p(t_q) + p_G(t_q)\right) + \frac{\Lambda_q}{3}c^2 \tag{8.7.58}$$

$$= -4\pi G_0 \frac{a_q^2}{l^2}\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right)^{-1}(p_q + p_{Gq}) + \frac{\Lambda_q}{3}c^2. \tag{8.7.59}$$

The second derivative $\ddot{\Theta}$, *i.e.* the pressure p_G, acts in the same way as $p(t_q)$. It leads to a deceleration. In the absence of Λ, negative values of p_G would be necessary to get acceleration, and this would be the case for $-p_G(t_q) > p(t_q)$. Yet, there is Λ_q, which possesses positive values.
According to the *Penrose–Hawking condition*, for $t = t_q$, if the strong and weak energy conditions are valid, then there must be an initial singularity for the primeval Universe, *i.e.* a Big Bang. In equation (8.7.2), taking $\varrho_T + 3p_T = \varrho + 3p + 3p_G + \Lambda c^2$, however, it is clear that such condition may be violated by

$$p_G(t_q) + \Lambda_q c^2 < p(t_q). \tag{8.7.60}$$

There is

$$p_q + \frac{1}{\kappa_0}\ddot{\xi}_q = (1-\hat{q})p_q - \frac{2}{\kappa_0 a_q^2}\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right), \tag{8.7.61}$$

so that, on the one hand, for $\hat{q} = 1$ and **(+)**, the *energy conditions would be broken* independently of p_q. For $\hat{q} = 0$, on the other hand, they would be broken for

$$p_q < \frac{2}{\kappa_0 a_q^2}\left(1 \mp \sqrt{1+\frac{a_q^2}{l^2}}\right) = \frac{1}{\kappa_0 l^2}\xi_q\,. \tag{8.7.62}$$

Neglecting Λ for **(-)**, the energy conditions appear not to be broken at all. Such breaking would depend on dynamics after $t = t_q$. However, the cosmological function should be taken into account, too, as a further term of the equation of state. The cosmological function leads to a new term dependent on ξ^2 (and hence on Λ). There is a term

$$p_\xi = -\frac{1}{6\kappa_0 l^2}\xi^2 < 0 \tag{8.7.63}$$

which acts antigravitationally. Furthermore, there is

$$p_G(t_q) + p_\xi(t_q) = \frac{\ddot{\xi}(t_q)}{\kappa_0} - \frac{1}{6\kappa_0 l^2}\xi^2(t_q) \tag{8.7.64}$$

$$= -\hat{q}p(t_q) - \frac{\xi(t_q)}{6\kappa_0 l^2}\left(6 + \xi(t_q)\right). \tag{8.7.65}$$

The total pressure would be

$$p_{tot}(t_q) = p(t_q) + p_G(t_q) + p_\xi(t_q) = (1-\hat{q})p(t_q) - \frac{\xi(t_q)}{6\kappa_0 l^2}\left(\xi(t_q) + 6\right). \tag{8.7.66}$$

The value of equation (8.7.66) is to be analyzed for the case of being positive and thus gravitationally interacting in the usual sense.
For $\hat{q} = 0$, there is

$$p(t_q) > \frac{\xi_q}{6\kappa_0 l^2}(\xi_q + 6) = \frac{1}{3\kappa_0 a_q^2}\left(1 \mp \sqrt{1 + \frac{a_q^2}{l^2}}\right)\left[\frac{2l^2}{a_q^2}\left(1 \mp \sqrt{1 + \frac{a_q^2}{l^2}}\right) + 6\right] \tag{8.7.67}$$

as condition for $p_{tot}(t_q) > 0$, which is the condition for $\ddot{a}(t_q) < 0$. For large length scales $l \gg a(t_q)$ and **(+)**, the latter condition yields

$$p(t_q) > \frac{2}{3\kappa_0 a_q^2}\left(\frac{4l^2}{a_q^2} + 6\right) > 0, \quad (l \gg a(t_q)), \tag{8.7.68}$$

and for $a(t_q) \gg l$,

$$p(t_q) > \frac{1}{3\kappa_0 a_q l}\left(\frac{2l}{a_q} + 6\right) \approx \frac{2}{a_q l \kappa_0} > 0\,.$$

Relatively high values of the initial pressure (which act counter-gravitationally) are necessary for deceleration to appear.
Let us take the two cases of $\hat{q}$:

(i) For the case **(-)** of $\hat{q} = 0$, the condition of $\ddot{a}(t_q) > 0$ for $a(t_q) \gg l$ yields

$$\begin{aligned} p(t_q) &> -\frac{1}{3\kappa_0 a_q l}\left(\frac{4l^2}{a_q^2} - 6\right) \quad (a(t_q) \gg l,\, (-)) \\ &\approx -\frac{2}{\kappa_0 a(t_q) l} < 0\,. \end{aligned} \tag{8.7.69}$$

Deceleration $\ddot{a}(t_q) < 0$ appears for $a(t_q) \ll l$ under following condition of pressure:

$$p(t_q) > -\frac{5}{12\kappa_0 l^2} < 0, \quad (a(t_q) \ll l\,(-)) \tag{8.7.70}$$

- **Hence, for (-) there may be acceleration only for negative initial pressures. On the other hand, for (+), all negative initial pressures $p(t_q)$ lead to acceleration and a breaking of the Penrose–Hawking condition. Furthermore, only relatively high negative initial pressures $p(t_q) < 0$ would make deceleration possible.**

(ii) For $\hat{q} = 1$, a positive total pressure $p_{tot}(t_q)$ is given by

$$-\frac{\xi(t_q)}{6\kappa_0 l^2}(\xi(t_q) + 6) > 0. \tag{8.7.71}$$

This is the case for $\xi(t_q) < 0$, *i.e.* for **(-)**. For **(+)**, however, there is always $p_{tot}(t_q) < 0$ for $\hat{q} = 1$ and **the Penrose–Hawking condition is easily broken in all cases of (+). Hence, a singularity is not necessary and a bounce is possible.**

8.8 The Planck-length Bounce

From Chapter 8.7 we know that the Hawking–Penrose condition is broken, especially for positive initial values of the scalar-field excitation. Furthermore, we know that following such breaking, initial singularities are not necessary and a Big Bang might rather be given by a bounce state with $a(t=0)\neq 0$. We know that when such is given as a statical case $\dot{a}(0)=0$ for vanishing initial densities $\varrho(0)=0$, the scalar field is statical at $t=0$. Now, let us analyze the properties at $t\approx 0$ with $t_q\approx 0$ for which we may give a value of a at the initial state.

Going forth in the redshift z, $a(z)$ becomes smaller and smaller. Within standard cosmology and with valid Penrose–Hawking conditions for a Big Bang, $a(t=0)$ then vanishes. Here, let a not vanish ($a\neq 0$) and be $\dot{a}(t_q)=0$. However, even though not vanishing, $a(t_q)$ shall be small. On the other hand, there is the length scale l. According to analyses for galaxies and Dark Matter in Chapter 7.8, l shall be of around the order of magnitude of the Galaxy's core.

Let us assume that the contraction of the Universe for higher redshifts go on until the Heisenberg uncertainty relation for energy,

$$\Delta E\Delta t=\hbar, \tag{8.8.1}$$

is valid. Typically, at this scale quantum mechanics becomes dominant and time itself is not exactly determined anymore, as classical mechanics lose their validity. Hence, let us assume that this point gives the initial singularity so that we take the Planck time $t_P\approx 0$. At this time, quantum fluctuations persist on the scale of the Planck length $l_P=ct_P$. From these two scales, further, the Planck mass $m_P=\varrho_P l_P^3$ is defined. Following the Friedmann equations, the Planck density ϱ_P is of the order $(G_0t_P^2)^{-1}$. Consequently,

$$\Delta E\Delta t\cong m_Pc^2t_P\cong \varrho_P(ct_P)^3c^2t_P\cong \frac{c^5t_P^4}{G_0t_P^2}\cong\hbar. \tag{8.8.2}$$

There is then (as commonly known)

$$t_P\cong\left(\frac{\hbar G_0}{c^5}\right)^{1/2}\cong 10^{-43}\text{s}. \tag{8.8.3}$$

The Planck length is then

$$l_P\cong ct_P\cong\left(\frac{G_0\hbar}{c^3}\right)^{1/2}\cong 1.7\cdot 10^{-33}\text{cm}. \tag{8.8.4}$$

The Planck length represents the order of magnitude of the cosmological horizon at $t=t_P$. Be the minimal scale of the Universe *ca.* the scale in which quantum fluctuations appear. Hence, be $t_q=t_P$. Then, there is $a(t_P)\equiv a_P\cong l_P$. However, given that $\varrho(t_q)$ is assumed as vanishing, the Planck mass shall be constituted by pressure terms $p(t_P)$ and scalar-field excitations $\xi(t_P)\equiv\xi_P$. Consequently, *sc.*

$$\varrho(t_P)\neq\varrho_P. \tag{8.8.5}$$

Take $a(t_P)=l_P$. Then, ξ_P is given by

$$\xi(t_P)\equiv\xi_P\cong\frac{2l^2c^3}{G_0\hbar}\left(1\mp\sqrt{1+\frac{G_0\hbar}{l^2c^3}}\right). \tag{8.8.6}$$

For $\xi_P\gg 1$:

$$\xi_P\cong\frac{4l^2c^3}{G_0\hbar}\cong\frac{4l^2}{l_P^2}\approx 10^{66}\text{cm}^{-2}\cdot l^2\,. \tag{8.8.7}$$

Actually, only the case **(+)** is possible since $\xi \geq -1$ and equation (8.8.7) are valid. For $l \cong 10^{22}$cm, for instance,[10] there is $\xi(t_P) \equiv \xi_P \cong 10^{110}$. For $l \sim 10^{28}$cm, on the other hand, there is $\xi_P \cong 10^{122}$. For $\breve{\alpha} \gg 1$, such a value leads to $\Lambda(\xi_P) \equiv \Lambda_P$ as

$$\begin{aligned}\Lambda_P =&\frac{3}{4l^2}\frac{\xi_P^2}{1+\xi_P}\\ \approx&\frac{3}{4l^2}\xi_P = \frac{3}{l_P^2}\\ \approx&10^{66}\text{cm}^{-2},\end{aligned} \tag{8.8.8}$$

independently of l. The same result is achieved directly from the first Friedmann equation (8.1.5) with

$$\frac{\Lambda_P}{3} = \frac{K}{l_P^2}\,. \tag{8.8.9}$$

$\epsilon(t_P)$ is taken as zero. However, we get an effective density of the system solely by $a(t_P) = l_P$ which is hence related to the Planck mass. There is according to the first Friedmann equation with effective density as ϱ_P, using equation (8.8.8),

$$\frac{c^2}{l_P^2} \cong \frac{8\pi G_0 \varrho_P}{3}. \tag{8.8.10}$$

This gives a density as

$$\varrho_P = \frac{3\,c^2}{8\pi G_0 l_P^2} \cong 10^{93}\,\frac{\text{g}}{\text{cm}^3}\,. \tag{8.8.11}$$

This is the order of the Planck density, indeed. The Planck density is usually defined by

$$\varrho_P \cong \frac{1}{G_0 t_P^2} \cong \frac{c^5}{G_0^2\hbar} \cong 4\cdot 10^{93}\text{g cm}^{-3}. \tag{8.8.12}$$

It leads to a Planck mass

$$m_P \cong \varrho_P l_P^3 \cong \left(\frac{\hbar c}{G_0}\right)^{1/2} \cong 10^{-5}\text{g}, \tag{8.8.13}$$

related to a Planck energy

$$E_P \cong m_P c^2 \cong 10^{19}\text{GeV}. \tag{8.8.14}$$

Thus, the first Friedmann equation is consistent with an initial density $\varrho(t_P)$ to be vanishing for $\dot{a}_P = 0$. The Planck density and hence the Planck mass are given by the scalar field at $t = t_P$, or more exactly by the scalar-field potential at the Planck time, given by the Planck length itself. The first Friedmann equation reads now

$$\frac{1}{l_P^2} = \frac{8\pi}{3}\frac{G_0}{c^4}\varrho_P = \frac{\Lambda_P}{3}\,. \tag{8.8.15}$$

Using the first Friedmann equation and hence equation (8.8.9), the second Friedmann equation reads

$$\frac{\ddot{a}_P}{l_P} - \frac{Kc^2}{l_P^2} = -\frac{l_P^2}{l^2}\left(\frac{\pi G_0 p(t_P)}{c^2} + \frac{\ddot{\xi}_P}{8}\right). \tag{8.8.16}$$

[10] This is about the order of magnitude of the length scale l for flat rotation curves, according to [20,24,50] and Chapters 6.3 and 7.8.

With

$$\ddot{\xi}_P = -\frac{c^2}{l^2}\xi_P - \frac{8\pi G_0}{c^2}\hat{q}p(t_P) \cong \ddot{\Theta}_P\,, \tag{8.8.17}$$

there is

$$\frac{\ddot{a}_P}{l_P} - \frac{Kc^2}{l_P^2} = \frac{l_P^2}{l^2}\left[-\frac{\pi G_0}{c^2}(1-\hat{q})p(t_P) + \frac{c^2}{8l^2}\xi_P\right]\,, \tag{8.8.18}$$

which yields after insertion of ξ_P for l relevantly higher-valued than l_P,[11]

$$\frac{\ddot{a}_P}{l_P} - \frac{Kc^2}{l_P^2} = -\frac{l_P^2}{l^2}\left[\frac{\pi G_0}{c^2}(1-\hat{q})p(t_P) - \frac{c^2}{2l_P}\right]. \tag{8.8.19}$$

Since $K = 1$, the latter equation may be rewritten as follows,

$$2\frac{\ddot{a}_P}{l_P} - \left(2 + \frac{l_P^2}{l^2}\right)\frac{c^2}{l_P^2} = -2\frac{l_P^2}{l^2}\frac{\pi G_0}{c^2}(1-\hat{q})p(t_P)\,. \tag{8.8.20}$$

For vanishing values of the initial pressure $p(t_P)$ or $\hat{q} = 1$, the right-hand side of equation (8.8.20) disappears and cosmic acceleration at $t = t_P$ is given. Given the low value of l_P and for $l_P \ll l$, the first term is dominant[12] and there is

$$\ddot{a}_P \sim 10^{53}\text{cm s}^{-2}, \quad (\text{for } l_P \ll l)\,. \tag{8.8.21}$$

As in the general case $t = t_q$, positive pressures pull acceleration down, since the pressure acts gravitationally. The pressure term, however, is dependent on the reciprocal value of the squared length scale l and on the squared value of the Planck length. Additionally, it possesses a G_0/c^2 dependence, in total a contribution $\ddot{a}_P \sim l^{-2} \cdot 10^{-128}\text{cm}^4\ \text{kg}^{-1}$. For $l \cong 10^{22}$cm, a pressure of about 10^{100}Pa would be necessary for the pressure term of the right-hand side of equation (8.8.20) to be dominant and hence for deceleration to appear and energy conditions to be valid. Pressures of the order of magnitude of $p(t_P) \sim 10^{200}$Pa for the pressure term (in case of $\hat{q} = 0$) are necessary for pressure terms to be dominant in the dynamics. Even for length scales of the order of magnitude of the Planck length, the pressure needed is extremely high. The pressure term is then of the order $10^{-50}\text{cm}^2\text{kg}^{-1}p(t_P)$, which may be compared with $c^2/l_P \sim 10^{55}\text{cm s}^{-2}$, which is the dominant term of $\ddot{a}_P$ for $\hat{q} = 1$ or relatively low pressures.
Consequently, for length scales relevantly larger than the Planck length, there is

$$\ddot{a}_P \approx \frac{c^2}{l_P}\,, \tag{8.8.22}$$

in good approximation, independently of $\hat{q}$, l and $p(t_P)$. This shows a highly accelerated state for the primeval Universe at $t \approx 0$. At this time, there is a very high cosmological function Λ_P which, acting antigravitationally, leads the accelerated expansion.

In Chapter 8.7 we had defined pressure terms p_G and p_ξ which depend on $\ddot{\Theta}$ and Λ, respectively (*cf.* equations (8.7.56) and (8.7.63)). Their value is related to the possibility of cosmic acceleration and the appearance of a bounce state which follows a breaking of the energy conditions of Penrose and Hawking. Now we

[11] This means, especially valid for $l \gg l_P$ but even in good approximation for l a few times larger (say twice) than l_P.

[12] For $l \cong 10^{22}$cm (*cf.* Chapters 2.4 and 7.8), there is $l_P c^2/l^2 \cong 10^{-57}\text{cm s}^{-2}$.

have a negative value of both p_G and p_ξ for $t = t_P$ according to the positive high values of ξ_P.
There are

$$p_G = \frac{1}{8\pi G_0}\ddot{\xi} \tag{8.8.23}$$

and

$$p_\xi = -\frac{\xi^2}{6\kappa_0 l^2}. \tag{8.8.24}$$

Further, there is at Planck time,

$$\ddot{\xi}_P = -2\frac{c^2}{l_P^2}\left[1 + \sqrt{1 + \frac{l_P^2}{l^2}}\right] - \hat{q}\kappa_0 p(t_P)c^2. \tag{8.8.25}$$

The term in parenthesis possesses a value between 2 and $1 + \sqrt{2}$. However, in good approximation (and especially for l relevantly larger than l_P), equation (8.8.25) reads

$$\ddot{\xi}_P = -2\frac{c^2}{l_P^2}\,, \tag{8.8.26}$$

almost independently on l, $\hat{q}$ and $p(t_P)$ unless p should tend to infinity for $t \approx 0$. $\ddot{\xi}_P = \ddot{\Theta}_P$ gives the statical value of the pressure term p_G coming from f. It yields for $l \gg l_P$,

$$p_G(t_P) = \frac{\ddot{\Theta}_P}{8\pi G_0} = -\frac{c^2}{2\pi G_0 l_P^2} - \hat{q}p(t_P) = p_T(t_P) - p(t_P)\,, \tag{8.8.27}$$

which violates Penrose and Hawking's strong-energy condition. Next to analyze would be the dynamics for $t > t_P$ in order to know if there appears some kind of rollover contraction (although at t_P there appears high acceleration) or if acceleration stays and leads to an inflationary epoch indeed.
According to the works [48] and [49], there should appear an inflationary state for both $\hat{q} = 1$ and $\hat{q} = 0$. This may be of New or Chaotic Inflation, which is dependent on the initial values of the scalar field. For $\hat{q} = 1$, there appears (*h.t.*) slow rollover dynamics before New Inflation ($\phi \ll v \to v$), and in the case of Chaotic dynamics ($\phi \gg v \to v$), Inflation appears automatically. Due to the relationship between masses of particle physics within GUT, parameters are not fine-tuned in order to predict acceptable values of reheating temperature and density (see [48]). For $\hat{q} = 0$, New Inflation needs of fine tuning but follows automatically after a short contraction era. Chaotic Inflation is achieved successfully, however best for high Higgs-particle masses (see [49]). The afore-mentioned analysis contributes to the possibility of the appearance of Chaotic Inflation since for a statical Universe at Planck time t_P, the scalar-field excitation is very high and thus, the scalar field is much larger than its ground-state value v. However, scalar-field dynamics should be further analyzed to compare dynamics of this scenario with those in [47]. **Yet, it may be concluded that the Penrose–Hawking condition does not hold and a singularity does not appear. Further, there appears an initial highly accelerated state at $0 \approx t = t_P$ in a statical Universe.**

Chapter 9

Results, conclusion and outlook

The theoretical relevance of the Higgs Mechanism and its universal properties cannot be questioned at all. Higgs particles in general appear effectively in all branches of physics; within Quantum Asthenodynamics they lead to the mass of elementary particles, and within mesoscopic physics they lead to the Meissner effect of superconductivity. Analogously, within Dual Quantum Chromodynamics they lead to dual superconductivity and hence to the confinement of quarks and color charges in hadrons. In this context, the first part of this work leads to the following conclusion:

- Dyon and monopole condensations with the Abelian Higgs Mechanism are equally capable of describing the superconducting QCD vacuum.
- Both dyons and monopoles lead to the Dual Meissner Effect and hence to confinement, however with different strengths.
- The magnetic permeability in such vacuum rises to infinity with vanishing momenta.

Given the universal properties of the scalar fields, the second part of this work further introduces into a model of General Relativity with Higgs Mechanism. For this, we have discussed the generalized concept of Higgs fields especially in the context of astrophysics. We have grounded our analysis on Bergmann–Wagoner models of scalar–tensor theories of induced gravity, and we have inserted Higgs fields as scalar fields whereas a Higgs potential has been chosen. Such leads to a cosmological function analog to the cosmological constant of General Relativity as well as to an effective gravitational coupling.
If the scalar field possesses a coupling to the fermionic Lagrange sector, Higgs fields then lose their source and cannot be generated in high-energy experiments. Without such coupling, though, they still couple analogously to cosmons of Dark Energy and Matter. The cosmological function and the effective coupling lead to new gravitational dynamics which have been analyzed here together with Maxwell-like equations of gravity.
We know that a scalar–tensor theory with Higgs potential is able to explain and contribute to the phenomenon of flat rotation curves (Dark Matter problem) for specific densities of galaxy's bulges. Hence, the third part of this work analyzes the issue of the metric components for central symmetry and the Friedmann–Robertson–Walker metric in detail. Conclusions out of the work related to Black Hole solutions for scalar fields with negligible mass are the following:

- The exact solution of the metric components for negligible scalar-field masses indicates that the metric components of the line element given by the equations correspond to the usual Schwarzschild metric which appears in this form only for the limiting case of the vanishing Higgs scalar-field excitations (*i.e.* $\xi = 0$).

- Higher values of A lead to a decrease in the gravitational potential ν through the exponent B/K.
- In fact, the metric and scalar field are regular everywhere with exception of $r = 0$ as naked singularity.
- There exists no Schwarzschild horizon except for the case of vanishing scalar-field excitations. Therefore, Black Holes (in the usual sense) do not appear for the case $A \neq 0$.
- For the general values of the excitation amplitude A, the qualitative results of minimally coupled scalar fields are valid and scalar fields thus act analogously to electric charges in a gravitational field. Black Holes go through to Grey Stars.

Further, we have investigated the singularities and Black Hole solutions with and without Higgs field excitations. We have considered two scenarios to solve the field equations with and without the vanishing of Higgs field mass, and the solutions have been further analyzed in view of the geodesic motion (in the case of scalar fields with fermionic source). In particular, the linear field equations with finite Higgs field mass have been solved to have a correct physical explantation of the parameters involved in the study as well as to discuss some aspects of the Black Hole solutions obtained. Further, in order to investigate the physical consequences of these solutions for both the cases, we have analyzed them in view of the geodesic motion.

- We have found the appearance of Reissner–Nordström-like Black Hole solutions for the case of non-vanishing field excitations in this formulation while in the vanishing limit of excitations we have the Schwarzschild geometry as usual in GR. It is shown that there are scalar-field terms which at low gravitational regimes act antigravitationally into a Reissner–Nordström-like metric acting as a generalized charge-like term.
- The terms corresponding to the pressure relevant from the scalar field and nonlinearities of the exact solution lead to a dynamical mass different to the luminous, bare mass from density. The behavior of the components of the metric is then described accordingly along with their physical consequences.
- Up to this order, the Schwarzschild horizon becomes weaker with stiffness w. The Schwarzschild radius changes with the effective mass as shown in the representative graphical plot. It is also shown that stiff matter acting repulsively in the metric component λ is an effect which appears especially for a negative effective, yet positive bare mass.
- The effective potential permits stable bounded orbits and angular velocities. The orbits are found qualitatively the same to those in the case of the Schwarzschild and Reissner–Nordström geometry in GR. The stability of the bound states is discussed from the viewpoint of the luminous and dynamical mass parameters.
- The assumption of stiff matter (relevant inner structure of matter) leads to relevant deviations from effective, measured astronomical masses to bare, luminous masses. Furthermore, it also leads to flattened curves of tangential velocity, which is shown in the concerning graphical plot marking a similarity at large distances to the flat rotation curve of a typical spiral galaxy.

We have further investigated the relation between the scalar-field excitations of induced gravity with a Higgs potential and the obligatory presence of finite pressure terms in energy density of gravitation, along with their appearance in linear solutions for solar-relativistic effects such as perihelion advance, and for flat rotation curves leading to Dark Matter phenomenology with scalar-field density components of the dark-matter profile. The important conclusions drawn from this study are summarized below:

- An energy density of gravitation following Maxwell-like equations may differ with its analogue of GR. Gravitational energy within induced gravity and GR are identical for $\hat{q} = 0$ (which denotes the coupling of the scalar field to the matter Lagrangian), and for $\hat{q} = 1$ (*i.e.* the absence of the coupling of the scalar field with the matter Lagrangian) they are the same only with the constraint on the equation of state parameter as $w = 1/5$.

- Finite values of pressure within vacuum solutions are expected from the nature of scalar-field excitations. This further leads to the notion of the dynamic and bare (luminous) masses in this model. The value of the dynamical mass is observed greater than that of the luminous mass, and the present formulation is thus useful to describe the signatures of unseen matter in nature.

- Perihelion shift is found the same as within GR for low-energetic systems with values of pressure as constraint by energy in view of the equation of state parameter as $w = 1/5$ .There is, further, more complicated dynamics for high-energetic systems with high coefficients of C_a per c and C_b of energy and momentum. Scalar fields are essential for low-range dynamics although they have newtonian behavior at such range.

- Flat rotation curves of galaxies lead to dark-matter profiles with baryonic and scalar-field components of density. Scalar-field densities are strongly related to pressure terms in this model and seems to provide a viable explanation of the Dark Matter contents of our Universe indeed.

- Dark Matter dominance leads to pressures related to an equation-of-state parameter of total energy of the same value as for weak fields in solar-relativistic ranges.

- The non-newtonian behavior of density appears as distances grow. Such non-newtonian behavior of scalar-field excitations leads to flat rotation curves. The contribution due to the scalar field in the energy density in fact acts as the dark-matter profile in view of the total energy density of the system.

Within Friedmann–Robertson–Walker cosmology, Friedmann equations are derived indeed, and these lead to negative equation-of-state parameters, of antistiff matter for the absence of matter.
The density parameters of cosmology within this model have been derived along with the deceleration parameter, together with its issues of cosmic acceleration. We have discussed possible interpretations of the structure of this model when compared to experimental results in the context of the SM. We have shown, hence, deSitter properties of the quantities which characterize Quintessence as well as Inflation for the primeval Universe. Especially, we conclude the following:

- Dominant values of the cosmological function may constrain length-scale and scalar-field excitation values. Both are highly important for corrections of the generalized Friedmann equations and for the energy density of usual matter. Furthermore, the field excitation plays a relevant role in the evolution of the Hubble parameter.

- Negative values of the deceleration parameter and of the equation-of-state parameter are possible within this model without taking negative usual pressures nor a cosmological constant. Cosmic acceleration (Quintessence) is possible, and scalar fields may act as part of dark sectors of matter and energy in form of further density parameters or as screening terms. A constant scalar field, however, would need of too high a length scale and negative excitations to account for both the dark sectors successfully. For dynamical fields, scalar-field excitations may be very small indeed and yet account for the dark sectors.

Further, we have analyzed the consequences of the scalar field for the primeval Universe, especially in the context of the Big Bang and Hawking–Penrose conditions of energy and Bounce scenarios. We conclude the following:

- The energy conditions may be broken in a primeval Universe. Hence, there exists the possibility of a Big Bounce as initial state. In a static state of the primeval Universe, there is a constant field excitation which is related to a negative scalar-field energy density which may break the Penrose–Hawking conditions and further lead to acceleration parting from a very condensed state of the Universe.
- For $\hat{q} = 0$ and positive initial scalar-field excitations, the initial state is accelerated unless there are high positive initial pressures. For $\hat{q} = 0$ and negative initial scalar-field excitations, negative initial pressures are necessary for acceleration. For $\hat{q} = 1$, there is acceleration for positive initial scalar-field excitations regardless initial pressure terms.

We have related this primeval state to the Planck time in a primeval, initial universe of Planck distance. At Planck time with a vanishing energy density of the static universe, the signature may be given by Planck values. The reciprocal value of the initial scale of the Universe gives Planck density and the initial value of the cosmological function, which is constant at $t = t_P$.

- At Planck time, high values of the scalar-field length scale which lead to flat rotation curves lead to high negative effective pressure terms which further break energy conditions and lead to a highly accelerated Bounce state. This shows a relation to a form of Chaotic Inflation.

However, many questions are still unanswered from the perspective of the present formulation. At galactic ranges, the estimation of the shift to intermediate behavior of scalar fields, *i.e.* the relation between scalar fields and galactic centers, is still unclear, and it would be a quite interesting problem to investigate. Furthermore, this might be valuable in relation to quintessential properties of scalar fields for galaxies within exact solutions, leading to the Reissner–Nordström-like behavior.
Further analyses related to new issues on solar-relativistic effects, especially generalized to galacto-relativistic effects are still unclear. Furthermore, the quintessential properties of scalar fields as well as primeval dynamics are still to analyze in more detail. Cosmological implications of induced gravity with Higgs potentials in terms of Quintessence and Dark Matter, as well as primeval dynamics are still to investigate in detail. Especially, the issues of Inflation after a Big Bounce are to discuss in detail. For instance, it is still unclear whether the high acceleration at $t = t_P$ goes through to a rollover contraction or whether it leads to Inflation, indeed.

Part IV

Appendix

Appendix A

General Relativity and Geometry

A.1 The Metrical Tensor

The **metrical tensor** $g_{\mu\nu}$ is a (4×4) 2^{nd}-rank tensor which physically gives the properties of spacetime. Shall it obey the requirements of

(i) **symmetry**

$$g_{\mu\nu} = g_{\nu\mu} \tag{A.1.1}$$

and

(ii) **unitarity**

$$g_{\mu\nu} g^{\mu\lambda} = \delta_{\nu}{}^{\lambda} \,, \tag{A.1.2}$$

respectively (while (A.1.2) defines the inverse of $g_{\mu\nu}$).

(iii) In the limiting case of vanishing spacetime dependence and thus of vanishing gravitational interactions, shall the metrical tensor possess the form of the (pseudo-euclidian) Minkowski metric $g_{\mu\nu} = \eta_{\mu\nu}$ of Special Relativity (SR). For it, we choose the signature (+,-,-,-) with

$$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1) \,. \tag{A.1.3}$$

Hence, we will count the indices with Greek letters from zero ($x^0 = ct$) through three ($x^3 = z$), and a constant metric (no curvature) will give **minkowskian spacetime** as found in the covariant form of electrodynamics or in usual quantum-relativistic mechanics. Latin indices shall count over spatial coordinates only.

The metrical tensor is in general a function of both space and time coordinates. With a^μ and b^μ as 4-vectors in $\mathbb{R}^4$, the metric defines the scalar product as

$$\sum_{\nu=0}^{3} a_\nu b^\nu = \sum_{\mu,\nu=0}^{3} g_{\nu\mu} a^\mu b^\nu \,. \tag{A.1.4}$$

Clearly, within notation, the metrical tensor is used for lowering and raising indices. However, usual notation convention is Einstein's one for summation. It mean that if in a sum an index appears twice, once as an upper (contravariant) index and once as a lower (covariant) one, it will be summed over it. Hence,

$$\sum_{\nu=0}^{3} a_\nu b^\nu \equiv a_\nu b^\nu = g_{\nu\mu} a^\mu b^\nu \tag{A.1.5}$$

$$= g_{\mu\nu} a^\mu b^\nu = a^\mu b_\mu . \tag{A.1.6}$$

Such convention will be used throughout this work unless explicitly mentioned elsewise.
A scalar product of 4-vectors (the length) is a scalar and as such it is invariant. The scalar product

$$\begin{aligned} ds^2 &= dx^\mu dx_\mu \\ &= g_{\mu\nu} dx^\mu dx^\nu \\ &= dx_\mu dx^\mu \end{aligned} \tag{A.1.7}$$

is called **line element**. It is usually used equivalently to the metrical tensor itself and therefore often called metric also.

A.2 Lorentz transformations

In analytical mechanics, there is the action

$$S = \int_A^B L ds \tag{A.2.1}$$

which, according to the Hamilton Principle of the Least Action, possesses a vanishing variation,

$$\delta S \equiv 0 \tag{A.2.2}$$

when keeping boundaries constant. Consequently, the Euler–Lagrange equations for the Lagrange function L follow,

$$\frac{\partial L}{\partial q_k} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = 0 , \tag{A.2.3}$$

with generalized (canonical) coordinates q_k and velocities $\dot{q}_k$ and defining generalized forces $\Phi_k = \frac{\partial L}{\partial q_k}$ and canonical momenta $p_k = \frac{\partial L}{\partial \dot{q}_k}$ conjugate to q_k.
For special relativity (SR), within minkowskian spacetime and with rest-mass m_0, 4-velocity $v^\mu = \frac{dx^\mu}{ds}$, electric charge e and 4-potential A^μ, the Lagrange function reads

$$L = \frac{1}{2} m_0 c^2 \eta_{\alpha\beta} v^\alpha v^\beta + e A_\alpha v^\alpha . \tag{A.2.4}$$

Hence, together with the Ricci identities of covariant derivatives, the Euler–Lagrange equation of the system gives the homogeneous and inhomogeneous Maxwell equations in covariant form,

$$F_{[\mu\nu,\lambda]} \equiv F_{\mu\nu,\lambda} + F_{\nu\lambda,\mu} + F_{\lambda\mu,\nu} = 0, \tag{A.2.5}$$

$$F^{\mu\nu}{}_{,\nu} = -\,4\pi j^\mu . \tag{A.2.6}$$

Here, $_{,\mu}$ gives the usual derivative upon the coordinate x^{ν} (see (A.3.6). Further, $F_{\mu\nu}$ is the energy–stress or field-strength tensor with a force density

$$k^{\mu} = m_0 c^2 v^{\mu}{}_{,\nu} v^{\nu} = F^{\mu}{}_{\nu} j^{\nu} = eF^{\mu}{}_{\nu} v^{\nu} \,. \tag{A.2.7}$$

Further, $j^{\mu} = e\{\rho, \frac{1}{c} j_i\}$ is the electric 4-current with a 4-vector $\{x_0, x_i\} \equiv \{x_0, x_1, x_2, x_3\}$. $F_{\mu\nu}$ is defined in terms of the 4-potential given by

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \,. \tag{A.2.8}$$

$A_{\mu} = \{\varphi, a_i\}$ is the gauge potential with a_i as vector potential and φ as scalar potential field, both known from electromagnetism. As already clear, this is the (specialrelativistic) electromagnetism with the magnetic strength pseudovector components $B_l = \frac{1}{2}\varepsilon_{ijk}\left(a_{k,i} - a_{i,k}\right)$ and the electric strength vector components $E_i = -\left(\varphi_{,i} + \frac{1}{c} a_{i,t}\right)$. ε_{ijk} is the completely skew symmetric 4-Levi–Civita tensor.

In the context of elementary particle physics, the theory can be quantized (*i.e.* put onto the form of explaining nature *via* interactions between quanta), and the gauge potential is then related to gauge photons as intermediate particles following from the gauge of the inner group. However, let us now focus on external transformations and introduce spacetime transformations $\Lambda^{\mu}{}_{\nu}$. These are given by

$$x^{\mu'} = \Lambda^{\mu}{}_{\nu} x^{\nu}, \quad \Lambda^{\mu}{}_{\nu} = \frac{\partial x^{\mu'}}{\partial x^{\nu}} \,, \tag{A.2.9}$$

which defines Lorentz transformations as far as

$$\left| \frac{\partial x^{\mu}}{\partial x^{\alpha'}} \right| \neq 0 \tag{A.2.10}$$

be given for the determinant. For them, the following be valid:

$$\Lambda^{\mu}{}_{\nu} = const., \quad \det \Lambda^{\mu}{}_{\nu} \neq 0. \tag{A.2.11}$$

Thus, Lorentz transformations are linear and occur between inertial systems. They conform the general linear group GL(4,$\mathbb{R}$).

The infinitesimal line element $ds^2 = dx_{\mu} dx^{\mu}$ is a scalar and hence

$$ds' = ds. \tag{A.2.12}$$

For vectors and tensors, however, homogeneous Lorentz transformations are like the following:

$$\begin{aligned}
v^{\mu'} &= \frac{\partial x^{\mu'}}{\partial x^{\nu}} v^{\nu} = \Lambda^{\mu}{}_{\nu} v^{\nu} \\
v_{\mu'} &= \frac{\partial x^{\nu}}{\partial x^{\mu'}} v_{\nu} = \left(\Lambda^{\mu}{}_{\nu}\right)^{-1} v_{\nu} \\
\eta_{\alpha'\beta'} &= \frac{\partial x^{\nu}}{\partial x^{\alpha'}} \frac{\partial x^{\mu}}{\partial x^{\beta'}} \eta_{\nu\mu} = \left(\Lambda^{\alpha}{}_{\nu}\right)^{-1} \left(\Lambda^{\beta}{}_{\mu}\right)^{-1} \eta_{\nu\mu} \\
\eta^{\alpha'\beta'} & \frac{\partial x^{\alpha'}}{\partial x^{\nu}} \frac{\partial x^{\beta'}}{\partial x^{\mu}} \eta^{\nu\mu} = \Lambda^{\alpha}{}_{\nu} \Lambda^{\beta}{}_{\mu} \eta^{\nu\mu}.
\end{aligned}$$

Lorentz transformations are defined such that

$$\begin{aligned}
\eta_{\alpha'\beta'} &= \eta_{\alpha\beta}, \\
\eta^{\alpha'\beta'} &= \eta^{\alpha\beta}
\end{aligned}$$

be valid. With equation (A.2.11), the latter leads to the orthogonality condition

$$(\Lambda^{\mu}{}_{\nu})^{-1} = (\Lambda^{\mu}{}_{\nu})^{T}.$$

Hence, these linear spacetime transformations are orthogonal. Further, for infinitesimal transformations there is

$$\Lambda^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + S^{\mu}{}_{\nu}, \quad \text{with } |S^{\mu}{}_{\nu}| \ll 1\,. \tag{A.2.13}$$

Consequently, the following is valid,

$$S^{\mu}{}_{\nu} = -S_{\nu}{}^{\mu}, \quad (\text{with } S^{T} = -S). \tag{A.2.14}$$

There are 6 linearly independent antisymmetric basis tensors $S^{i\mu}{}_{\nu}$. They are called generators of the group GL(4,$\mathbb{R}$). Thus, one may write

$$S^{\mu}{}_{\nu} = \lambda_i S^{i\mu}{}_{\nu}, \quad i = 1, \ldots 6, \tag{A.2.15}$$

with λ_i as real-valued constants. λ_i are group elements and transformation parameters. There are 3 for the Euler angles and 3 for velocity components of inertial systems in relation to each other. Since they are continuous, the group is a Lie group with 1-element $\lambda_i = 1$.
$S^{i\mu}$ are the generators of the group. For finite transformations, exponentiating leads to the transformation with following form:

$$\Lambda^{\mu}{}_{\nu} = e^{\lambda_i S^{i\mu}{}_{\nu}}. \tag{A.2.16}$$

There is the commutator of the Lie algebra

$$[S^i, S^j] = f^{ij}{}_{k} S^k, \quad f^{ij}{}_{k} \neq 0\,, \tag{A.2.17}$$

with f^{ij} as a nonvanishing structure constant. Thus, the Lorentz group is not abelian. Furthermore, it may be generalized with

$$x^{\mu'} = \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu}, \quad \Lambda^{\mu}{}_{\nu} a^{\mu} = const. \tag{A.2.18}$$

This is a 10-parametrical group of so-called inhomogeneous Lorentz transformations, or simply Poincaré transformations.

A.3 The local gauge of the Lorentz group

Lorentz transformations are an example of global gauge transformation of the system. Under local gauge, on the other hand, there is $\lambda_i = const. \rightarrow \lambda_i = \lambda_i(x^{\alpha})$, *i.e.* the transformation parameters acquire spacetime dependence.
Take only homogeneous Lorentz transformations. Then, there is

$$\Lambda^{\mu}{}_{\nu} = \frac{\partial x^{\mu'}(x^{\alpha})}{\partial x^{\nu}}. \tag{A.3.1}$$

Equation (A.3.1) is usually known as the relativity principle:

- There exists a uniform transformation between both systems x and x' so that the same principles should be valid between them.

Using it, length of 4-vectors is to stay constant after transformations, *i.e.* $v^{\mu'} v_{\mu'} \equiv v^{\mu} v_{\mu}$, independently of the coordinate system. For the metric, which now becomes spacetime dependent, on the other hand, there is

$$g_{\mu'\nu'}(x^{\nu'}) = \frac{\partial x^{\alpha}}{\partial x^{\mu'}} \frac{\partial x^{\beta}}{\partial x^{\nu'}} g_{\alpha\beta}(x^{\nu}) \tag{A.3.2}$$

and

$$g^{\mu'\nu'}(x^{\nu'}) = \frac{\partial x^{\mu'}}{\partial x^{\alpha}} \frac{\partial x^{\nu'}}{\partial x^{\beta}} g^{\alpha\beta}(x^{\nu}). \tag{A.3.3}$$

Physically, the functional values of the metric mean transformations not only between inertial systems. As a consequence, inertial forces as Coriolis and centrifugal forces appear, and these are related to the spacetime dependence of the metrical tensor. This is the grounding of General Relativity as a geometrized theory of gravitation. Therefore, (local) gauging of the Poincaré group leads to a classical theory of gravitation which is a theory of inertial forces. The equation of motion is given by that of free particles along a geodisical line. This is called the **geometrization of gravitation**.

Within a Poincaré gauge theory, the derivative of vectors and tensors transforms differently to vectors and tensors themselves. There is

$$\begin{aligned} v_{\mu',\nu'} =& \left(v_{\alpha} \frac{\partial x^{\alpha}}{\partial x^{\mu'}} \right)_{,\beta} \frac{\partial x^{\beta}}{\partial x^{\nu'}} \\ =& v_{\alpha,\beta} \frac{\partial x^{\alpha}}{\partial^{\mu'}} \frac{\partial^{\beta}}{\partial x^{\nu'}} + v_{\alpha} \frac{\partial^2 x^{\alpha}}{\partial x^{\mu'} \partial^{\nu'}}, \end{aligned} \tag{A.3.4}$$

where only the first term represents the behavior of tensorial transformations. Hence, covariant derivatives are needed so that the following is valid,

$$v_{\mu';\nu} \equiv v_{\alpha;\beta} \frac{\partial x^{\alpha}}{\partial^{\mu'}} \frac{\partial^{\beta}}{\partial x^{\nu'}}. \tag{A.3.5}$$

A semicolon represent the covariant derivative while a subscripted coma represent the usual derivative with

$$_{,\nu} \equiv \partial_{\nu} \simeq \partial_{\nu} \delta_{\lambda}{}^{\alpha} = \frac{\partial}{\partial x^{\nu}} \delta_{\lambda}{}^{\alpha}. \tag{A.3.6}$$

If only gravitation is taken into account, the covariant derivative is defined by

$$_{;\nu} \equiv D_{\nu} \simeq D_{\nu\lambda}{}^{\alpha} = \partial_{\nu} \delta_{\lambda}{}^{\alpha} - \Gamma^{\alpha}_{\nu\lambda} \tag{A.3.7}$$

and

$$v_{\lambda;\nu} = D_{\nu\lambda}{}^{\alpha} v_{\alpha} \tag{A.3.8}$$

for generally curved lorentzian manifolds (for quantum mechanics, gauge fields A_{μ} take the place of Chirstoffel symbols. Furthermore, for dual symmetry also dual gauge fields A'_{μ} are to be taken into account; all with the respective coupling constant).

Equation (A.3.6) is usually written in simplified manner. In this work, the derivative is written so that there be

$$D_{\nu} u^{\mu} \equiv D_{\nu\lambda}{}^{\mu} u^{\mu} \tag{A.3.9}$$

for the covariant derivative.
An affine connection is a geometrical object on a smooth manifold which connects nearby tangent spaces and so permits tangent vector fields to be differentiated as if they were functions on the manifold with values in a fixed vector space. The choice of the affine connection is equivalent to prescribing a way of differentiating vector fields which satisfies several properties. This is to lead to covariant derivatives and to covariant behavior, and it is equivalent to a notion of parallel transport, which is a method for transporting tangent vectors along curves. The connection coefficients of the affine connection of lorentzian manifolds, $\Gamma^{\alpha}_{\nu\lambda} = \Gamma^{\alpha}_{\lambda\nu}$, are called Christoffel symbols. They transform in the following way:

$$\Gamma^{\mu'}_{\rho'\nu'} = \Gamma^{\sigma}_{\beta\alpha} \frac{\partial x^{\alpha}}{\partial x^{\nu'}} \frac{\partial x^{\mu'}}{\partial x^{\sigma}} \frac{\partial x^{\beta}}{\partial x^{\rho'}} - \frac{\partial^2 x^{\mu'}}{\partial x^{\beta} \partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x^{\nu'}} \frac{\partial x^{\beta}}{\partial x^{\rho'}} \tag{A.3.10}$$

$$= \Gamma^{\sigma}_{\beta\alpha} \left(\Lambda^{\nu}{}_{\alpha}\right)^T \Lambda^{\mu}{}_{\sigma} \left(\Lambda^{\rho}{}_{\sigma}\right)^T - \frac{\partial}{\partial x^{\beta}} \Lambda^{\mu}{}_{\alpha} \left(\Lambda^{\nu}{}_{\alpha}\right)^T \left(\Lambda^{\rho}{}_{\beta}\right)^T . \tag{A.3.11}$$

Only then, the derivative (A.3.7) will possess the transformation behavior in equation (A.3.5). Else would lead to coordination effects which would not be physical.
General higher-order tensors transform in the following way,

$$T'^{i_1 \ldots l i_m}_{j_1 \ldots j_n} = \frac{\partial x'^{i_1}}{\partial x^{k_1}} \cdots \frac{\partial x'^{i_m}}{\partial x^{k_m}} \frac{\partial x^{l_1}}{\partial x'^{j_1}} \cdots \frac{\partial x^{l_n}}{\partial x'^{j_n}} T^{k_1 \ldots k_n}_{l_1 \ldots l_n} . \tag{A.3.12}$$

This is the tensor-transformation law.
Christoffel symbols possess an inhomogeneous transformation behavior and are therefore no tensors. Their transformation may be made to vanish through coordinate transformations [74].
Christoffel symbols give the affine connection for GR. This gives a rule which describes how to legitimately move a vector along a curve of a manifold without changing its direction. Therefore, the Christoffel symbol should account for curvature of spacetime and parallel transport. Furthermore, they are related to the metrical tensor $g_{\mu\nu}$ by

$$\Gamma^{\mu}_{\beta\nu} = \frac{1}{2} g^{\mu\alpha} (g_{\beta\alpha,\nu} + g_{\alpha\nu,\beta} - g_{\beta\nu,\alpha}), \tag{A.3.13}$$

so that

$$v^{\mu}{}_{;\nu} v^{\nu} = 0 \tag{A.3.14}$$

is valid. Furthermore, the affine connection satisfies Riemann's requirement that an object should be independent of its description in a particular coordinate system. The covariant derivative represents especially a differential operator for an additive linear transformation which obeys the product rule. Hence, following be valid for the covariant derivative of a product:

$$(v_{\mu} b_{\nu})_{;\alpha} = v_{\mu,\alpha} b_{\nu} + v_{\mu} b_{\nu,\alpha} - \Gamma^{\sigma}_{\mu\alpha} v_{\sigma} b_{\nu} - \Gamma^{\sigma}_{\nu\alpha} v_{\mu} b_{\sigma}$$

$$= v_{\mu;\alpha} b_{\nu} + v_{\mu} b_{\mu;\alpha} . \tag{A.3.15}$$

For a 2^{nd}-rank tensor $A_{\mu\nu}$, there be

$$A_{\mu\nu;\alpha} = A_{\mu\nu,\alpha} - \Gamma^{\sigma}_{\mu\alpha} A_{\sigma\nu} - \Gamma^{\sigma}_{\nu\alpha} A_{\mu\sigma} . \tag{A.3.16}$$

For upper (contravariant) indices, there is

$$v^{\mu}{}_{;\nu} = v^{\mu}{}_{,\nu} + \Gamma^{\mu}_{\beta\nu} v^{\beta} , \tag{A.3.17}$$

and given equation (A.3.17), it can directly be proven that

$$v^{\lambda}{}_{;\lambda} = \frac{(\sqrt{-g}\, v^{\mu})_{,\lambda}}{\sqrt{-g}} \tag{A.3.18}$$

is valid, with g as the determinant of the absolute value of the metrical tensor $g_{\mu\nu}$. Following the calculation, there is

$$\frac{(\sqrt{-g})_{,\lambda}}{\sqrt{-g}}\, v^{\lambda} \equiv \Gamma^{\mu}_{\lambda\mu} v^{\lambda} \tag{A.3.19}$$

for the Christoffel symbol.
Since in curved space inversion of the first and second derivatives of a vector does not lead to the same mathematical object, a measure of such permutation loss may be defined by the Ricci identities as a gravitational field strength. They give a tensor of 4^{th} rank with

$$\begin{aligned} -[D^{\nu}, D^{\alpha}]u^{\mu} &= (u^{\mu}{}_{;\nu;\alpha} - u^{\mu}{}_{;\alpha;\nu}) && \text{(A.3.20)} \\ &= (u^{\mu}{}_{,\nu} + \Gamma^{\mu}_{\sigma\nu}u^{\sigma})_{;\alpha} - (u^{\mu}{}_{,\alpha} + \Gamma^{\mu}_{\sigma\alpha}u^{\sigma})_{;\nu} \\ &= -\left(\Gamma^{\mu}_{\beta\alpha}\Gamma^{\beta}_{\sigma\alpha} - \Gamma^{\mu}_{\beta\alpha}\Gamma^{\beta}_{\sigma\nu} + \Gamma^{\mu}_{\sigma\alpha,\nu} - \Gamma^{\mu}_{\sigma\nu,\alpha}\right) \\ &= -R^{\mu}{}_{\sigma\alpha\nu}u^{\sigma} && \text{(A.3.21)} \end{aligned}$$

which is known as the Riemann (curvature) tensor. Further terms which are dependent on the coordinates cancel, given Schwartz's theorem for usual derivatives. Furthermore, the trace of $R_{\mu\nu\lambda\sigma}$ is called Ricci tensor $R_{\mu\nu}$. This rank-2 tensor is gotten by

$$\begin{aligned} R_{\mu\nu\lambda\sigma}g^{\mu\sigma} &= R^{\sigma}{}_{\nu\lambda\sigma} \\ &\equiv R_{\nu\lambda}. && \text{(A.3.22)} \end{aligned}$$

Thus, there is

$$\begin{aligned} -[D_{\sigma}, D_{\alpha}]u^{\sigma}u^{\alpha} &= (u^{\sigma}{}_{;\sigma;\alpha} - u^{\sigma}{}_{;\alpha;\sigma})u^{\alpha} && \text{(A.3.23)} \\ &= u^{\sigma}{}_{;\sigma;\alpha}u^{\alpha} - u^{\sigma}{}_{;\alpha;\sigma}u^{\alpha} \\ &= R_{\mu\alpha}u^{\mu}u^{\alpha}\,. && \text{(A.3.24)} \end{aligned}$$

Further, the Ricci or Riemann scalar is then given by

$$\begin{aligned} R_{\mu\nu}g^{\nu\mu} &= R_{\mu}{}^{\mu} && \text{(A.3.25)} \\ &\equiv R. \end{aligned}$$

The Jacobi identity with the covariant derivatives leads to the Bianchi identities

$$\begin{aligned} R_{\mu\lambda[\nu\sigma;\rho]} &= R_{\mu\lambda\nu\sigma;\rho} + R_{\mu\lambda\rho\nu;\sigma} + R_{\mu\lambda\sigma\rho;\nu} && \text{(A.3.26)} \\ &= 0\,. \end{aligned}$$

These are the (homogeneous) Yang–Mills equations of gravity (*cf.* equation (A.2.5) for the ones of electromagnetism, *viz* Maxwell equations). Multiplying equation (A.3.26) by $g^{\lambda\sigma}$ and $g^{\mu\rho}$ then leads to

$$\left(R_{\mu}{}^{\nu} - \frac{1}{2}R\delta_{\mu}{}^{\nu}\right)_{;\nu} = 0\,, \tag{A.3.27}$$

which can be taken as a definition of a tensor

$$G_{\mu}{}^{\nu} = R_{\mu}{}^{\nu} - \left(\frac{1}{2}R - \Lambda_0\right)\delta_{\mu}{}^{\nu}\,, \tag{A.3.28}$$

with $G_{\mu\nu}$ known as Einstein (curvature) tensor. It is divergence-free, *i.e.*

$$G_{\mu}{}^{\nu}{}_{;\nu} = 0\,, \tag{A.3.29}$$

and it possesses a constant term Λ_0 called cosmological constant. Equation (A.3.28) with (A.3.27) gives the geometrical (left-hand side) part of the equations of motion of gravitation in GR. The right-hand side, which is the one of matter, is presented in Chapters 6.1 and A.4. It is model-dependent.

A.4 Einstein equations and matter

Within SR, relativistic mass increases with velocity as well as particle density does, following Lorentz transformations. Hence, energy density

$$\epsilon = \varrho\, c^2, \tag{A.4.1}$$

whereas ϱ give matter density and c the speed of light, possesses a quadratical dependence to Lorentz transformations. Hereby, density reads $\epsilon = mnc^2$, with mass m, particle density n and speed of light c. Thus, the metrical energy–stress tensor

$$T^{\mu\nu} = \epsilon v^{\mu} v_{\nu} \tag{A.4.2}$$

with $v^{\mu} = dx^{\mu}/ds$ replaces mass and energy density when using the covariant formalism.
Within GR gravitational interaction (understood as curving of spacetime) is caused by all kinds of mass and energy densities. On the other hand, Bianchi identities as field equations of curvature show the Einstein tensor as essential property of gravitation. Therefore, Einstein's field equations, first derived in 1915 [81], read

$$G_{\mu\nu} = -\kappa_N\, T_{\mu\nu}\,. \tag{A.4.3}$$

Here, the Einstein tensor reads

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda_0 g_{\mu\nu}, \tag{A.4.4}$$

with the Ricci tensor $R_{\mu\nu}$, the Ricci scalar R, the metrical tensor $g_{\mu\nu}$ with determinant g and a cosmological constant Λ_0.

$$\kappa_N = 8\pi G_N/c^4 \tag{A.4.5}$$

is a strength-coupling constant gotten by comparison of the linearized field equations with Newton's theory. Therefore, it is related to Newton's gravitational constant G_N and the speed of light c, which GR postulates as constant.

To ground Einstein's general theory of relativity, the fundament of the theory is the Hilbert–Einstein (HE) action,[1]

$$S_{HE} = \int \left[\frac{1}{\kappa_N} (R + \Lambda_0) + \mathcal{L}_M \right] \sqrt{-g}\, d^4x. \tag{A.4.6}$$

The Hamilton Principle of the Least Action leads correctly to newtonian as well as to quantum mechanics, electromagnetism and to current models of elementary particle physics, for instance (see [22]). The model, of course, grounds on the Lagrangian (or Lagrange density $\mathcal{L}$) chosen under the integral of the action as

$$S = \int_a^b \mathcal{L} \sqrt{-g} d^4x,$$

with a and b as constant boundaries.

Equation (A.4.6) is the fundament of usual GR, and it defines all gravitational interactions.

Furthermore, the energy–momentum tensor $T_{\mu\nu}$ is related to the Lagrangian of matter $\mathcal{L}_M$, which is the fundament of a theory where (especially fermionic) matter is defined in terms of the wave function given by the state ψ, in accordance with quantum mechanics. However, it is quantum mechanics which indeed leads to the idea of field theories instead of only theories for the dynamics of particle systems. Within quantum theories, trajectories are no longer defined. However, their analog can be found in the quantum mechanical state, as the system "blurred" in space. As eigenvector of an observable, the state gives the probability of qualities to be measured.

With equation (A.4.6), there is explicitly,

$$\delta \int (R + \Lambda_0) \sqrt{-g} d^4x = -\kappa_N \int \frac{\partial \left(\mathcal{L}_M \sqrt{-g} \right)}{\partial g_{\alpha\beta}} \delta g_{\alpha\beta} d^4x. \tag{A.4.7}$$

The differential quotient on the right-hand side of equation (A.4.7) is tensorial. With the ansatz

$$\frac{\partial \left(\mathcal{L}_M \sqrt{-g} \right)}{\partial g_{\alpha\beta}} = -\sqrt{-g} T^{\alpha\beta}, \tag{A.4.8}$$

there is then

$$T^{\alpha\beta} = -\frac{1}{\sqrt{-g}} \frac{\partial \left(\mathcal{L}_M \sqrt{-g} \right)}{\partial g_{\alpha\beta}}. \tag{A.4.9}$$

So, the Lagrangian of matter give the metrical energy–stress tensor as a source of the metric itself. There is

$$\delta \left(\int R + \Lambda_0 \right) \sqrt{-g} d^4x = -\kappa_N \int T^{\alpha\beta} \delta g_{\alpha\beta} \sqrt{-g} d^4x. \tag{A.4.10}$$

On the other hand, the left-hand side of equation (A.4.7) may be written as

$$\begin{aligned} \delta \int (R + \Lambda_0) \sqrt{-g} d^4x &= \int \sqrt{-g} \delta R d^4x + \int (R + \Lambda_0) \delta \sqrt{-g} d^4x \\ &= \int \sqrt{-g} \delta R d^4x + \frac{1}{2} \int (R + \Lambda_0) g^{\alpha\beta} \delta g_{\alpha\beta} \sqrt{-g} d^4x. \end{aligned} \tag{A.4.11}$$

There is

$$\begin{aligned} \delta R =& \delta \left(R_{\alpha\beta} g^{\alpha\beta} \right) \\ =& R_{\alpha\beta} \delta g^{\alpha\beta} + \delta R_{\alpha\beta} g^{\alpha\beta}. \end{aligned}$$

[1] After David Hilbert (1862-1943) and Albert Einstein (1879-1955, Nobel prize 1921).

Therein,

$$\delta g^{\alpha\beta} = -g^{\alpha\sigma} g^{\beta\lambda} \delta g_{\sigma\lambda}. \tag{A.4.12}$$

Therefore, there is

$$\delta R = -R^{\alpha\beta} \delta g_{\alpha\beta} + \delta R_{\alpha\beta} g^{\alpha\beta}.$$

The variation of the Ricci tensor is given by

$$\delta R_{\alpha\beta} = (\delta \Gamma^\sigma{}_{\sigma\beta})_{;\alpha} - (\delta \Gamma^\sigma{}_{\alpha\beta})_{;\sigma}.$$

Be metricity postulated, *i.e.*

$$g_{\mu\nu;\alpha} = 0. \tag{A.4.13}$$

Then we have

$$\delta R = -R^{\alpha\beta} \delta_{\alpha\beta} + (g^{\alpha\beta} \delta \Gamma^\sigma{}_{\sigma\beta})_{;\alpha} - (g^{\alpha\beta} \delta \Gamma^\sigma{}_{\alpha\beta})_{;\sigma}.$$

Consequently, equation (A.4.11) yields

$$\begin{aligned}
\delta \int (R + \Lambda_0) \sqrt{-g} d^4x = & - \int \left(R^{\alpha\beta} + \Lambda_0 g^{\alpha\beta}\right) \delta g_{\alpha\beta} \sqrt{-g} d^4x + \\
& + \int \left(\left(g^{\alpha\beta} \delta \Gamma^\sigma{}_{\sigma\beta}\right)_{;\alpha} - \left(g^{\alpha\beta} \delta \Gamma^\sigma{}_{\alpha\beta}\right)_{;\sigma} \right) \sqrt{-g} d^4x + \frac{1}{2} \int R g^{\alpha\beta} \delta g_{\alpha\beta} \sqrt{-g} d^4x \\
= & - \int \left(R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} + \Lambda_0 g^{\alpha\beta} \right) \delta g_{\alpha\beta} \sqrt{-g} d^4x.
\end{aligned} \tag{A.4.14}$$

The integral terms over the variation of the Christoffel symbols vanish by means of the Gauß (in the following *Gauss*) theorem, since it is integrated over two vectorial divergences, and both the variation of the metric and the one of its first derivative are to vanish at the boundaries. Furthermore, since equation (A.4.14) is to be valid for general $\delta g_{\alpha\beta}$, then there is equation (A.4.3) with

$$R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} + \Lambda_0 g^{\alpha\beta} = -\kappa_N T^{\alpha\beta}. \tag{A.4.15}$$

These are the Einstein equations of General Relativity, explicitly derived for seek of completeness. Λ_0 is again the cosmological constant. For it we use a subscript zero to stress its constant character and to differ between it and a functional term $\Lambda(\xi) \equiv \Lambda$ as introduced in Chapter 6.1. This constant was first added by Einstein in [82] with the idea of a closed Universe which would be statical. Hence, it acts against gravitation. However, Einstein's statical universe is unstable as was shown by deSitter [222]. No physical or mathematical property, though, has shown so far why Λ_0 should be exactly zero. $\Lambda \equiv 0$ is preferred according to simplicity postulates. Modern studies, however, lead to small but nonvanishing values of the same [198]. This is related to the problem of Dark Energy which is discussed in Chapter 2.4

Appendix B

Wave function and elementary particles

B.1 QM state and Spin-Magnetic interaction, QM postulates and measurement

Within the formal derivation of Einstein's equations of gravitation in Appendix A.4, the wave function ψ was introduced in the context of the Lagrangian of matter. A wave equation or better said the quantum mechanical state is the one mathematical object in which the whole information for a measurement is kept. In situations where the maximally possible amount of information is gotten, a closed quantum system is given at each time t by its state vector ψ (postulate 1 of quantum mechanics). Outside of measuring processes in the nonrelativistic case, its dynamical development (for particle systems with mass m) is given by the Hamilton operator $\hat{H}$ of the system (postulate 4 of quantum mechanics), within nonrelativistic limites by means of the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \hat{H}\psi(\vec{r},t). \tag{B.1.1}$$

Within SR, further, the specialrelativistic Schrödinger (Klein–Gordon) equation,

$$\left(\Box - \frac{m^2c^2}{\hbar^2}\right)\psi = 0\,, \tag{B.1.2}$$

is taken, or else the Dirac equation (6.2.7), which introduces Spin, formally by the square-root of (B.1.2)). Under special not highly relativistic regimes, for instance, the Pauli equation may be used. It takes into account the interaction of the particle's spin with the electromagnetic field in the non-relativistic limit. Hence, the Pauli equation presents the Stern–Gerlach term with a $\hbar Q\vec{\sigma}\cdot\vec{\varphi}\vec{B}/(2m)$ added to the Schrödinger equation which already possesses $\vec{j}\vec{A}$ terms from the covariant derivative. Q is the electric charge, $\vec{\sigma}$ are the Pauli matrices, $\vec{B}$ is the magnetic field-strength and $\vec{\varphi}$ is the Dirac vector $|\psi>$ with spinor components. $\hbar$ is the reduced Planck action and m is the system's mass. The spin operator is given by $\hat{S} = \hbar\vec{\sigma}/2$.

The quantization of angular momentum of spin (Nobel-prize awarded 1943) was experimentally proven by O. Stern and W. Gerlach in 1922: Spin momentum leads to a quantized magnetic moment of a semiclassical spinning dipole. Consequently, torque exerted by a magnetic field leads to precession of the dipole. However, because of quantization of spin, an atom beam under a magnetic field splits into two beams related to the two different orientations of electron spin. Further, a state transition can be induced so that one of these beams vanishes if an alternating field with the resonance frequency (Larmor frequency) is used. This is the Rabi experiment [204] of Electron Paramagnetic (EPR) or Electron Spin resonance (ESR), Nobel prize

awarded in 1946. Further, nuclear spin leads to analogous, yet far weaker effects from the coupling between nuclear momenta and the electromagnetic field.
The expansion of Rabi's technique by F. Bloch and E.M. Purcell [28, 203] for the nuclear spin was Nobel-prize awarded in 1952. It lead to the Nuclear Magnetic Resonance (NMR). Here, the Larmor frequency (characteristic of the isotope in question) is directly proportional to the strength of the applied magnetic field and to the energy splitting between both nuclear spins. The proportionality factor is the gyromagnetic ratio of the isotope. The so-called Rabi oscillations are the working mechanism for (Nuclear) Magnetic Resonance Imaging (MRI) (or NMR Tomography) presently used in radiology to visualize detailed internal structure providing contrast between the different soft tissues of the body:[1] Radio frequencies are used to systematically alter the alignment of the magnetization. This causes the hydrogen nuclei to produce a rotating magnetic field detectable by the scanner. An electromagnetic pulse causes the nuclei to absorb energy and radiate this back out at the Larmor frequency. Hence, different nuclides may be contrasted by means of their moment in the magnetic field. Momenta are quantum numbers of the quantum state.

Given that trajectories lose their meaning within quantum mechanics, another approach for dynamics is the path integral. If a particle with amplitude (state) $\psi(\vec{r}_1, t_1)$ propagates from $A(\vec{r}_1, t_1)$ to $B(\vec{r}_2, t_2)$, its dynamics may analogously be described by a wave function as follows [76],

$$\psi(\vec{r}_2, t_2) = \int d^3\vec{r}_1 K(\vec{r}_2, t_2|\vec{r}_1, t_1)\psi(\vec{r}_1, t_1). \tag{B.1.3}$$

Its amplitude (kernel, propagator, *cf.* Chapter 3.3) as

$$K(B|A) = \int dC \phi_{BA}[C], \tag{B.1.4}$$

with C as the path taken to get from A to B, and with $\phi_{BA}[C]$ as amplitude of the path, commonly taken to be

$$\phi_{BA}[C] = e^{\frac{i}{\hbar}S[C]}, \tag{B.1.5}$$

with the action S. The sum in equation (B.1.4), which is known as Feynman path, has to be taken over all paths from A to B. With equation (B.1.5), the Feynman propagator yields naïvely,

$$K(\vec{r}_2, t_2; \vec{r}_1, t_1) = \int_{r(t_1)=r_1}^{r(t_2)=r_2} [d\vec{r}(t)] e^{\frac{i}{\hbar}S(\vec{r}(t), \dot{\vec{r}}(t); t)}. \tag{B.1.6}$$

Its main contribution for $\hbar \to 0$ is the classical one with S_{cl} for the classical path $\vec{r}$ since the path of minimum action dominates. Further contributions appear from the deviation from the path and the expansion of the action (or Lagrange function) around such path. Interestingly, reverse Wick rotation $it \to t$ of the Feynman propagator leads directly to the partition function for an action (see Chapter 4.2).
Since trajectories are not defined (hence the deviation from the classical path and the sum over all paths in Feynman's formulation above), the equations of motion are given for states (ψ), treating matter as fields.[2]

[1] Within functional imaging even hemodynamic (blood) responses related to neural activity and blood oxygen levels may be contrasted, *sc.* [184].

[2] For instance, the electrons of a single isolated atom occupy atomic orbitals which form a discrete set of energy levels. In a molecule, these orbitals split into a number of molecular orbitals proportional to the number of atoms. For a high amount of atoms, these orbitals form (quasi) continuous energy bands. In case of (semi)conductors and insulators, energy bands each correspond to a large number of discrete quantum states of the electrons.

Dynamics is then given by some of the latter formulations, and operations on the fields are given by operators which act on the states. A state to an eigenvalue of an operator is called eigenstate.
Physical, measurable properties are given by (hermitean) operators $\hat{A}$ which possess real-valued eigenvalues. They are called observables (postulate 2). Their eigenvalues should be the average expectation value for measurements (postulate 3). The measurement problem itself, though (and hence QM's postulates themselves), is a matter of research, and it is related to the so-called *collapse* of the wave function [125]. The observer is understood as part of the whole quantum mechanical system, and as such there is an intrinsic interaction between him and the analyzed subsystem. This interaction leads to loss of information in form of the "collapse" to the basis of the observed property with eigenvalues a, which are the statistical mean value of measurements. If the eigenvalues of $\hat{A}$ and of a further observable $\hat{B}$ possess different eigenvectors, the measurement of both is not commutative ($\hat{A}\hat{B} \neq \hat{B}\hat{A}$) and a change in the order of measurement leads to different mean results. The quantum mechanical state, which gives the properties of the analyzed system, changes after the first measurement. A second measurement represents an interaction with a different system where information was lost. Actually, this is nearly related with Heisenberg's *uncertainty relation*, often given for the average measurement of a momentum $\hat{p}_x$ (*i.e.* $< \hat{p}_x >$) and of a coordinate $\hat{x}$ which a particle may possess as properties at a specific moment ($< \hat{x} >$). As canonical conjugate operators, $\hat{p}_i$ and $\hat{x_i}$ are orthogonal to each other and thus possess a different basis. Consequence: it is not possible to know both the exact momentum (velocity) and place of say an electron at the same time,

$$\sqrt{< \Delta x_i^2 >< \Delta p_j^2 >} \geq \frac{\hbar}{2}\delta_{ij} \, . \tag{B.1.7}$$

Furthermore, this is valid for the measurement of operator $\hat{A}$ with canonical conjugate operator P_A in general. More generally, though, it is valid for general observables $\hat{A}$ and $\hat{B}$ so that

$$\sqrt{< \Delta A^2 >< \Delta B^2 >} \geq \frac{1}{2}| < [\hat{A}, \hat{B}] > |. \tag{B.1.8}$$

This is the general form of the uncertainty principle. Without relevance of order, both observables can be simultaneously measured only if their commutator vanishes. This is the case if they possess the same basis. Is this not the case, then the product of the average deviations ΔA and ΔB will not vanish. This quantum mechanical property is very useful to give length scales associated to masses of elementary particles. Since t, however, is not an operator in QM, uncertainty is not derived in such a formal manner as for operators. Nevertheless, there is

$$\Delta t \, \Delta E \gtrsim \hbar. \tag{B.1.9}$$

The quantity Δt is the minimal measure-time to determine the energy of a wave packet of width ΔE passing through a detector. Given that mass and energy are related and passed time and distances as well (by means of uncertainty relations), massive mediative particles can be related to distances in which interactions are mediated by these particles (*i.e.* distances which these particles are able to move through before they vanish again). With a distance $R = c\Delta t$ and an energy ΔE given by the mass of a mediated particle with $\Delta E = mc^2$, the length scale in which this particle interacts is acquired. Particles with an energy of 140 MeV, for instance, have length scales R of the order of magnitude of a femtometer. This is the case for the particles which H. Yukawa predicted in 1939 [246] (the pions, discovered in 1947 [149]), and which may move only distances smaller than the width of a nucleon in which they interact during the short time before decaying in other particles.
Furthermore, since a realistic quantum system is never isolated, the interaction of the state with its environment is important. There are quantum correlations between them, and these interactions may be understood as a sort of measurement, again related to a collapse but especially to the so-called *decoherence* [97].

Through it, superpositions of the wave functions, a fundamental property of quantum physics (mathematically founded in the linearity of the Hilbert space, in which quantum mechanical states exist), vanish.[3]
The idea is that classical mechanics should be recovered from quantum mechanics by means of the quantum properties themselves, especially for large sizes and masses of the observed system. Quantum properties cancel out, leading to the classical world. The dynamics that will explain the collapse and define a complete theory of measurement has not yet been completely explained, though. It is related to what is called "the problem of definite outcomes" and the one of the "preferred basis". Together they form *the measurement problem* (*cf.* [141]), and their future research relies on quantum information theory.
Research on quantum information, related for instance with quantum and non-linear optics, leads to many new and classically unexplainable effects such as entanglement (from the so-called EPR paradox of Einstein, Podolsky and Rosen [83]) and quantum teleportation [16] (experimental fact since Zeilinger's experiment in 1997 [34]!), which is fundamental to the concept of quantum computers [35] and which should further be explained in relativistic contexts (where *a priori* information of the state to be teleported seems necessary to label identical particles to make them effectively distinguishable [148]). However, neither usual (Schrödinger's -nonrelativistic- nor even Dirac's - special-relativistic for particles with spin) quantum mechanics alone nor General Relativity can describe the nature of matter itself. This is rather fulfilled within the context of (special-relativistic as well as quantum theoretic) elementary particle and high energy physics. The latter evolved out of nuclear physics with the desire to discover the foundations of matter and its fundamental dynamics. Hence, the quantum mechanical state may be related to isospin, and the Heisenberg's uncertainty relation takes a fundamental role for interpretations of length scales of interactions and masses of particles in fundamental dynamics.

B.2 On the Yang–Mills theory

The Yang–Mills theory is a non-abelian (non-commutative) theory with SU(N) transformations (*i.e.* unitary matrix-valued transformations for N dimensions and determinant +1 for the transformation operator or matrix) and thus with self-interactions that generalize the Maxwell equations of (abelian U(1)-) electrodynamics (where photons as gauge bosons -mediators- do not self-interact) to the so-called and analog Yang–Mills equations. With the Ricci identities,

$$F_{\mu\nu a}{}^{b} \equiv \frac{1}{ig}[D_{\mu a}\,, D_{\nu c}{}^{b}], \tag{B.2.1}$$

there is the field-strength tensor $F_{\mu\nu a}$ for the isospin component a of the isospin vector ψ_a and with the general form

$$F_{\mu\nu i} = (A_{\nu i,\mu} - A_{\mu i,\nu}) - gA_{\mu k}A_{\nu l}f^{kl}{}_{i}, \tag{B.2.2}$$

following the covariant derivative defined as

$$D_{\mu a}{}^{b} = \delta_a{}^{b}\partial_\mu + igA_{\mu a}{}^{b}\,, \tag{B.2.3}$$

with the gauge field $A_{\mu a}$ which is related to gauge bosons[4] and is analogue to the potential A_μ entailing scalar (φ) and vector potential ($\vec{A}$) in Special Relativity and electromagnetism.

[3] A well-known example of superpositions in quantum mechanics is *Schrödinger's cat*. It is described as alive and death at the same time until measurement (observation) "decides" its classical status.

[4] Within electroweak interactions, the mass eigenstates of gauge fields are gotten through a further orthogonal transformation called Weinberg mixture, related to the vanishing mass of photons. Further, the current of measurable gauge fields appears from superposition

The gauge field appears by means of local gauge transformation of the gauge parameters λ_i of the transformation group

$$\text{SU(N)} \equiv U_a{}^b = e^{i\lambda_i \tau^i{}_a{}^b}. \tag{B.2.4}$$

The gauge parameters λ_i are spacetime dependent under local gauge transformations. Hence, since field equations are to remain invariant, they lead to the necessity of replacing usual by covariant derivatives.
Further, τ^i are hermitean transformation matrices with $i = 1, \ldots N^2 - 1$ with N isotopic fermionic components ψ_a under the interactions given by the force which is reduced to gauge potentials A_μ.
According to the Bianchi identities for SU(N), homogeneous Yang–Mills systems are given by

$$\partial_{[\lambda} F_{\mu\nu]i} - gA_{k[\lambda} F_{\mu\nu]j} f^{kj}{}_i = 0, \tag{B.2.5}$$

with Bach parenthesis $a_{[i}\, b_{k]} = \frac{1}{2}(a_i b_k - a_k b_i)$ and structure constants $f^{kj}{}_i$ of the gauge group. Furthermore, with adjoint fields

$$\mathcal{F}_{\mu\nu} = F_{\mu\nu i}\tau^i, \quad \text{with} \quad F_{\mu\nu a}{}^b = F_{\mu\nu i}\tau^i{}_a{}^b, \tag{B.2.6}$$

with a further definition

$$\mathcal{D}_\lambda \mathcal{F}_{\mu\nu} \equiv [D_\lambda, \mathcal{F}_{\mu\nu}] = \mathcal{F}_{\mu\nu,\lambda} + ig[\mathcal{A}_\lambda, \mathcal{F}_{\mu\nu}] \tag{B.2.7}$$

and with adjointly represented gauge fields

$$(\mathcal{A}_\mu)_a{}^b = A_{\mu i}(\tau^i)_a{}^b \tag{B.2.8}$$

with

$$\mathcal{F}_{\mu\nu} = \mathcal{A}_{\nu,\mu} - \mathcal{A}_{\mu,\nu} + ig[\mathcal{A}_\mu, \mathcal{A}_\nu] = \mathcal{F}^\dagger_{\mu\nu}, \tag{B.2.9}$$

there is a matrix-valued covariant derivative

$$\mathcal{D}_\mu = \underline{1}\partial_\mu + ig\mathcal{A}_\mu \tag{B.2.10}$$

so that the homogeneous Yang–Mills system may be written as follows,

$$\mathcal{D}_\lambda \mathcal{F}_{\mu\nu} + \mathcal{D}_\mu \mathcal{F}_{\nu\lambda} + \mathcal{D}_\nu \mathcal{F}_{\lambda\mu} = 0. \tag{B.2.11}$$

This a form analogue to the homogeneous Maxwell system of electrodynamics.
Euler–Lagrange equations for each isospin component ψ_a yield the following set of equations

$$\begin{aligned} \left(\frac{\partial \mathcal{L}}{\partial \psi_{aA,\mu}}\right)_{,\mu} - \left(\frac{\partial \mathcal{L}}{\partial \psi_{aA}}\right) &= 0\,, \\ \left(\frac{\partial \mathcal{L}}{\partial \bar{\psi}^{aA}{}_{,\mu}}\right)_{,\mu} - \left(\frac{\partial \mathcal{L}}{\partial \bar{\psi}^{aA}}\right) &= 0\,, \end{aligned} \tag{B.2.12}$$

For general Lagrangians, there is the canonical energy–stress tensor which is described within Yang–Mills theories for massless fermions as follows,

$$T_\mu{}^\nu = \frac{\partial \mathcal{L}}{\partial \psi_{aA,\nu}} \psi_{aA,\mu} + \frac{\partial \mathcal{L}}{\partial \bar{\psi}^{aA}{}_\nu} \bar{\psi}^{aA}{}_{,\mu} - \mathcal{L}\delta_\mu{}^\nu. \tag{B.2.13}$$

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of the different iso-components of the gauge fields in the form $W^\pm_\mu = \frac{1}{2}(W_{\mu 1} \mp iW_{\mu 2})$ (for $A_{\mu i} = W_{\mu i}$ of weakons), and analogously for gluons of strong interactions. The field-strength tensor of W^+ bosons, for instance, is then related to decay channels such as $\nu_L \to e_L$ plus a mass term of the gauge fields themselves (see [74]).

Analogously to electrodynamics, invariance of the Lagrangian under global SU(N) transformations in isospin space leads to conservation of 4-current densities,

$$j^{\mu i} = \frac{1}{i}\left(\frac{\partial \mathcal{L}}{\partial \psi_{aA,\mu}}\tau^i{}_a{}^b\psi_{bA} - \frac{\partial \mathcal{L}}{\partial \bar{\psi}^{aA}{}_{,\mu}}\bar{\psi}^{bA}\tau^i{}_b{}^a\right), \tag{B.2.14}$$

Massless fermionic multiplets (*i.e.* with spin) are described by a Dirac Lagrangian of following form,

$$\mathcal{L} = \frac{i}{2}\bar{\psi}^{aA}\gamma^\mu{}_A{}^B\psi_{aB,\mu} - \frac{i}{2}\bar{\psi}^{aA}{}_{,\mu}\gamma^\mu{}_A{}^B\psi_{aB}\,, \tag{B.2.15}$$

with Dirac matrices γ^μ following the Clifford algebra

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}\underline{1}, \tag{B.2.16}$$

with the Minkowski metric $\eta^{\mu\nu}$ of Special Relativity. Hence, Euler–Lagrange equations lead to so-called Dirac equations of massless multiplets with the following form,

$$i\bar{\psi}^{aA}{}_{,\mu}\gamma^\mu{}_A{}^B = 0, \tag{B.2.17}$$

$$i\gamma^\mu{}_A{}^B\psi_{aB,\mu} = 0. \tag{B.2.18}$$

The canonical energy–stress tensor reads now explicitly,

$$T_\mu{}^\nu = \frac{i}{2}\bar{\psi}^{aA}\gamma^\nu{}_A{}^B\psi_{aB,\mu} - \frac{i}{2}\bar{\psi}^{aA}{}_{,\mu}\gamma^\nu{}_A{}^B\psi_{aB}. \tag{B.2.19}$$

Furthermore, there is the covariant 4-current density which reads explicitly

$$j^{\mu i} = \bar{\psi}^{aA}\gamma^\mu{}_A{}^B\tau^i{}_a{}^b\psi_{bB}. \tag{B.2.20}$$

The consequences of gauge on the Lagrangian may be written as the addition of an additional interaction term

$$\mathcal{L}_{int} = gj^{\mu i}A_{\mu i} \tag{B.2.21}$$

which relates to a general form of the jA coupling of electrodynamics. The gauge potential leads to dynamics of non-free systems as related to appearing forces analogous to newtonian forces or electromagnetic strength. Analogously to electrodynamics, it gives a generalized (invariant) Lorentz force density by means of

$$F_{\mu\nu i}j^{\mu i} = k_\nu. \tag{B.2.22}$$

With further Lagrangian terms from the field-strength tensor itself, Euler–Lagrange equations further lead, analogously to electrodynamics, to inhomogeneous Yang–Mills equations,

$$D_\nu F^{\mu\nu i} = -4\pi\hbar c g j^{\mu i}(\psi_a), \tag{B.2.23}$$

which in adjoint formulation read

$$\mathcal{D}_\nu\mathcal{F}^{\mu\nu} = -4\pi\hbar c g\mathcal{J}^\mu(\psi_a), \tag{B.2.24}$$

with current conservation given by

$$\mathcal{D}_\mu\mathcal{J}^\mu(\psi_a) = 0, \tag{B.2.25}$$

following

$$\mathcal{D}_\mu \mathcal{D}_\nu \mathcal{F}^{\mu\nu} = 0. \tag{B.2.26}$$

For $N = 1$, the Yang–Mills theory indeed reduces to electromagnetism, and field-strength $F_{\mu\nu}$, related to $\vec{E}$ and $\vec{B}$ fields (see Chapter 4.2), is given by derivatives of the isoscalar gauge potential A_μ (itself related to the scalar (φ) and the vector ($\vec{A}$) potentials). For $N > 1$, there are self-interactions of the gauge fields themselves, entailing that the gauge particles related to the gauge field A_μ self-interact. There are $N^2 - 1$ gauge bosons for a transformation group SU(N), whereas gauge bosons mediate some forces related to the potentials. Within elementary-particle physics, these mediated forces are crucial. The formalism above, however, is only valid for massless fermions and bosons related to gauge potentials. Simple addition of mass terms to the Lagrangian is not possible taking into account parity violations in weak interactions. Parity violation was first proposed by Lee and Yang in 1956 [150] and it represents indeed an experimental fact since Wu's works of 1959 [244]. Furthermore, such terms as simply added masses lead to singularities. A per-hand-massive Yang–Mills theory is not renormalizable. To achieve a physical theory, it seems necessary to introduce scalar fields and the concept of symmetry breaking so that masses appear in an indirect way by means of new parameters (see Chapter 3).

B.3 Electroweak doublet of the SM

The isodoublet of electroweak interactions reads in its general form as follows,

$$\psi_{L/R}{}^{mf} = \begin{pmatrix} \psi^{lf} \\ \psi^{qf} \end{pmatrix}_{L/R},$$

It may be decomposed in the following way (see Chapter 2.2):

- Leptonic fields:
  For $m = 1$, *i.e.* for the wave function of leptons, the index f takes into account the three families (or generatons) ($f = 1 \ldots 3$) of electron-like particles. Hence, for left- and right-handed states, $e^{f=1}$ represents the electron, while for $f = 2$, the isospin component represents muons μ. For $f = 3$, finally, the represented lepton is the tauon τ.
  Given parity-symmetry breaking, the second isospin component for $m = l$ is non-existent for right-handed particles. The state is represented by isoscalars which transform under the transformation group U(1). Left-handed states represent the three generations of neutrinos related to each electron-like particle, *i.e.* ν_e, ν_μ and ν_τ (it is assumed that they exist only left-handed!). Further, parity non-conservation exists in form of CP breaking with C for conjugation. Hence, for antiparticles, left-handed states are isoscalar, with positrons e^+_L for $f = 1$, anti-muons μ^+_L for $f = 2$ and anti-tauons τ^+_L for $f = 3$ as isoscalar components. For the right-handed isodoublet, on the other hand, antielectron-particles (e^+_R, μ^+_R and τ^+_R) are represented by one component of the state while the related antineutrinos (right-handed) are represented by the other. Left-handed antineutrino states vanish.

- Quark fields:
  The quark doublet for both right- and left-handed states possesses up- ($a = 1$) and down-type ($a = 2$)

quarks as isospin components, all of them as part of a triplet, given the three quark families or generations $f = f_q$.[5] Each iso-pair (and anti-pair) of the generation is given by a pair analog to the proton–neutron pair as isovector in nuclear-physics models, however here with fractional electric charge. A component (up-type) with charge $Q = +(2/3)e$ and another (down-type) with $Q = -(1/3)e$. Further, the difference between generations consists, as for leptons, in their masses. Higher-generation quarks possess higher masses. The first generation (up u and down d) consists of the least massive quarks with $m_{up}/m_{down} \approx 0.56$ and m_{up} of about 2 MeV/c^2. The second one consists of the doublet of charm c and strange s quarks. The third and most massive generation finally consists of top t and bottom b quarks. Top quarks (proven experimentally only until 1995 at Fermilab [46]) possess a mass of *ca.* 171 GeV/c^2: about 1000 times more massive than (composite) pions and with almost twice the mass of weakons!

For both left- and right-handed states, the quark multiplet reads as follows,

$$\psi_{L/R}{}^{qf} = \begin{pmatrix} u_f \\ d_f \end{pmatrix}_{L/R} ,$$

with

$$u_1 = \text{up}, \quad u_2 = \text{charm}\,(c), \quad u_3 = \text{top}\,(t) \tag{B.3.1}$$

and

$$d_1 = \text{down}, \quad d_2 = \text{strange}\,(s), \quad d_3 = \text{bottom}\,(b)\,. \tag{B.3.2}$$

Baryons as particles with a baryonic number (as quantum number) 1 or -1 are composite particles out of two kinds of partons: quarks (actually 3) and gluons. However, in a general sense, quarks may be regarded as baryons with the baryonic number of 1/3 and -1/3 for antiquarks, such that hadrons in general possess a baryonic number of 1, 0 or -1, whereas mesons, composite particles from quarks, antiquarks and gluons, possess the vanishing baryonic number. For hadrons, the baryonic number is conserved, however not for each family. Especially high-family baryonic particles may decay in less massive (lower-family) particles as long as the total amount of baryons is conserved. Hence, protons, which are the least massive baryonic particles, do not decay within the SM.

Given that baryonic number is not conserved for each flavor, there exists a mixture between them. On the one hand, here are the mass eigenstates d, s and b. On the other hand, there are the flavor states d', s' and b'. The flavor state is consequence of the Glashow–Iliapoulos–Maiami (GIM) mechanism (a generalized Cabibbo transformation), which describes a unitary transformation U(3) between the different down quarks. The transformation matrix is the Kobayashi–Maskawa matrix, also known as Cabibbo matrix.[6] The transformed state is the one which has to appear within the spinor.

Under weak-interaction processes, the leptonic number (as quantum number) is conserved for each family indeed (see [157], pg. 240), however only as long as neutrino mass is neglected. Such a mass would lead to neutrino oscillations [77]. Neutrino mixing, however, leads to “neutrino flips” which break family-wise lepton conservation. Such can, however, only occur if neutrinos have a finite mass (see [157] pg. 294). Then, on the one hand, there exist flavor states of neutrinos (ν_i), and on the other, there exist mass eigenstates of the same (ν_i'), analogously to quarks within the GIM mechanism.

[5] According to the work of M. Kobayashi and T. Maskawa, Nobel-prize awarded 2008 together with Y. Nambu, there have to be at least three quark families in nature.

[6] See Nobel prize 2008.

B.4 Strong triplet of the SM

Within quantum mechanical interactions, neutrinos couple only weakly while electron-like particles couple electroweakly, *i.e.* weakly and electromagnetically. Quarks, on the other hand, couple electroweakly but also within strong interactions. Hence, quarks have to appear as an isovector within strong-interaction transformations also. The isospin vector of SU(3)$_C$ reads

$$\psi_a{}^f = \begin{pmatrix} r_a{}^f \\ g_a{}^f \\ b_a{}^f \end{pmatrix},$$

with the subscript a counting the color charge ($a = 1, 2, 3$).
Unlike hadrons, quarks do not seem to possess an inner structure, and especially the SM assumes none. What makes them differ from each other is simply called *flavor* f, of which there are 6, together with 6 anti-flavors (divided onto the 3 generations found in electroweak states). Hence, there is, for instance, $r_2{}^3 = d_b$ for the down-quark with blue color charge.
The flavor, which be the strong-interaction family of quarks, counts from one through six for up, down, charm, strange, top and bottom (and analogously for antiquarks). This takes into account the appearance of high-generation quarks which was historically defined by new (flavor) quantum numbers within nuclear physics. These numbers are called charmn C, strangeness S and so on (and they come from times before quark theory). They are related to new types of quarks which are more massive than *up* and *down* quarks (hence the strong relation to flavor). These numbers are conserved during strong and electromagnetic processes but not during weak-interaction ones. Consequently, the lightest particles containing a strange quark, for instance, cannot decay by strong-interaction processes, and must decay *via* the much slower weak interaction [84].

B.5 The SM system

The fermionic (Dirac) Lagrange density for electroweak interactions reads (without symmetry-breaking terms)

$$\begin{aligned}\mathcal{L} = \sum_{m,f,\mu,A,B} \Bigg[& \frac{i}{2}\bar{\psi}_L^{mfA}\gamma^\mu{}_A{}^B\psi_{LmfB;\mu} - \frac{i}{2}\bar{\psi}_{L;\mu}^{mfA}\gamma^\mu{}_A{}^B\psi_{LmfB} + \\ & + \frac{i}{2}\bar{\psi}_R^{mfA}\gamma^\mu{}_A{}^B\psi_{RmfB;\mu} - \frac{i}{2}\bar{\psi}_{Rmf,\mu}^{A}\gamma^\mu{}_A{}^B\psi_{RmfB} - \\ & - \frac{1}{16\pi\hbar c}\left(F_{(2)}{}^i{}_{\mu\lambda}F_{(2)i}{}^{\mu\lambda} + F_{(1)\mu\lambda}F_{(1)}^{\mu\lambda}\right)\Bigg],\end{aligned} \tag{B.5.1}$$

wherein g_i are the coupling constants of each interaction and γ^μ are the Dirac matrices fulfilling the Clifford algebra, while a bar means Dirac-conjugation ($\bar{a} = a^\dagger\gamma^0$). Dirac conjugation responds to non-hermitean properties of Dirac matrices. $F_{(j)}{}^k{}_{\mu\nu}$ stays for the energy–stress tensors for each symmetry group, and it is closely related to the covariant derivative which reads as follows,

$$D_\mu \psi_L^{lf} = \left(\partial_\mu + \frac{i}{2} g_1 B_\mu + i g_2 W_\mu^i \tau_i\right) \psi_L^{lf} \quad \text{for left-handed leptons,} \tag{B.5.2}$$

$$D_\mu \psi_L^{qf} = \left(\partial_\mu - \frac{i}{6} g_1 B_\mu + i g_2 W_\mu^i \tau_i\right) \psi_L^{gf} \quad \text{for left-handed quarks,} \tag{B.5.3}$$

$$D_\mu \psi_R^{lf} = \left(\partial_\mu + i g_1 B_\mu\right) \psi_R^{lf} \quad \text{for right-handed leptons,} \tag{B.5.4}$$

$$D_\mu u_R^f = \left(\partial_\mu - \frac{2}{3} i g_1 B_\mu\right) u_R^f \quad \text{for right-handed up-quarks,} \tag{B.5.5}$$

$$D_\mu d^{f'}{}_R = \left(\partial_\mu + \frac{1}{3} i g_1 B_\mu\right) d^{f}{}'_R \quad \text{for right-handed down-quarks.} \tag{B.5.6}$$

Further, Euler–Lagrange equations give for electroweak interactions the homogeneous (Dirac) equations for multiplets,

$$i\gamma^\mu D_\mu \psi_L^{mf} = 0\,, \quad h.c. \tag{B.5.7}$$

$$i\gamma^\mu D_\mu \psi_R^{mf} = 0\,, \quad h.c. \tag{B.5.8}$$

and to the inhomogeneous Yang–Mills equations of the Glashow–Salam–Weinberg model,

$$F_{(2)i}^{\mu\lambda}{}_{,\lambda} + g_2 \varepsilon_i{}^{jk} F_{(2)j}^{\mu\lambda}{}^{\mu\lambda} W_{\mu k} = 4\pi j_{(2)i}{}^{\lambda}, \tag{B.5.9}$$

$$F_{(1)}{}^{\mu\lambda}{}_{,\mu} = 4\pi j_{(1)}{}^{\lambda}. \tag{B.5.10}$$

Herewith, $W_{\mu k}$ are weak gauge potentials related to weakons as physical particles.
While field-strength tensors are given by covariant derivatives and thus by gauge potentials (Ricci identities),

$$\mathcal{F}_{(2)\mu\nu} = F_{(2)\mu\nu i} \tau^i \equiv [D_\mu^{(2)}, D_\nu^{(2)}], \quad F_{(2)\mu\nu i} = W_\nu i, \mu - W_{\mu i\,\nu} - g_2 \varepsilon_i{}^{jk} W_{\mu j} W_{\nu k}, \tag{B.5.11}$$

$$\mathcal{F}_{(1)\mu\nu} = F_{(1)\mu\nu} Y \equiv \frac{-1}{i g_1/2} [D_\mu^{(1)}, D_\nu^{(1)}], \quad B_{\nu,\mu} - B_{\nu,\mu}, \tag{B.5.12}$$

weak and hypercharge current densities read

$$j_{(2)i}^\lambda = g_2 \bar{\psi}_L \gamma^\lambda \tau_i \psi_L, \quad j_{(1)}^\lambda = g_1 \left(\frac{1}{2} \bar{\psi}_L \gamma^\lambda \psi_L + \bar{\psi}_R \gamma^\lambda \psi_R\right). \tag{B.5.13}$$

$\tau^i = \frac{1}{2}\sigma^i$ is valid for the generators of SU(2). They commute with hypercharges, given that Y is proportional to the unit matrix, and Pauli matrices are trace-free.
Hypercharges (Y_x) are the generators of transformations of U(1), and their eigenvalues differ for left- and right-handed particles and for different isospin. For SU(2)$_R$, the eigenvalues of the generator vanish. For SU(2)$_L$ the generators are multiples of Pauli matrices. The third one of them, $\sigma^3 = diag(1, -1)$, is related to the so-called isospin tensor τ^3.
Electric charge as an eigenvalue of an operator Q, with isospin operator T^3 and hypercharge operator Y, is given by the Gell-Mann–Nishijima equation,

$$Q = T^3 + \frac{1}{2} Y\,, \tag{B.5.14}$$

which can further be generalized for charmness, strangeness and so on. It finally relates both hypercharge and isospin values Y and T^3 with the electric charge Q (with the dimension e). It then leads to electrodynamics, which are else mixed with weak interactions in U(1)$_Y$.

On the one hand, within electrodynamics, photons are mediated in electromagnetic interactions. Photons are massless and, thus, electromagnetism is related to long-range interactions. On the other hand, given $N = 2$ for electroweak interactions, there are three gauge bosons expected as intermediate particles of weak interactions. These are the weakons $W^{\pm}$ and Z^0, all related to photonic states by reasons of diagonalization of the mass matrix after breaking the symmetry of the theory and with all three of them acting only for left-handed states (from which physical photon states are derived also), since weak processes are left-handed. Further, since there appear self-interactions given by $[A_\mu, A_\nu] \neq 0$ of the gauge potentials, symmetry breaking (*cf.* Chapter 3.3) leads to a nonvanishing mass of the weakons, which were experimentally discovered in 1983 [10]. Actually, mass of $W^{\pm}$ bosons lies around 80 GeV/c^2 while Z^0 possesses a mass of about 91 GeV/c^2. Weak interactions are short-ranged and dominate only within nuclear ranges.
For strong interactions, the Lagrange density reads

$$\mathcal{L} = \sum_{a,f,\mu} \left[\frac{i}{2} \bar{\psi}^a_f \gamma^\mu \psi^f_{a;\mu} - \frac{i}{2} \bar{\psi}^a_{f;\mu} \gamma^\mu \psi^f_a - \frac{1}{16\pi\hbar c} F_{\mu\nu i} F^{\mu\nu i} - \bar{\psi}^a_f m_f \psi^f_a \right], \tag{B.5.15}$$

wherein a counts the isospin (color) and f counts flavor.
The covariant derivative reads

$$D_\mu = \partial_\mu + i g_3 G_{\mu i} \tau^i, \tag{B.5.16}$$

and the field-strength tensor is given by

$$\mathcal{F}_{\mu\nu} = \frac{1}{i g_3} [D_\mu, D_\nu] \tag{B.5.17}$$

$$= F_{\mu\nu i} \tau^i, \quad F_{\mu\nu i} = G_{\nu i,\mu} - G_{\mu i,\nu} - g_3 f_i^{jk} G_{\mu j} G_{\nu k}. \tag{B.5.18}$$

τ^i ($i = 1, \ldots 8$) are now 3×3 matrices called Gell-Mann matrices. They are the generators of SU(3)$_C$, and f_i^{jk} are structure constants describing their algebra. Further, the inhomogeneous Yang–Mills equations read [74]

$$F^{\mu\nu i}{}_{,\mu} + g_3 f^{ijk} F_j^{\mu\nu} G_{\mu k} = 4\pi j^{\nu i}, \quad i = 1, \ldots 8, \tag{B.5.19}$$

and the (Dirac) field equations for quarks read (without Einstein convention)

$$i\gamma^\mu D_\mu \psi^f_a - m^f \psi^f_a = 0 \quad , h.c. \tag{B.5.20}$$

Gauge fields G_μ are related to gluons as gauge bosons. So-called color-currents (eight types of them exist) between same-colored and differently-colored quark states are given following the scheme,

$$j^{\mu 1} = \frac{1}{2} g_3 (\bar{g}_f \gamma^\mu r^f + \bar{r}_f \gamma^\mu g^f), \tag{B.5.21}$$

$$j^{\mu 2} = \frac{1}{2} g_3 (\bar{g}_f \gamma^\mu r^f - \bar{r}_f \gamma^\mu g^f), \tag{B.5.22}$$

$$j^{\mu 3} = \frac{1}{2} g_3 (\bar{r}_f \gamma^\mu r^f - \bar{g}_f \gamma^\mu r^f), \tag{B.5.23}$$

$$\vdots \tag{B.5.24}$$

$$j^{\mu 8} = \frac{1}{2\sqrt{3}} g_3 (\bar{r}_f \gamma^\mu r^f + \bar{g}_f \gamma^\mu g^f - 2\bar{b}_f \gamma^\mu b^f), \tag{B.5.25}$$

Given gauge invariance, the eight color currents of gluons are covariantly conserved. Furthermore, while quarks are "colored" *i.e.* possess a color charge, the resulting superposition of all free particles in nature is assumed to be "colorless", which in this context means "white"-charged, following the analogy to color theory. This is related with the problem of *confinement* and *asymptotic freedom*: quarks move freely within hadronic ranges but cannot be detected as free particles since strong-interaction (color) forces should augment with distance. The prediction of the interaction between the color-mediating gluons and quarks in hadrons, first discovered in the early 1970s, lead to the Nobel Prize for Gross, Wilczek and Politzer in 2004. Within a hadron (femtometer scale), however, quarks would move freely. The problem of confinement is here treated within Dual QCD in Chapter 4.

B.6 Schematic properties of fermions, bosons and their interactions

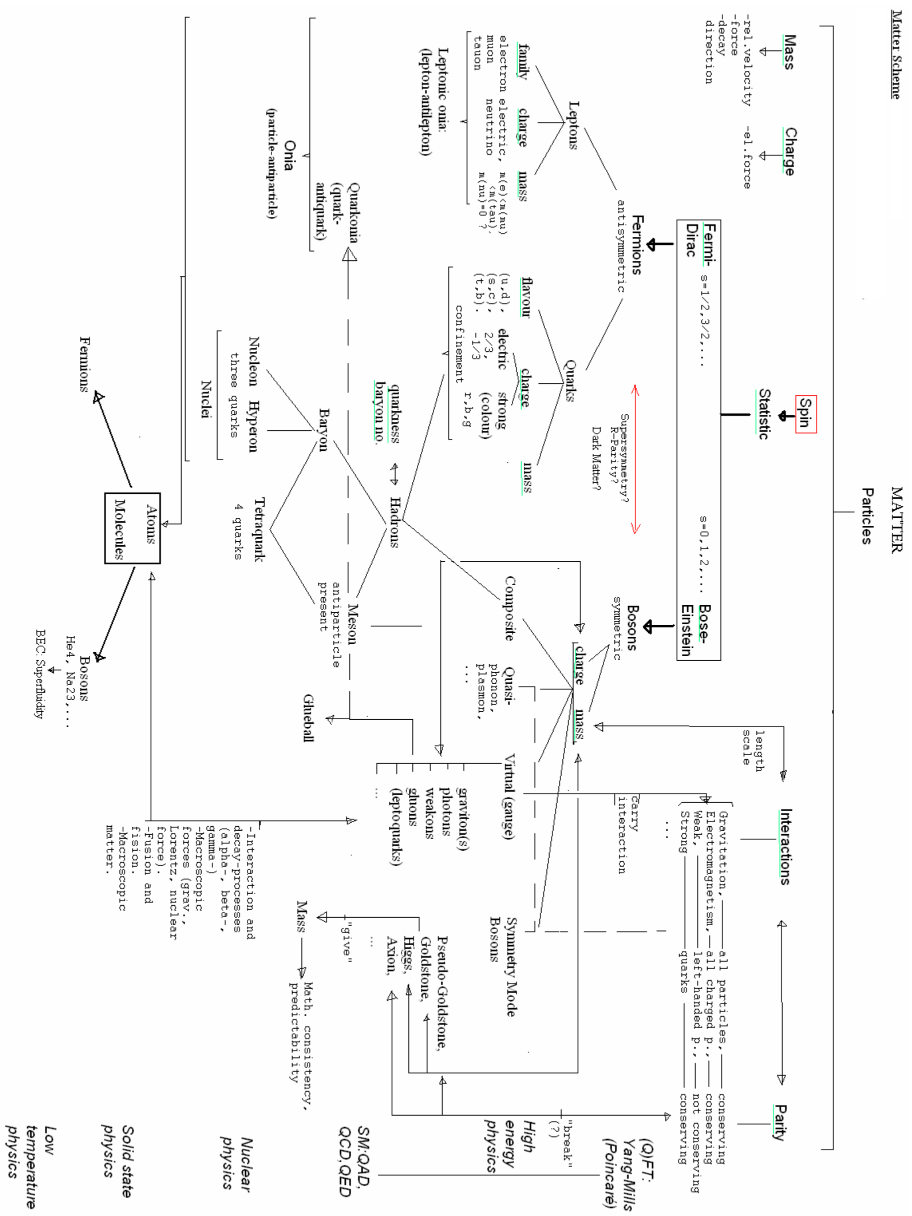

Figure B.1: Schematics on the properties of fermions, bosons and their interactions

Appendix C

Cosmology

C.1 Spherical symmetry and the ideal liquid

Spherical symmetry or central symmetry is given by the following line element,

$$ds^2 = e^{\nu}(cdt)^2 - e^{\lambda}dr^2 - r^2 d\Omega^2\,, \tag{C.1.1}$$

with the 4-vector x^μ={$x^0 = ct, x^1 = r, x^2 = \vartheta, x^3 = \varphi$}. The metric components ν and λ are functions of the r and t coordinates only and $d\Omega^2 = \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)$ is the metric of a 2-dim unit sphere. Furthermore, let us take now $c \neq \hbar \neq 1$ in the equations.
For ideal liquids, the energy–momentum tensor reads

$$T_{\mu\nu} = (\epsilon + p)\, u_\mu u_\nu - p g_{\mu\nu}, \quad u^\mu u_\mu = 1, \tag{C.1.2}$$

with a 4-velocity u^μ, pressure p and energy-density distribution ϵ.
With $u_\mu = (u_0, u_1, 0, 0)$ and $u_1 := u_0 \frac{v_1}{c}$ (velocity v_1), there is for spherical symmetry,

$$u_0^2 = \left[e^{-\nu} - \left(\frac{v_1}{c}\right)^2 e^{-\lambda}\right]^{-1}. \tag{C.1.3}$$

The nonvanishing components of $T_{\mu\nu}$ yield

$$\begin{aligned}
T_{00} =& u_0^2\left[\epsilon + \left(\frac{v_1}{c}\right)^2 p e^{\nu-\lambda}\right],\\
T_{01} =& u_0^2\,(\epsilon + p)\,\frac{v_1}{c},\\
T_{11} =& u_0^2\left[\left(\frac{v_1}{c}\right)^2 \epsilon + p e^{\lambda-\nu}\right],\\
T_{22} =& p r^2,\\
T_{33} =& p r^2 \sin^2\vartheta,
\end{aligned}$$

with the trace

$$T = \epsilon - 3p \tag{C.1.4}$$

In case of barotropic matter, pressure will be given by $p = w\epsilon$, whereas w is called equation-of-state parameter, given by the ratio of pressure (p) to the energy density (ϵ), taken as a constant which is independent

of time.
In curvature coordinates for central symmetry, the Christoffel symbol,

$$\Gamma^{\sigma}_{\mu\sigma} = \frac{\partial}{\partial x^{\sigma}} \log\sqrt{-g}, \tag{C.1.5}$$

as connection of the form, has the following nonvanishing components:

$$\begin{aligned}
&\Gamma^0_{00} = \frac{1}{2} g_{00,0} g^{00} = \frac{\dot\nu}{2}, \quad \Gamma^0_{01} = \frac{1}{2} g_{00,1} g^{00} = \frac{1}{2}\nu', \quad \Gamma^0_{11} = -\frac{1}{2} g_{11,0} g^{00} = \frac{1}{2}\dot\lambda e^{\lambda-\nu},\\
&\Gamma^0_{22} = -\frac{1}{2} g_{22,0} g^{00} = \frac{\dot\lambda}{2} r^2 e^{\lambda-\nu}, \quad \Gamma^0_{33} = -\frac{1}{2} g_{33,0} g^{00} = \frac{\dot\lambda}{2} r^2 \sin^2\vartheta e^{\lambda-\nu},\\
&\Gamma^1_{00} = \frac{1}{2} g_{00,1} g^{11} = \frac{1}{2}\nu' e^{\nu-\lambda}, \quad \Gamma^1_{10} = \frac{1}{2} g_{11,0} g^{11} = \frac{\dot\lambda}{2}, \quad \Gamma^1_{11} = \frac{1}{2} g_{11,1} g^{11} = \frac{\lambda'}{2},\\
&\Gamma^1_{22} = -\frac{1}{2} g_{22,1} g^{11} = -r - \frac{\lambda'}{2} r^2, \quad \Gamma^1_{33} = -\frac{1}{2} g_{33,1} g^{11} = -r\sin^2\vartheta e^{-\lambda},\\
&\Gamma^2_{02} = \frac{1}{2} g_{22,0} g^{22} = \frac{\dot\lambda}{2}, \quad \Gamma^2_{12} = \frac{1}{2} g_{22,1} g^{22} = \frac{1}{r}, \quad \Gamma^2_{33} = -\frac{1}{2} g_{33,2} g^{22} = -\sin\vartheta\cos\vartheta,\\
&\Gamma^3_{13} = \frac{1}{2} g_{33,1} g^{33} = \frac{1}{r}, \quad \Gamma^3_{23} = \frac{1}{2} g_{33,2} g^{33} = \cot\vartheta, \quad \Gamma^3_{30} = \frac{1}{2} g_{33,0} g^{33} = \frac{1}{2}\dot\lambda.
\end{aligned}$$

The Riemann tensor is constructed as follows,

$$R^{\tau}{}_{\mu\nu\sigma} = \Gamma^{\alpha}_{\mu\nu}\Gamma^{\tau}_{\alpha\sigma} - \Gamma^{\alpha}_{\mu\sigma}\Gamma^{\tau}_{\alpha\sigma} + \Gamma^{\tau}_{\mu\nu,\sigma} - \Gamma^{\tau}_{\mu\sigma,\nu}, \tag{C.1.6}$$

with the trace

$$R_{\mu\nu} = R^{\sigma}{}_{\mu\nu\sigma} = \Gamma^{\alpha}_{\mu\nu}\Gamma^{\sigma}_{\alpha\sigma} - \Gamma^{\alpha}_{\mu\sigma}\Gamma^{\sigma}_{\alpha\sigma} + \Gamma^{\sigma}_{\mu\nu,\sigma} - \Gamma^{\sigma}_{\mu\sigma,\nu}. \tag{C.1.7}$$

The nonvanishing components of the Ricci tensor $R_{\mu\nu}$ read exactly (including time-dependence),

$$\begin{aligned}
R_{00} &= -e^{\nu-\lambda}\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} + \frac{\nu'}{r}\right) + \frac{1}{c^2}\frac{\ddot\lambda}{2} + \frac{1}{c^2}\frac{\dot\lambda^2}{4} - \frac{1}{c^2}\frac{\dot\lambda\,\dot\nu}{4}, && \text{(C.1.8)}\\
R_{10} &= \frac{1}{c}\frac{\dot\lambda}{r}, && \text{(C.1.9)}\\
R_{11} &= -\frac{1}{c^2}\,e^{\lambda-\nu}\left(\frac{\ddot\lambda}{2} + \frac{\dot\lambda^2}{2} - \frac{\dot\lambda\dot\nu}{4}\right) + \frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} - \frac{\lambda'}{r}, && \text{(C.1.10)}\\
R_{22} &= e^{-\lambda}\left[1 + \frac{r}{2}\left(\nu' - \lambda'\right)\right] - 1, && \text{(C.1.11)}\\
R_{33} &= \sin^2\vartheta\, R_{22}. && \text{(C.1.12)}
\end{aligned}$$

C.2 Scalar-field equation with central symmetry

After symmetry breaking, the scalar-field equation (6.3.21) reads as follows (see Chapter 5),

$$\xi^{,\mu}{}_{;\mu} + \frac{\xi}{l^2} = \frac{1}{1+\frac{4\pi}{3\breve\alpha}} \cdot \left(\frac{8\pi G_0}{3}\hat q \hat T + \frac{4}{3}\Lambda_0\right).$$

Explicitly, for the geometrical part we have for spherical symmetry the following components with Christoffel symbols $\Gamma^{\mu}_{\nu\lambda}$:

$$\begin{aligned}
\xi^{,1}{}_{;1} = (\xi_{,1}g^{11})_{;1} &= (-\xi_{,1}e^{-\lambda})_{;1} \\
&= (-\xi' e^{-\lambda})_{,1} + \Gamma^1_{11}\xi^{,1} + \Gamma^1_{10}\xi^{,1} \\
&= (-\xi' e^{-\lambda})_{,1} + \Gamma^1_{11}(-\xi' e^{-\lambda}) + \Gamma^1_{10}\dot{\xi}e^{-\nu} \\
&= -\xi'' e^{\lambda} + \lambda'\xi' e^{-\lambda} + \frac{\lambda'}{2}(-\xi' e^{-\lambda}) + \frac{\dot{\lambda}}{2}\dot{\xi}e^{-\nu} \\
&= -e^{-\lambda}\xi'' + \frac{\lambda'}{2}\xi' e^{-\lambda} + \frac{\dot{\lambda}}{2}\frac{\dot{\xi}}{c^2}e^{-\nu},
\end{aligned}$$

$$\begin{aligned}
\xi^{,0}{}_{;0} = (\xi_{,0}g^{00})_{;0} &= (\xi_{,0}e^{-\nu})_{;0} \\
&= (\dot{\xi}e^{-\nu})_{,0} + \Gamma^0_{01}(-\xi' e^{-\lambda}) + \Gamma^0_{00}\dot{\xi}e^{-\nu} \\
&= \frac{\ddot{\xi}}{c^2}e^{-\nu} - \frac{\nu'}{2}\xi' e^{-\lambda} - \frac{\dot{\xi}}{c^2}\frac{\dot{\nu}}{2},
\end{aligned}$$

$$\xi^{,2}{}_{;2} = \Gamma^2_{21}\xi^{,1} = -\frac{1}{r}\xi' e^{-\lambda},$$

and

$$\xi^{,3}{}_{;3} = \Gamma^3_{30}\xi^{,1} = -\frac{1}{r}e^{\lambda}\xi'. \tag{C.2.1}$$

Hence, for vacuum, there is for equation (6.3.21),

$$\begin{aligned}
\xi^{,\mu}{}_{;\mu} + \frac{\xi}{l^2} &\equiv \sum \xi^{,\mu}{}_{;\mu} + \frac{\xi}{l^2} \\
&= -\left[\xi'' - \frac{(\lambda' - \nu')}{2}\xi' + \frac{2}{r}\xi'\right]e^{-\lambda} + \frac{1}{l^2}\xi - \left[\frac{1}{c^2}\frac{(\dot{\nu} - \dot{\lambda})}{2}\dot{\xi} - \frac{1}{c^2}\ddot{\xi}\right]e^{-\nu} = 0.
\end{aligned} \tag{C.2.2}$$

Let us take equation (6.3.21). Further assume $\xi = \bar{\xi}(r)\cdot h(t)$ and write $h(t) \equiv h$ and $\bar{\xi}(r) \equiv \bar{\xi}$. Now, in the linear case, we may rewrite it as

$$\frac{\ddot{h}}{h} = \frac{c^2}{\bar{\xi}}\Delta\bar{\xi} - \frac{c^2}{l^2} + \hat{q}\frac{8\pi G_0}{3\bar{\xi}c^2}(\epsilon - 3p). \tag{C.2.3}$$

Let us here take $\hat{q} = 0$. We have

$$\ddot{h} + \left(\frac{c^2}{l^2} - \lambda\right)h = 0 \tag{C.2.4}$$

and

$$c^2\Delta\bar{\xi} - \left(\frac{c^2}{l^2} + \lambda\right)\bar{\xi} = 0, \tag{C.2.5}$$

with the eigenvalue λ. Further, we may define the eigenfrequency

$$\omega^2 = \frac{c^2}{l^2} - \lambda \tag{C.2.6}$$

and hence write the time-dependent solution as

$$h(t) = a_1 \cos[\omega(t - t_0)] + a_2 \sin[\omega(t - t_0)] \quad (\text{for } \hat{q} = 0). \tag{C.2.7}$$

There is $h(t_0) = a_1$.
For the radial equation (C.2.5), there is

$$\Delta = \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) - \frac{1}{r^2} \hat{l}^2 \right). \tag{C.2.8}$$

Here, $\hat{l}$ is a momentum operator which is given by

$$\hat{l}^2 = -\left[\frac{1}{\sin^2 \vartheta} \frac{\partial^2}{\partial \varphi^2} + \frac{1}{\sin^2 \vartheta} \frac{\partial}{\partial \vartheta} \left(\sin \vartheta \frac{\partial}{\partial \vartheta} \right) \right]. \tag{C.2.9}$$

Analogously to the wave equations in nonrelativistic QM, the scalar field may be solved by

$$\bar{\xi}(r) = \xi_0 f(r) Y_{\bar{l}m}(\vartheta, \varphi). \tag{C.2.10}$$

$Y_{\bar{l}m}(\vartheta\varphi) \equiv Y_{\bar{l}m}$ is the spherical harmonics with the following eigenvalue problem,

$$\hat{l}^2 Y_{\bar{l}m} = \bar{l}(\bar{l}+1) Y_{\bar{l}m}. \tag{C.2.11}$$

If we define

$$E^2 = \lambda, \tag{C.2.12}$$

equation (C.2.5) may be written onto a Sturm–Liouville equation as follows for $f \equiv f(r)$,

$$-(r^2 f')' + \frac{E^2 r^2}{c^2} f - \bar{l}(\bar{l}+1) f = 0. \tag{C.2.13}$$

This is a modified Bessel differential equation.
For $E \neq 0$, (C.2.13) is solved by

$$f = \frac{1}{\sqrt{r}} \left\{ C_1 J_{\frac{1}{2}+\bar{l}} \left(-\frac{iEr}{c} \right) + C_2 Y_{\frac{1}{2}+\bar{l}} \left(-\frac{iEr}{c} \right) \right\}, \tag{C.2.14}$$

whereas C_1 and C_2 are integration constants and $J_n(z)$ is the Bessel function of 1^{st} kind. $Y_n(z)$ is the Bessel function of second kind.
For z real-valued, there is $E^2 < 0$, and thus $\lambda \bar{l}^2 < -c^2$. $E^2 < 0$ implies

$$E := i\tilde{E} \quad \text{with} \quad \tilde{E} \in \mathbb{R}. \tag{C.2.15}$$

This is required for the solutions not to be exponentially increasing for $r \to \infty$. In other words, the spectrum $\{\lambda\}$ is to be determined, as within QM, through asymptotical boundary conditions for $r \to \infty$.
For $\bar{l} = 0$ (monopole), for instance, we have

$$J_{\frac{1}{2}} \left(-i\frac{Er}{c} \right) = \frac{\sqrt{2} \sin(-iEr/c)}{\sqrt{-i\frac{Er}{c}\pi}}, \tag{C.2.16}$$

$$Y_{\frac{1}{2}} \left(-i\frac{Er}{c} \right) = -\frac{\sqrt{2} \cos(-iEr/c)}{\sqrt{-i\frac{Er}{c}\pi}}. \tag{C.2.17}$$

This gives a monopole radial solution of the scalar field for the assumption of separability.

C.3 RN-like parameters

Let us write down the parameters of Chapter 7.5 for $c = 1$. The series-expansion (7.5.1)

$$\tilde{g} = \sum_{n=1}^{\infty} \frac{C_n}{r^n} = \frac{1}{r}\left[C_1 + \frac{C_2}{r} + \frac{C_3}{r^2} + \frac{C_4}{r^3} + \dots\right]$$

has the constants (7.5.2)

$$\left.\begin{array}{c} C_1 = 1 \\ C_2 = 2A + B \\ C_3 = (2A+B)^2 + \frac{AB}{4} \\ C_4 = (2A+B)^3 + \frac{2AB}{3}(2A+B) \\ C_5 = (2A+B)^4 + \frac{29AB}{24}(2A+B)^2 + \frac{3(AB)^2}{32} \end{array}\right\} .$$

which are multiplicative factors or A and B (see Chapter 7.5).
Near to $r = 2\tilde{M}G_N$ and $r = \sqrt{|\tilde{Q}^2|}$, higher-order corrections of r^{-n} are necessary for an analysis of the behavior of the metric components. Furthermore, they may be used for indications about exact behavior. Up to the 10^{th} order of approximation, there are further constant terms

$$\begin{aligned}
C_6 =&(2A+B)^5 - \frac{37AB}{20}(2A+B)^3 + \frac{2(AB)^2}{5}(2A+B) \\
C_7 =&(2A+B)^6 - \frac{103AB}{40}(2A+B)^4 + \frac{751(AB)^2}{720}(2A+B)^2 - \frac{5(AB)^3}{128} \\
C_8 =&(2A+B)^7 - \frac{118AB}{35}(2A+B)^5 + \frac{676(AB)^2}{315}(2A+B)^3 - \frac{8(AB)^3}{35}(2A+B) \\
C_9 =&(2A+B)^9 - \frac{2369AB}{560}(2A+B)^6 + \frac{17151(AB)^2}{4480}(2A+B)^4 - \\
&- \frac{6959(AB)^3}{8960}(2A+B)^2 + \frac{35(AB)^4}{2048} \\
C_{10} =&(2A+B)^{10} - \frac{2593AB}{504}(2A+B)^7 + \frac{1787(AB)^2}{288}(2A+B)^5 - \\
&- \frac{4549(AB)^3}{2268}(2A+B)^3 + \frac{8(AB)^2}{63}(2A+B)
\end{aligned}$$

For higher-order corrections $n \geq 3$ which are related to $X(A,B;r^{-n})$ of equation (7.5.4), mass–charge, and charge–charge couplings appear. There is up to the 9^{th} order,

$$\begin{aligned}
X(A,B;r^{-n}) =&\frac{1}{2r^2} + \frac{4\,(2A+B)}{3r^3} + \frac{29\,(2A+B)^2}{6r^4} + \frac{3\,AB}{8r^4} + \frac{37(2A+B)^3}{5r^5} + \\
&+ \frac{8AB}{5r^5}(2A+B) + \frac{103(2A+B)^3}{10r^6} + \frac{751AB}{90r^6}(2A+B)^2 + \frac{5(AB)^2}{32r^6} + \\
&+ \frac{472(2A+B)^5}{35r^7} + \frac{2704AB}{315r^7}(2A+B)^3 + \frac{32AB}{35r^7}(2A+B) + \\
&+ \frac{2369(2A+B)^6}{280r^8} + \frac{17151AB}{2240r^8}(2A+B)^4 + \frac{6959(AB)^2}{4480r^8}(2A+B)^2 + \\
&+ \frac{35(AB)^3}{884r^8} + \frac{2593(2A+B)^7}{252r^9} + \frac{1787AB}{144r^9}(2A+B)^5 + \\
&+ \frac{4549(AB)^2}{1134r^9}(2A+B)^3 + \frac{16(AB)^3}{63r^9}(2A+B)\,.
\end{aligned} \tag{C.3.1}$$

Further X_i terms are defined exactly as before since they are all defined in terms of X.
In terms of $\tilde{M}$ and $\tilde{Q}^2$, up to 4^{th} order, there appear further terms $8\tilde{M}G_N/(3r^3) + 29(\tilde{M}G_N)^2/(3r^4) + 3\tilde{Q}^2/(8r^4)$ of $X(A,B;r^{-n})$ which lead to the following correction terms for e^λ,

$$\frac{\tilde{Q}^2\tilde{M}G_N}{r^3} + \frac{2\tilde{Q}^2\left(\frac{8}{3}(\tilde{M}G_N)^2 - \tilde{Q}^2\right)}{r^4}, \tag{C.3.2}$$

and for e^ν (under the parenthesis),

$$\frac{7\tilde{Q}^2\tilde{M}G_N}{3r^3} + \frac{\frac{17}{3}\tilde{Q}^2(\tilde{M}G_N)^2 - 9\tilde{Q}^4}{r^4}. \tag{C.3.3}$$

C.4 Standard Friedmann cosmology

- **Vanishing cosmological constant:**

Let us shortly discuss the simple limiting case of vanishing scalar-field excitations ($\xi_0 = 0$) and derivatives of the same, with energy density $\epsilon = \epsilon_0$ and scale factor $a = a_0$. Then, for $w = 0$ we have from (8.6.1) an Einstein–deSitter Universe with

$$\left(\frac{\dot{a}}{a_0}\right)^2 - \frac{8\pi}{3}\frac{G_0}{c^2}\epsilon\left(\frac{a}{a_0}\right)^2 = H_0^2\left(1 - \frac{\epsilon_0}{\epsilon_{0c}}\right) = H_0^2\left(1 - \Omega_{0\epsilon}\right) = -\frac{Kc^2}{a_0^2}. \tag{C.4.1}$$

The subscript 0 of the density parameter defines it as the present one with the Hubble parameter $H \equiv H(t_0) \equiv H_0$. The latter equation shows the already defined critical density ϵ_c as being the one needed indeed for the curvature to be $K = 0$ and thus the Universe to be flat. A smaller density means $K = -1$ and a higher one means $K = 1$.

The Einstein–deSitter Universe is mainly a one-fluid model. In $\Omega_{0\epsilon}$, however, are matter and radiation terms, and other types of matter may be defined in it as well.

Let us take again an Einstein–deSitter Universe, but this time with time dependence ($\epsilon \sim a^{-3(1+w)}$). There is, using (8.6.3),

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_{0w}\left(\frac{a_0}{a}\right)^{1+3w} + (1 - \Omega_{0w})\right]. \tag{C.4.2}$$

Let us assume a flat Einstein–deSitter Universe, *i.e.* $K = 0$. Without scalar field and derivatives of the same, there is

$$\begin{aligned}\left(\frac{\dot{a}}{a_0}\right)^2 &= H_0^2\Omega_{0w}\left(\frac{a_0}{a}\right)^{1+3w} \\ &= H_0^2(1+z)^{1+3w}.\end{aligned} \tag{C.4.3}$$

For only matter fluids, there is $w = 0$, while for radiative-fluid dominance there is $w = 1/3$. The total energy density gives $\Omega_{total} = 1$.

(C.4.3) can be integrated immediately to obtain (*cf.* [56])

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{2/3(1+w)}. \tag{C.4.4}$$

Furthermore, there is

$$t = t_0(1+z)^{-3(1+w)/2}, \tag{C.4.5}$$

which relates time t to redshift z. Further, there would be

$$H \equiv \frac{\dot{a}}{a} = \frac{2}{3(1+w)t} = H_0\frac{t_0}{t} = H_0(1+z)^{3(1+w)/2}, \tag{C.4.6}$$

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1+3w}{2} = const. = q_0, \tag{C.4.7}$$

$$t_{0w} \equiv t_0 = \frac{2}{3(1+w)H_0}. \tag{C.4.8}$$

For a dust model ($w = 0$), the relation (C.4.1) yields in general

$$\left(\frac{\dot{a}}{a_0}\right)^2 = H_0^2\left(\Omega_0\frac{a_0}{a} + 1 - \Omega_0\right). \tag{C.4.9}$$

The latter equation is solved parametrically for open models ($K = -1$) [56],

$$a(\psi) = a_0 \frac{\Omega_0}{2(1-\Omega_0)}(\cosh\psi - 1), \tag{C.4.10}$$

$$t(\psi) = \frac{1}{2H_0}\frac{\Omega_0}{(1-\Omega_0)^{3/2}}(\sinh\psi - \psi). \tag{C.4.11}$$

These relations then give

$$t_0 = \frac{1}{2H_0}\frac{\Omega_0}{(1-\Omega)^{3/2}}\left[\frac{2}{\Omega_0}(1-\Omega_0)^{1/2} - \cosh\left(\frac{2}{\Omega_0} - 1\right)\right] > \frac{2}{3H_0}\,, \tag{C.4.12}$$

and for $\Omega_0 \ll 1$,

$$t_0 \approx \frac{1}{H_0}(1 + \Omega_0 \ln\Omega_0). \tag{C.4.13}$$

For a one-fluid closed universe ($K = 1$), there is [56]

$$a(\vartheta) = a_0 \frac{\Omega_0}{2(\Omega_0 - 1)}(1 - \cos\vartheta), \tag{C.4.14}$$

$$t(\vartheta) = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}(\vartheta - \sin\vartheta). \tag{C.4.15}$$

The scale factor grows in time for $0 \leq \vartheta \leq \vartheta_m = \pi$. The maximum value of the same is

$$a_m = a(\vartheta_m) = a_0 \frac{\Omega_0}{\Omega_0 - 1}. \tag{C.4.16}$$

It occurs at a time t_m given by

$$t_m = t(\vartheta_m) = \frac{\pi}{2H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}. \tag{C.4.17}$$

Furthermore, there is (*op.cit.*)

$$t_0 = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0 - 1)}\left[\cos^{-1}\left(\frac{2}{\Omega_0} - 1\right) - \frac{2}{\Omega_0}(\Omega_0 - 1)^{1/2}\right] < \frac{2}{3H_0}. \tag{C.4.18}$$

t_0 for a closed-universe model is smaller than for $K = 0$.

- **Three-fluid system with cosmological constant:**
  Let us now take a three-fluid system with dust, radiation and a cosmological constant. There is

$$\left(\frac{\dot{a}}{a_0}\right)^2 = H_0^2\left[\Omega_{0M}\left(\frac{a_0}{a}\right) + \Omega_{0R}\left(\frac{a_0}{a}\right)^2 + \Omega_{0\Lambda}\left(\frac{a_0}{a}\right)^{-2} + (1 - \Omega_{0M} - \Omega_{0R} - \Omega_{0\Lambda})\right]. \tag{C.4.19}$$

The Universe is flat when $\Omega_{0M} + \Omega_{0R} + \Omega_{0\Lambda} = 1$. Furthermore, $\Omega_{0\Lambda} = \frac{H^2}{H_0^2}\Omega_\Lambda$ is valid for the energy-density parameter. This means that the cosmological constant, or equally the energy density of the cosmological constant, is constant indeed. In the case of $\xi = const. \neq 0$, the cosmological constant is given by the scalar-field excitations. In that case (see Chapters 2.4 and 8.4), Ω_{0i} are effective (screened) values (Ω^*) of the bare parameters of true densities, as defined in equation (8.6.10). Here, we take the standard approach $\xi = 0$, which means a further true cosmological constant Λ_0 of unknown nature.

A closed-form expression of the equation (C.4.19) for flat Universes $K = 0$, containing only dust and Λ_0, is available. Then there is with $\Omega_{0\epsilon} = \Omega_{0M} \equiv \Omega_0$ [56],

$$t_0 = \frac{2}{3H_0}\left[\frac{1}{2\sqrt{1-\Omega_0}}\log\frac{1+\sqrt{1-\Omega_0}}{1-\sqrt{1-\Omega_0}}\right]. \tag{C.4.20}$$

A nonvanishing curvature will generally lead to accelerating contributions and thus to an increase in the age of the Universe in comparison to those with $K = 0$.

In matter-dominated Friedmann models, the age of the Universe is given to a good approximation by (*cf.* [56])

$$t_0 = F(\Omega_0)H_0^{-1} \approx 0.98 \cdot 10^{10} F(\Omega_0)h^{-1}\text{years}. \tag{C.4.21}$$

The function F is dependent on the curvature K, with

$$F(\Omega_0) = \frac{\Omega_0}{2}(\Omega_0 - 1)^{-3/2}\cos^{-1}\left(\frac{2}{\Omega_0} - 1\right) - (\Omega_0 - 1)^{-1} \quad \text{for} \quad \Omega_0 > 1, \tag{C.4.22}$$

$$F(\Omega_0) = \frac{2}{3} \quad \text{for} \quad \Omega_0 = 1, \tag{C.4.23}$$

$$F(\Omega_0) = (1 - \Omega_0)^{-1} - \frac{\Omega_0}{2}(1 - \Omega_0)^{-3/2}\cosh^{-1}\left(\frac{2}{\Omega_0} - 1\right) \quad \text{for} \quad \Omega_0 < 1. \tag{C.4.24}$$

For limiting cases, there is

$$F(\Omega_0) \approx \frac{1}{2}\pi\Omega_0^{-1/2} \quad \text{for} \quad \Omega_0 \gg 1, \tag{C.4.25}$$

$$F(\Omega_0) \approx 1 + \Omega_0 ln\Omega_0 \quad \text{for} \quad \Omega_0 \ll 1. \tag{C.4.26}$$

As a constraints on density there is

$$0.01 < \Omega_0 < 2 \tag{C.4.27}$$

so that for the age of the Universe, there is

$$t_{0H} \approx (6.5 - 10) \cdot 10^9 h^{-1}\text{years}. \tag{C.4.28}$$

There is the Hubble constant $H_0 = h \cdot 100\,\text{km} \cdot s^{-1}\text{Mpc}^{-1}$ from the Hubble law $v_i = H_0 x_i$ for irrotational velocity fields and isotropic spaces, with the reduced Hubble parameter h which is observationally set between 0.5 and 1.

- **Observational constraints of age:**
  Let us take the values in [56]. As observational constraints, galaxy formation needs about 1 to $2 \cdot 10^9$ years. Globular clusters are thought to be around $1.3 - 1.4 \cdot 10^{10}$ years old or even older.
  Another constraint can be found by the relative abundances of long-lived radioactive nuclei and their decay products. These nuclei are synthesized in processes involving the absorption of neutrons by heavy nuclei such as iron, and processes of this type are thought to occur in supernovae explosions. Stars that become supernovae are short lived and with a life stem of about 10^7 years.
  Nucleocosmochronology helps determining the time at which stars and galaxies were formed. If our galaxy find its origin at $t \approx 0$, time at which an era of nucleosynthesis of heavy elements occurred,

then this happened during $t = T$. This interval is followed by a time Δ in which the solar system became isolated from the rest of the galaxy. Isolation time should then be followed by a period t_s corresponding to the age of the solar system itself. The estimate of the age of the Universe thus yields

$$t_n = T + \Delta + t_s. \tag{C.4.29}$$

The times t_s as well as $T + t_s$ can be traced back following the decay channels and mean lifetime of elements such as ^{238}U into ^{206}Pb or ^{87}Rb into ^{87}Sr (with mean lifetimes of about 10^9 years and $6.6 \cdot 10^{10}$years, respectively). In this way, the solar system is concluded to be about $4.6 \cdot 10^9$ years old, and $\Delta = (1-2) \cdot 10^8$ years. Furthermore, the constraints on the age of the Universe give

$$t_n \approx (0.6 - 1.5) \cdot 10^{10}\text{years}. \tag{C.4.30}$$

- **The energy density:**
The present Universe is well-approximated by a dust or matter-dominated model, with a total energy density[1]

$$\epsilon_{0\epsilon} = \epsilon_{0M} + \epsilon_{0R} + \epsilon_{0\nu} \approx \epsilon_{0M}. \tag{C.4.31}$$

The pressure is

$$p_0 = p_{0M} + p_{0R} + p_{0\nu} \approx \epsilon_{0M}\frac{k_B T_{0M}}{m_p} + \frac{1}{3}\epsilon_{0R} \approx \epsilon_{0R} \ll \epsilon_{0\epsilon}. \tag{C.4.32}$$

The constraints on the galactic contribution to density (Ω_g) are considerably uncertain but around

$$\Omega_g = \frac{\epsilon_{0g}}{\epsilon_{0c}} \approx 0.03. \tag{C.4.33}$$

This should give the amount of mass concentrated in galaxies.
On the other hand, gravitational dynamics of large-scale objects show a contribution of

$$\Omega_{dyn} \approx 0.2 - 0.4 \tag{C.4.34}$$

for dynamical matter. The discrepancy between (C.4.33) and (C.4.34) leads to the already mentioned assumption of the existence of non-luminous, dark, matter. This matter (or its dynamics) plays an important role in structure formation.
The first modern studies of possible "missing" mass go back to Öpik's in 1915 [185], related to the dynamical density of the dynamics of our galaxy and our solar vicinity, and later to Oort [186] and others starting 1932. Data does not suggest discrepancies between dynamical and observational mass in the solar vicinity, though.
1933, there came first evidence of missing, "invisible" mass through Zwicky's work on the dynamics in the Coma cluster [253]. Evidence later accumulated [79, 138, 187] and independent determination of rotation velocities of galaxies at large galactocentric distances confirmed the presence of dark matter in form of halos around the galaxies [211, 212].
The assumption that the dominant part of dark matter is non-baryonic (called cold, CDM) was made 1982 by Blumenthal *et al.* [30].

[1] $\Omega_{0\nu} \approx \Omega_{0R} \approx 10^{-5}h^{-2}$.

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Extended list of mathematical symbols

Symbol	Name / Description	Definition / Example
0	Current-time subscript	$a_0,\, t_0, \dots$
$*$	Complex conjugate	$\mathbb{C} \ni x = a + ib \implies x^* = a - ib$
$\dagger$	Hermitean transpose, dagger	$(A^\dagger)_{ij} = A^*_{ji} \iff A = A^\dagger$
T	Transpose	$(A^T)_{ij} = (A)_{ji}, \quad (AB)^T = B^T A^T$
		Hermitean matrix: $A_{ij} = A^*_{ji} \iff A = \sum_i \lambda_i u_i u_i^\dagger$ with $\lambda_i \in \mathbb{R}$
$\wedge$	Wedge operator	$(a \wedge b)^{\mu\nu} = a^\mu b^\nu - a^\nu b^\mu$
$\vec{\nabla}$	Nabla operator	$\vec{\nabla} = \sum_i \partial_i \vec{e}_i$
$\Box$	d'Alembert operator	$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$
$\|v>,\, \|w> \dots$	Dirac vectors (kets)	Scalar product: $< f\|g > \hat{=} \int f^* g \, d\vec{r}$
		Matrix element: $A_{kl} =< k\|\hat{A}\|l >=< k\|\hat{A}l >$
		Decomposition: $\hat{A} = \sum_{k,l} \|k > A_{kl} < l\|$
		Decomposition of unity: $\sum_k \|k >< k\| = \underline{1}$
		Average on ψ basis (expectation value of $\hat{A}$):
		$< \psi\|\hat{A}\|\psi >=< \psi\|\hat{A}\psi >=< \hat{A} >_\psi$
$< X >$	Mean square	
$\hat{A},\, \hat{B}, \dots$	Operators	Hamilton operator: $\hat{H}\|\psi >= E_n\|\psi >$
$A_{(i}\, B_{k)}$	Antisymmetric Bach parenthesis	$A_{(i}\, B_{k)} = \frac{1}{2}(A_i B_k + A_k B_i)$
$A_A{}^B,\, B_A{}^B$	2^{nd} rank Spinors	$A_A{}^B = \frac{1}{2}\,\gamma_{\mu A}{}^B A^\mu$
$A^\mu,\, B^\mu, \dots$	Gauge potentials (fields)	Minimal coupling: $D_\mu = \partial_\mu + igA_\mu$
		Z boson: $Z^\mu = W_3{}^\mu \, \cos\vartheta_W + A^\mu \, \sin\vartheta_W$
		Photon: $B^\mu = -W_3{}^\mu \, \sin\vartheta_W + A^\mu \, \cos\vartheta_W$
$A'_\mu,\, B'_\mu, \dots$	Dual gauge fields	$\vec{B} = \vec{\nabla} \times \vec{A} + \vec{A}'$
$\tilde{A}_\mu,\, \tilde{B}_\mu, \dots$	Transformed fields	$(\tilde{\vec{A}},\, \tilde{\vec{B}})^T = R(\vartheta)(\vec{A},\, \vec{B})^T$
$\mathcal{A}_\mu,\, \mathcal{B}_\mu, \dots$	Gauge potentials (matrix notation)	$\mathcal{A}_\mu = A_{\mu i}\tau^i$
$a^\mu, b^\mu, \dots$	4-vectors	Components: $x^\mu \in \{x^0, x^1, x^2, x^3\},\ \mu = 0, 1, 2, 3$
$x^{\mu'},\, x^{\nu'}$	Lorentz transformed coordinate	$x^{\mu'} = \Lambda^\mu{}_\nu x^\nu$
$\bar{\psi}$	Adjoint conjugate	$\bar{\psi} = \psi^\dagger \gamma^0$

Symbol	Name / Description	Definition / Example
α	Phase	$\tilde{v} = ve^{i\alpha}$
α	RN-like charge term	$\alpha = -\dfrac{AB}{2} = -\tilde{Q}^2$
$\breve{\alpha}$	Gravitational strength	$\breve{\alpha} \cong (M_P/M_B)^2 \gg 1$
α'	Regge slope parameter	$\alpha' = (2\pi\sigma)^{-1}$
$\Gamma^{\mu}_{\beta\nu}$	Christoffel symbol	$\Gamma^{\mu}_{\beta\nu} = \dfrac{1}{2}g^{\mu\alpha}(g_{\beta\alpha,\nu} + g_{\alpha\nu,\beta} - g_{\beta\nu,\alpha}) = \Gamma^{\mu}_{\nu\beta}$
		GR: $v^{\mu}{}_{;\nu} = v^{\mu}{}_{,\nu} + \Gamma^{\mu}_{\beta\nu}v^{\beta}$
		$\Gamma^{\mu}_{\alpha\mu} = (\sqrt{-g})_{,\alpha}/\sqrt{-g}$
γ	Charge–coupling ratio	$\gamma = \dfrac{Q}{g}$
γ	Polytropic index	$p = w_P\epsilon^{\gamma}$
$\gamma^{\mu}{}_{A}{}^{B}$	Dirac matrices (spinor)	Clifford algebra: $\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}\underline{1}, \quad \mu, \nu = 0, \dots 3$
		$\gamma^{\mu\dagger} = \pm\gamma^{\mu}$
		Spinor: $A_A{}^B = \dfrac{1}{2}\gamma_{\mu A}{}^{B}A^{\mu}$
γ^4	Projector operator	$\gamma^4 = i\dfrac{1}{4!}\varepsilon_{\alpha\beta\mu\nu}\gamma^{\alpha}\gamma^{\beta}\gamma^{\mu}\gamma^{\nu}, \quad (\gamma^4)^2 = \underline{1}, \quad \{\gamma^4, \gamma^{\mu}\} = 0$
Δ	Standard deviation (RMS)	$\Delta X = \sqrt{< (X- < X >)^2 >}$
Δ	Density ratio	$\Delta \equiv \hat{\epsilon}/\epsilon^*$
Δ	Difference	$\Delta x = x_2 - x_1$
Δ	Laplace operator	$\Delta = \nabla^2 = \sum_i \partial_i^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{2}{r}\dfrac{\partial}{\partial r}$
δ	Variation	$\delta S(q, \dot{q}) = \int_{t_0}^{t_1} \left(\dfrac{\partial L}{\partial q_k}\delta q_k + \dfrac{\partial L}{\partial \dot{q}_k}\delta(\dot{q}_k)\right)$
δ		$\delta \equiv \left(\dfrac{a(t_q)}{l}\right)^2 \ll 1$
δ_{ij}	Kronecker symbol	$\delta_{ij} = 0$, for $i \neq j$, $\delta_{ii} = 1$
∂_{μ}	Partial derivative	$\partial_{\mu} = \dfrac{\partial}{\partial x^{\mu}}$
ϵ	Energy density	$\epsilon = \varrho c^2$
		Effective: $\epsilon^* = \epsilon/(1 + \xi)$
		Newtonian (baryonic): $\epsilon^* = \dfrac{v_t^2}{4\pi G_N r^2}$
		Scalar-field: $\epsilon_{\xi} = \dfrac{\xi c^2}{8\pi G_N l^2}$
		Dark Matter profile: $\hat{\epsilon} = \epsilon^* + \epsilon_{\xi} = \epsilon_{DM}$
		Critical: $\epsilon_c = 3H_0^2c^2/(8\pi\tilde{G})$
		Total-energy: $\epsilon_T = \epsilon + \epsilon_{\Lambda}$
		Cosmological-term: $\epsilon_{\Lambda} = V - \dfrac{3c^2}{8\pi G_0}\dfrac{\dot{a}}{a}\dot{\xi} + \dfrac{v^2}{8}\dfrac{\dot{\xi}}{1+\xi}$
		SF-derivative: $\epsilon_I = \epsilon_{\Lambda} - V$
		Matter: $\epsilon_M \approx \epsilon_{\epsilon} + \epsilon_R + \epsilon_{\nu}$
		Bayonic: ϵ_B
		Radiation: ϵ_R
		Neutrino: ϵ_{ν} (current: $\nu_{0\nu} \approx 10^{-5}h^{-2}$)
		Galactic: ϵ_g

Symbol	Name / Description	Definition / Example
ϵ_0	Minimum energy–density	Higgs: $\epsilon_0 = -\check{V} = \epsilon_{min}$
ε	Permittivity	Vacuum: $\varepsilon_0 \approx 8.8542 \cdot 10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}$
		$1/(\mu\varepsilon) = c^2$
ε	Geodesic parameter	$g_{\mu\nu}g^{\mu\nu} = -\varepsilon$
		Spacelike: $\varepsilon = 1$
		Lightlike (null): $\varepsilon = 0$
		Timelike: $\varepsilon = -1$
ε	Integration constant of Kepler orbit	$0 < \epsilon < 1$: Ellipse
ε_{ijk}	(Levi–Civita) total skew symmetric tensor	$\varepsilon_{123} = 1,\, \varepsilon_{321} = -1,\, \varepsilon_{112} = 0, \ldots$
$\varepsilon^{\mu\nu}{}_{\kappa\lambda}$	4^{th} rank Levi–Civita tensor	
$\eta_{\mu\nu}$	Minkowski metric	$\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$
ϑ_W	Weinberg angle	$\sin^2\vartheta_W \approx 0.21$, $\tan\vartheta_W = g_1/g_2$
κ	Ginzburg–Landau parameter	$\kappa = \dfrac{m_\Phi}{m_V}$
κ	Gravitational coupling parameter	$\kappa = \dfrac{8\pi G}{c^4}$
		Effective: $\tilde{\kappa} = \dfrac{8\pi\tilde{G}}{c^4}$
Λ	Cosmological term	Function: $\Lambda(\xi) = 4\pi G_{eff}V(\xi)$
		$\Lambda^* = \Lambda + \Lambda_0$
		Constant: Λ_0
		Planck: $\Lambda_P = \dfrac{3K}{l_P^2}$
$\Lambda^\mu{}_\nu$	Lorentz transformation	$x^\mu \to x^{\mu'} = \Lambda^\mu{}_\nu x^\nu$
		$\Lambda^\mu{}_\nu = \dfrac{\partial x^{\mu'}}{\partial x^\nu}$
λ	Gauge parameter	Gauge: $\psi' = e^{i\lambda\tau i}\psi$
λ	Higgs parameter	$V(\phi) = \dfrac{\mu^2}{2}\phi^\dagger\phi + \dfrac{\lambda}{4!}\left(\phi^\dagger\phi\right)^2, \quad v = \pm\sqrt{-\dfrac{6\mu^2}{\lambda}}$
λ	Metric component (gravitational potential)	$ds^2 = e^\nu(dct)^2 - e^\lambda dr^2 - r^2 d\Omega^2$
λ_i	Eigenvalue	$f(u_i) = \lambda_i u_i, \quad \lambda_i \in \mathbb{C},\, u_i \in \mathbb{C}^\ltimes,\, i \in \{1, \ldots n\}$
μ	Higgs parameter	$V(\phi) = \dfrac{\mu^2}{2}\phi^\dagger\phi + \dfrac{\lambda}{4!}(\phi\phi)^2$
		Ground-state value: $v = \pm\sqrt{-\dfrac{6\mu^2}{\lambda}}$
μ	Permeability	Vacuum: $\mu_0 = 4\pi \cdot 10^{-7}$ N/A^2
		Dual: $\mu^2(p^2, \Phi_0) = 1 - p^2 m_V^{-2}$
μ	f=2-lepton: muon	$e^2 = \mu$
$\mu(a/a_0)$	MOND parameter	$\mu(a/a_0) = 1\,(= a/a_0)$ for $a \gg a_0\,(a \ll a_0)$
ν	Neutrino	
ν	Metric component (gravitational potential)	$ds^2 = e^\nu(dct)^2 - e^\lambda dr^2 - r^2 d\Omega^2$
		Central symmetry (linear): $\nu = -\dfrac{r_{dyn}}{r}$
		Potential: $\Phi = \nu/c^2$

Symbol	Name / Description	Definition / Example
ξ	(Square-root) scalar-field excitation	$\xi = \frac{\phi^\dagger\phi}{v^2} - 1$
		$\xi = \frac{G(v) - \tilde{G}}{\tilde{G}}$
Π	Polarization	Dual: $\tilde{\Pi}(p^2, \Phi_0) = -m_V^2/p^2$
ϱ	Density	Baryonic density: ϱ_B
		Critical: $\varrho_c = 3H_0^2/(8\pi\tilde{G})$
		Planck: $\varrho_P \cong \frac{c^5}{G_0^2\hbar} \cong 10^{93}$g/cm^3
ρ^2	Square scalar field ϕ	$\phi = \rho UN$
		After unitary gauge: $\phi^\dagger\phi = \rho^2, \quad \phi = \rho N$
σ	Magnetic charge density	Dual: $\vec{\nabla} \cdot \vec{B} = \sigma$
σ	String tension of flux tube	$\alpha' = (2\pi\sigma)^{-1}$
$\tilde{\sigma}$	Higgs component	$\phi^0 = (\tilde{\sigma} + i\chi)/\sqrt{2}, \quad <\tilde{\sigma}>= v$
σ_i	Pauli matrices	Spin: $\hat{S}_i = \frac{\hbar}{2}\sigma_i,$
		Clifford algebra: $\{\sigma_i, \sigma_j\} = 2\delta_i$
τ	f=3-lepton (tauon)	$e^3 = \tau$
τ	Volume	Dual: $\int \sigma d\tau = g$
τ	Eigentime (affine parameter)	
τ^i	Group generator	$U = e^{i\lambda\tau^i},$
		$[\tau^i, \tau^j] = if^{ij}{}_k\tau^k, \quad \{\tau^i, \tau^j\} = c^{ij}\underline{1} + d^{ij}{}_k\tau^k$
Φ	Potential	$\vec{\nabla} \cdot \Phi = \vec{F}$
		Gravitational: $\Phi = \frac{2\nu}{c^2}$
Φ, ϕ	Scalar field	Symmetry-broken: $\phi = \rho N = v\sqrt{1+\xi}N$
		Ground-state: Φ_0, ϕ_0, $\phi_{a(0)}$
$\hat{\phi}$	Scalar-field excitation	$\phi = vN + \hat{\phi}$
$\hat{\phi}$	BW scalar field	$\mathcal{L}(\hat{\phi})_{BW} = \frac{\sqrt{-g}}{16\pi}\hat{\phi}R$
ϕ_{BA}	Path amplitude from A to B	$\phi_{BA}[C] = e^{\frac{i}{\hbar}S[C]}$
φ	Scalar-field excitation	$\phi_a = (1+\varphi)\phi_{a(0)},$
φ	Angle	Kepler: $u_0 = \frac{\tilde{r}_S}{2C_b^2}c^2(1 + \varepsilon\cos\varphi)$
χ	Goldstone component	$\phi^0 = (\tilde{\sigma} + i\chi)/\sqrt{2}, \quad U = e^{i\chi}$
χ	Covariant distance	
Ψ	Gravitational potential (Newton)	$\Psi = \Phi + \frac{c^2}{2}\xi$
Ψ	Flux	Color-electric charge: $\Psi_E = \int \vec{E} \cdot dSn\Psi_0$
		$\Psi_0 = 2\pi/Q$

Symbol	Name / Description	Definition / Example
ψ	Wave function / Quantum state	Spinor: ψ_A
		Isospinor: ψ_a
$\psi_{\mu\nu}$	Linear gravitational field	$\psi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}$, $h = h_{\mu\nu}\eta^{\mu\nu}$, $\lvert h_{\mu\nu}\rvert \ll 1$
Ω	Unit sphere	$d\Omega^2 = d\vartheta^2 + \sin^2\vartheta d\varphi^2$
Ω	Angular velocity	$\Omega = \frac{d\varphi}{dr}$
Ω	Density parameter	$\Omega_i = \frac{8\pi G(\xi)}{3H^2}\varrho_i = \frac{\epsilon_i}{\epsilon_c}$
		Matter: $\Omega_M = 0.127h^{-2}$
		Baryons: $\Omega_B = 0.0223h^{-2}$
		DM: $\Omega_{DM} = 0.105h^{-2}$
		Neutrinos: $\Omega_\nu < 0.007h^{-2}$
		Cosmological-constant: $\Omega_\Lambda = 0.76$
		Total: $\Omega_T = 1.003^{0.013}_{0.017}$
		Density term: $\Omega_\epsilon = \frac{8\pi G(\xi)}{3H^2c^2}\epsilon = \frac{\Omega_B}{1+\xi}$
		Pressure term: $\Omega_p = \frac{8\pi G(\xi)}{H^2c^2}p$
		$\Omega = \Omega_\epsilon + \Omega_p$
Ω_Λ	Cosmological term	$\Omega_\Lambda = \frac{8\pi G(\xi)}{3H^2c^2}V = \frac{\Lambda}{3H^2}$
		$\Omega^*_\Lambda = \frac{8\pi G(\xi)}{3H^2c^2}\epsilon_\Lambda$
Ω_I		$\Omega_I = \frac{8\pi G(\xi)}{3H^2c^2} = \frac{\epsilon_I}{\epsilon_c} = -\frac{\dot{\xi}}{H(1+\xi)}$
Ω_{II}		$\Omega_{II} = \frac{8\pi G(\xi)}{3H^2c^2}(p_\Lambda + V) - \frac{2}{3}\Omega_I$
ω	JBD coupling parameter	
A	Scalar-field amplitude (exact)	$A = -\frac{2}{3}\frac{G_0}{c^2}\int T\sqrt{-g}d^3x, \quad (l \to \infty)$
A	Spinor index	$\psi_A, \quad \gamma^\mu{}_A{}^B$
AU	Astronomical unit	1 AU $= 149,597,870\,\text{km}$
a	Isospin index	Electroweak: $\psi_a = \begin{pmatrix} \nu_e^f \\ e^f \end{pmatrix}$
a	Scale parameter (factor)	$a^2 = \frac{\dot{a}^2 + Kc^2}{\Omega_T H^2}$
		Current: $a(t_0) = a_0$
		Primeval (statical): $a(t_q) = a_q$
		Planck: $a(t_P) = a_P$
a	Scale radius (of spherical system)	
a_0	Critical acceleration (MOND)	$a_0 = 1.2 \cdot 10^{-20}\text{m s}^{-2}$
a_H	Hubble distance	$a_H = Hc$

Symbol	Name / Description	Definition / Example
B	Baryon subscript	$\Omega_B,\ \epsilon_B \ldots$
B	Newtonian field amplitude	$B = \frac{2M_{dyn}G_N}{c^2}$
$\vec{B}$	Magnetic field / magnetic induction	EM: $\vec{\nabla} \cdot \vec{B} = 0$
B^μ	Photon field (em gauge boson)	$B^\mu = -{W_3}^\mu \sin\vartheta_W + A^\mu \cos\vartheta_W$
B_{ik}	Magnetic tensor	$B_{ik} = \frac{i}{g}[D_i, D_k]$
		EM: $B_{kj,i} + B_{ik,j} + B_{ji,k} = 0$
b	Blue (strong color charge)	QCD: ${\psi_a}^1 = {b_a}^u$
b	Bottom-quark (flavor)	GSW: ${\psi_L}^{q=6} = b_L$
C	Color (charge) subscript	$\mathrm{U(3)}_C$
C	Integration constant	$\xi = \frac{C}{r}e^{-r/l}$
C	Integration constant for minkoskian limit	$C = \left(\frac{\sqrt{K^2+4\alpha}+K}{\sqrt{K^2+4\alpha}-K}\right)$
C_a	Parametrized energy constant	Geodesics: $C_a = \left(1 - \frac{\tilde{r}_S}{r}\right)^{r_{dyn}/\tilde{r}_S} \frac{dct}{d\tau} = const.$
C_b	Momentum constant	$L = mC_b$
$\tilde{C}_{\mu\nu}$	Dual field-strength tensor to $F_{\mu\nu}$	$\tilde{C}_{\mu\nu} = \tilde{A}'_{\nu,\mu} - \tilde{A}'_{\mu,\nu}$
c	Lightspeed (lat. *celeritas*)	SI: $c = 2.99792458 \cdot 10^8$ m/s
		natural: $c \equiv 1$
c	Charm-quark (flavor)	${\psi_L}^{q3} = c_L$
${c^{ij}}_k$	Structure constant	$\{\tau^i, \tau^j\} = c^{ij}\underline{1} + {d^{ij}}_k\tau^k$
D	Dyon subscript	Dyonic action: S_D
D	Integration constant	$\nu = -\frac{C}{r}e^{-r/l} - \frac{D}{r}$
$\mathcal{D}$	Differential: summation over all paths	$Z_{AHM} = \int \mathcal{D}\tilde{C}_\mu \Phi e^{-S_{AHM}[\tilde{C}_\mu,\Phi]}$
$\vec{D}$	Electric displacement field	$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$
D_μ	Covariant derivative	$D_\mu = \frac{D}{Dx^\mu} = {}_{;\mu}$
		SM: $D_\mu = \partial_\mu + igA_\mu$
		GR: $D_\mu = \partial_\mu - \Gamma^\alpha_{\nu\lambda}$
		Dual: $D_\mu = \partial_\mu - ieA_\mu - igA'_\mu$
$\mathcal{D}_\mu$	Covariant derivative (matrix notation)	SM: $\mathcal{D}_\mu = \underline{1}\partial_\mu + ig\mathcal{A}_\mu$
DM	Distance modulus	$DM = m - M = -5 + 5\,\mathrm{log}d_L(\mathrm{pc})$
d	Down-quark (flavor)	GSW: ${\psi_L}^{q2} = d_L$
d_L	Luminosity distance	$d_L = a_0^2\frac{r}{a} = \frac{c}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2 + \ldots\right]$
ds^2	Line element	$ds^2 = \sum_\mu dx_\mu dx^\mu = \sum_\mu dx^\mu dx_\mu = \sum_{\mu,\nu} g_{\mu\nu}dx^\mu dx^\nu$
$d\vec{s}$	Element of area	Dual: $\int_S \vec{A}' \cdot d\vec{s} = g$
${d^{ij}}_k$	Structure constant	$\{\tau^i, \tau^j\} = c^{ij}\underline{1} + {d^{ij}}_k\tau^k$

Symbol	Name / Description	Definition / Example
E	Integration constant, energy	Geodesics: $\dot{t} = Ee^{-\nu}$
$\mathcal{E}$	Effective energy	Geodesics: $\mathcal{E} = \left(\frac{dr}{dt}\right)^2 + V_{eff}$
E_P	Planck energy	$E_P \cong m_P c^2 \cong 10^{19}$ GeV
eV	electronvolt	$1\,\text{eV} \approx 1.602 \cdot 10^{-19}$ J
eV/c^2	energy–mass equivalent	$1\,\text{eV/c}^2 \approx 1.783 \cdot 10^{-36}$ kg
F	4-Force	
$F_{\mu\nu}$	Field-strength tensor	$F_{\mu\nu a}{}^{b} = \{A_{\nu i,\mu} - A_{\mu i,\nu} - gA_{\mu k}A_{\nu l}f^{kl}{}_{i} ig[A_\mu, A_\nu]\}(\tau^i)_a{}^b$ $F_{\mu\nu a}{}^{b} = F_{\mu\nu i}\tau^i{}_a{}^b$
$F^*_{\mu\nu}$	Dual field-strength tensor	$F^{\mu\nu *} = \frac{1}{2}\varepsilon^{\mu\nu}{}_{\kappa\lambda}F^{\kappa\lambda}$
$\mathcal{F}_{\mu\nu}$	Field-strength tensor (matrix notation)	$\mathcal{F}_{\mu\nu} = \mathcal{A}_{\nu,\mu} - \mathcal{A}_{\mu,\nu} + ig[\mathcal{A}_\mu, \mathcal{A}_\nu]$ $\mathcal{F}_{\mu\nu} = F_{\mu\nu i}\tau^i = \mathcal{F}^\dagger_{\mu\nu}$
$\tilde{F}_{\mu\nu}$	Field-strength of grav. Maxwell eqs.	$\tilde{F}_{\mu\nu} = (u_{\sigma,\mu} - u_{\mu,\sigma}) = H_{\nu\mu} - H_{\mu\nu}$
$f(t)$		$f = \frac{1}{2}(f_2 - f_1)$
$f(\chi)$		$f \in \{\sin\chi,\, \xi,\, \sinh\chi\}$ for $K \in \{1,\, 0,\, -1\}$
$f_1(r/l)$		$f_1 = \frac{1}{4}\left(1 - e^{-r/l}\right)$
$f_1(t)$		$f_1 = -\frac{\dot{a}}{a}\frac{\dot{\xi}}{1+\xi} + \frac{\pi}{3\breve{\alpha}}\frac{\dot{\xi}^2}{(1+\xi)^2}$
$f_1(G)$		$f_1 = \frac{\dot{G}(\xi)}{G(\xi)}\left[\frac{\dot{a}}{a} + \frac{\pi}{3\breve{\alpha}}\frac{\dot{G}(\xi)}{G(\xi)}\right]$
$f_2(r/l)$		$f_2 = \frac{1}{4}\left[1 + \left(1 + \frac{r}{l}e^{-r/l}\right)\right]$
$f_2(t)$		$f_2 = -\frac{\ddot{\xi}}{1+\xi} - 2\frac{\dot{a}}{a}\frac{\dot{\xi}}{1+\xi} - \frac{\pi}{\breve{\alpha}}\frac{\dot{\xi}^2}{(1+\xi)^2}$
$f_2(G)$		$f_2 = 2\frac{\dot{a}}{a}\frac{\dot{G}(\xi)}{G(\xi)} - \frac{\dot{G}(\xi)^2}{G(\xi)^2}\left(2 + \frac{\pi}{\breve{\alpha}}\right) + \frac{\ddot{G}(\xi)}{G(\xi)}$
G	Gravitational coupling	Function: $G(\phi) \equiv \frac{1}{\breve{\alpha}\phi^\dagger\phi}$ Constant: $G_0 \equiv G(v) = \frac{1}{\breve{\alpha}v^2} - \frac{1}{\breve{\alpha}}\frac{\lambda}{6\mu^2}$ Newton's: $G_N \approx 6.674 \cdot 10^{-11}\,\text{m}^3/(\text{kg}\cdot\text{s}^2)$ $\hat{q} = 1:\ G_N = \frac{4}{3}G_0$ Effective: $\tilde{G} = G(\xi)\frac{G_0}{1+\xi}$
G_μ	Gluon field (strong gauge boson)	QCD: $D_\mu = \partial_\mu + ig_3 G_{\mu i}\tau^i$
G_F	Fermi's constant	$\frac{G_F}{(\hbar c)^3} = \frac{\sqrt{2g^2}}{(8m_W^2)} \approx 1.166 \cdot 10^{-5}\,\text{GeV}^{-2}$
$G_{\mu\nu}$	Einstein tensor	$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}(+\Lambda_0 g_{\mu\nu})$
GL$(4,\mathbb{R})$	General linear real 4-dim group	

Symbol	Name / Description	Definition / Example
g	Coupling constant	$g_{em} = e/\sqrt{\hbar c}$
		Dual magnetic charge: $\int_\tau \vec{\nabla} \cdot \vec{B} d\tau \int \sigma d\tau = g$
		Dirac quantization: $e \cdot g = \frac{n}{2}\hbar$
g	Metric determinant	$g = \det g_{\mu\nu}$
g	Green (strong color charge)	QCD: $\psi_{L2}{}^{q=1} = g_L{}^u$
$\tilde{g}$	Exponential metric parameter	$\tilde{g} \equiv \tilde{g}(r) = \frac{e^{(\lambda-\nu)/2}}{1+\xi}$
		$r\tilde{g} = \alpha\tilde{g}^3 - K\tilde{g}^2 - \tilde{g}$
$g_{\mu\nu}$	Metrical tensor (physical)	$\sum_\nu a_\nu b^\nu = \sum_{\mu,\nu} g_{\nu\mu} a^\mu b^\nu$
H	Higgs subscript	$\mathcal{L}_H = \frac{1}{2}\phi_{,,\mu}{}^\dagger \phi^{,\mu} - \frac{\mu^2}{2}\phi^\dagger\phi - \frac{\lambda}{4!}\left(\phi^\dagger\phi\right)^2$
H	Hubble parameter	$H = \frac{\dot{a}}{a} = 100h$ km/s/Mpc
$\vec{H}$	Magnetic field, magnetizing field	$\vec{H} = \vec{B}/\mu_0 - \vec{M}$
$H_{\mu\nu}$	Symmetric (grav.) field-strength part	$H_{\mu\nu} = u_{\mu;\mu} - u^\alpha{}_{;\alpha} g_{\nu\mu}$
h	Reduced Hubble constant	$h \approx 0.74$
h	Planck's action quantum	$h \approx 6.626 \cdot 10^{-34}$ Js $\approx 4.136 \cdot 10^{-15}$ eVs
h	Linear metric parameter	$h = \frac{1 - f_2 + 2\gamma(1+2f_2)}{1 - f_1 + \frac{3}{2}\gamma\left(1 - \frac{1}{2}f_1\right)}$
		$r \ll l$: $h = \frac{1+8w}{2+3w}$
$\hbar$	Reduced Planck's action quantum	$\hbar = h/(2\pi) \approx 1.055 \cdot 10^{-34}$ Js $\approx 6.582 \cdot 10^{-16}$eV · s
$h_{\mu\nu}$	Deviation from Minkowski metric	Weak fields: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \lvert h_{\mu\nu}\rvert \ll 1$
i	Imaginary unit	$i^2 = -1$
J	Spin	$J = \alpha_0 + \alpha' M_J^2$
J^μ, j^μ	Current	4-current: $j^\mu = (\rho, \vec{j})$
		Magnetic current: $\vec{j}_s = \vec{\nabla} \times \tilde{\vec{E}}$
$\mathcal{J}^\mu$	Current (matrix notation)	$\mathcal{J}_\mu = J_{\mu i}\tau^i$
K	Curvature constant	$K \in \{-1, 0, 1\}$
K	Mass parameter	$K = 2A + B$
K_g	Gluon-field energy	DME: $K_g = \tilde{E}^2/2$
$K[b,a]$	Feynman propagator, kernel, path integral	$K[B\lvert A] = \int [dC]\phi_{BA}[C]$
K^μ	Force	$K^\mu = \int k^\mu d^3x$
$\vec{k}$	Magnetic current density	Dual: $\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = -\vec{k}$
		Color-force density of flux tube: $\vec{k}_s = \vec{\nabla} \cdot \tilde{\vec{P}} = \vec{j}_s \times \tilde{\vec{E}}$
k_B	Boltzmann constant	$k_B \approx 1.3807 \cdot 10^{-23}$ m^{2}kg s^{-2}K^{-1}
k^μ	Force density	$K^\mu = \int k^\mu d^3x$

Symbol	Name / Description	Definition / Example
L	Absolute luminosity	
L	Angular momentum	Conjugate momentum: $L = mr^2 \dfrac{d\varphi}{d\tau}$
L	Lagrangian (Lagrange function)	$L = T - V$
L	Left-handed subscript	$\psi_{a,L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = \frac{1}{2}(\underline{1} - \gamma^5) \begin{pmatrix} \nu_e \\ e \end{pmatrix}$
$\mathcal{L}$	Lagrangian (Lagrange density)	$L = \int \mathcal{L}\sqrt{-g}d^3x$
λ	Penetration depth	ME: $\lambda = (ev)^{-1}$
l	(Compton) length scale (penetration depth)	$l = \dfrac{\hbar}{Mc} = \left[\dfrac{1 + \frac{4\pi}{3\breve{\alpha}}}{16G_0(\mu^4/\lambda)}\right]^{1/2}$
l	Lepton index	$l = \{e^f, \nu^f\}$
l	Apparent luminosity	$l = \dfrac{L}{4\pi q_0^2 r^2}\left(\dfrac{a}{a_0}\right)$
l_a	Length scale per spherical length	$l_a = \dfrac{l}{a}$
ly	Lightyear	1 ly $= 9,460,730,472,580.8\,$km
M	Higgs mass	$M = \left[-\dfrac{8\pi}{3}\dfrac{\mu^2}{\breve{\alpha}}\dfrac{c}{\hbar}\right]^{1/2}$
M_H	Standard Higgs mass	$M_H^2 = -2\mu^2$
M_1	System mass	
M_1^*	Dynamical mass for $p = 0$	$M_1^* = M_1 = M_1(1 - f_1)$
M_{dyn}	Dynamical mass	$2M_{dyn}G_N = M_{dyn} = 2M_1G_N\left[1 - f_1 + \dfrac{3}{2}\gamma\left(1 - \dfrac{1}{2}f_1\right)\right]$
		$r \ll l$: $M_{dyn} = \left(1 + \dfrac{3}{2}w\right)M_1$
$\tilde{M}$	Effective mass	$\tilde{M}G_N = A + \dfrac{B}{2}$
$M^{(i)}$	Mass-square matrix (diagonal)	$M^{(i)} = 2\sqrt{\pi\hbar c}gv\sqrt{(\tau^i N)^\dagger(\tau^i N)}$
		SU(2): $(M_{(2)}^2)_{ij} = \pi g_2^2\hbar cv^2\delta_{ij} = M_W^2\delta_{ij}$
M_W	W-weakon mass	$M_W = \pi g_2^2\hbar cv^2 = 80.398 \pm 0.23\,$GeV/c^2
M_Z	Z-weakon mass	$M_Z = M_W/cos\vartheta_W = 91.1876 \pm 0.0021\,$GeV/c^2
$\vec{M}$	Magnetization	$\vec{B} = \mu_0(\vec{H} + \vec{M})$
m	Apparent magnitude	
m	Fermionic index	$m = l,\, m = q$
m	Mass	Planck: $\cong \varrho_P l_P^3 \cong 10^{-5}$
		Fermionic: $m_f = G_f v\left(N^\dagger\hat{x} + \hat{x}^\dagger N\right)$
		(Dual) vector mass: $m_V = Q\sqrt{n_s(\Phi)}$
		(Dual) scalar mass: $m_\Phi = 2\sqrt{\lambda n_s(\Phi)}$
		Fermionic mass operator: $\hat{m} = G_f v(N^\dagger\hat{x} + \hat{x}^\dagger N)$
N	Gauge fixing parameter	$N = (1,\, 0,\, \ldots)^T,\, \Rightarrow\, \phi_{10} = v,\, \phi_{20} = \phi_{30} = \ldots = 0$
N	Isotopic parameter	SU(N): $2N^2 - 1$ gauge bosons
N	Newton subscript	$G_N \approx 6.674 \cdot 10^{-11}\,\mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$
$n_s(\Phi)$	Dyon / monopole density	$n_s(\Phi) = \|\Phi\| = \Phi_0^2$

Symbol	Name / Description	Definition / Example
O	Order	maximal order: $r^3 \Leftrightarrow O(r^3)$
O(3)	(Orthogonal) rotation group	$\vec{x} \to \vec{x}' = R\vec{x}, \quad R^T = R^{-1}$ for all $R \in$ O(3)
O(3)/SO(3)	Factor group of Lorentz group	O(3)/SO(3) $= \{1, -1\}$
P	Perihelion subscript	$\Delta\phi_P = \dfrac{6\tilde{M}G_N}{C_b^2}\pi$
$\vec{P}$	Polarization vector	$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$
p	Momentum	$p_k = \dfrac{\partial L}{\partial \dot{q}_k}$
p	Pressure	Barotropic pressure: $p = w\varrho$
		Effective: $p^* = p/(1+\xi)$
		Scalar-field: $p_\Lambda = -\dfrac{1}{3}V + \dfrac{c^2}{8\pi G_0}\left(\ddot{\xi} + 2\dfrac{\dot{a}}{a}\dot{\xi} - \dfrac{\pi}{3\breve{\alpha}}\dfrac{\dot{\xi}^2}{1+\xi}\right)$
		Bounce: $p_G = \dfrac{1}{8\pi G_0}\ddot{\xi}(t_q)$
pc	Parallax of arcsecond (parsec)	1 pc $= 1\mathrm{AU}/\tan 1'' = 3.2615668\,\mathrm{ly} = 30.856776 \cdot 10^{15}\mathrm{m}$
Q	Charge	Dyons: $Q = \sqrt{e^2 + g^2}$
$\tilde{Q}^2$	Charge parameter (generalized)	$\tilde{Q}^2 = \dfrac{AB}{2}$
$Q_{\mu\nu}$		$Q_{\mu\nu} = H_{\nu\mu} + H_{\mu\nu}$
q	Deceleration parameter (cosmic)	$q = -\dfrac{a\ddot{a}}{a}$
		Effective: $\tilde{q} = \dfrac{q}{1 + \frac{Kc^2}{a}}$
q	(Logarithmic) scalar-field excitation	$q = \ln(1+\xi)$
$\tilde{q}$	Effective deceleration parameter	$\tilde{q} = \dfrac{q}{1 + \frac{Kc^2}{\dot{a}^2}} = \dfrac{1}{2}\left(1 + 3\dfrac{p_T}{\epsilon_T}\right)$
$\hat{q}$	Matter-Lagrangian coupling	Fermionic coupling: $\hat{q} = 1$
		Quintessential coupling: $\hat{q} = 0$
R	Ricci curvature scalar	$R \equiv R_{\mu\nu}g^{\nu\mu}$
$R_{\mu\nu}$	Ricci curvature tensor	$R_{\mu\nu} \equiv R_{\mu\nu\lambda\sigma}g^{\mu\sigma}$
$R^\lambda{}_{\mu\nu\kappa}$	Riemann curvature tensor	$R^\mu{}_{\sigma\alpha\nu} = -[D_\nu, D_\alpha]$
$R_0,\ R_1$	Galactic core (bulge)	
$R(\vartheta)$	Symmetry operator	$(\tilde{\vec{E}}, \tilde{\vec{B}})^T = R(\vartheta)(\vec{E}, \vec{B})^T$
$\mathbb{R}^4$	4-dimensional real-component field	
r	Radius, distance	Halo radius: r_H
		Dynamical Schwarzschild: $r_{dyn} = \dfrac{2M_{dyn}G_N}{c^2}$
		Charge-parameter: $r_Q = \lvert\tilde{Q}^2\rvert = \dfrac{\lvert A\tilde{r}_S\rvert}{2}$
		Schwarzschild: $r_S = \dfrac{2M_1G_N}{c^2}$
		Eff. Schwarzschild: $\tilde{r}_S = 2A + r_{dyn} \approx h(w)r_{dyn}$
r_a	Distance per scale	$r_a = \dfrac{r}{a}$
S	Action	$S = \displaystyle\int_{t_1}^{t_2} L dt$
S	Area	Dual: $\displaystyle\int_\tau \left(\vec{\nabla}\cdot\vec{B}\right)d\tau = \int_S \vec{B}d\vec{s}$

Symbol	Name / Description	Definition / Example
$S^{\mu}{}_{\nu}$	Generator of GL(4,$\mathbb{R}$)	$S^{\mu}{}_{\nu} = \lambda_i S^{i\mu}{}_{\nu}$
		$L^{\mu}{}_{\nu} = e^{\lambda_i S^{i\mu}{}_{\nu}}$
SO(3)	Special rotation group	SO(3) $\subset$ O(3), $\det R = +1$ for all $R \in$ SO(3)
SO(3,1)	Lorentz group	Generalized orthogonal Lie group
SO^{+}(3,1)	Restricted Lorentz group	$\det\Lambda = 1$, $\Lambda_0{}^0 \geq 1$
SU(N)	N-dimensional special unitary group	$U_a{}^b = e^{i\lambda_i \tau^i{}_a{}^b}$, $i = 1, \ldots N$
s	Energy density	Gravity: $s = s_\mu u^\mu$
s	Strange-quark (flavor)	GSW: $\psi_L{}^{q=s4} = s_L$
s_μ	Energy–momentum tensor	Gravitation: $\tilde{F}_\mu{}^{\lambda}{}_{;\lambda} = 2\tilde{\kappa}(j_m u + s_\mu)$
T	Energy–momentum trace	$T = \frac{i}{2}\bar{\psi}\gamma^{\mu}_{L,R}\psi_{;\mu} + h.c. = \sqrt{1+\xi}\bar{\psi}\hat{m}\psi$
		Ideal gas: $T = \epsilon - 3p$
T	Kinetic energy	$L = T - V$
		Gell-Mann–Nishijima: $Q = T^2 + \frac{1}{2}Y$
T	Total-subscript	$\Omega_T = 1 \iff K = 0$
T	Transpose sign	
		Ideal gas: $T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu - p g_{\mu\nu}$
$\mathcal{T}$	Kinetic energy density	$T = \int \mathcal{T}\sqrt{-g}d^3x$
T^3	Isospin operator	$T^3\psi = \tau^3\psi, \quad \tau^i = \frac{1}{2}\sigma^i$
$T_{\mu\nu}$	Energy–stress tensor	$T^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial\psi_{aA,\nu}}\psi_{aA,\mu} + \frac{\partial\mathcal{L}}{\partial\bar{\psi}^{aA}{}_{,\nu}}\bar{\psi}^{aA}{}_{,\mu} - \mathcal{L}\delta_\mu{}^\nu$
t	Top-quark (flavor)	GSW: $\psi_L{}^{q=5} = t_L$
t	Time	Primeval time: t_q
		Planck time: t_P
U	BW cosmological term	$U(\hat{\phi}) = \frac{1}{\breve{\alpha}\phi^\dagger\phi}[8\pi V^*(\phi^\dagger\phi)]$
u	Up-quark (flavor)	GSW: $\psi_L{}^{q=1} = u_L$
u	Potential-term mixture	$u = \lambda + \nu$
u	Energy density of em fields	$u = \frac{1}{2}\left[\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right]$
u	Reciprocate distance	$u = 1/r$
		Newtonian Kepler orbit: $u_0 = \frac{\tilde{r}_S}{2C_b^2}c^2(1 + \varepsilon\cos\varphi)$
		Second-order approximation: u_1
u	Up-quark (flavor)	GSW: $\psi_L{}^{q=1} = u_L$
u_i	Eigenvector	$f(u_i) = \lambda_i u_i, \quad f \in L(U,U),\ n:$ dimension of U
u^μ	4-velocity	$u^\mu = \frac{dx^\mu}{ds}$
		$u_0 = \left[e^{-\nu} - \left(\frac{v_1}{c^2}\right)^2 e^{-\lambda}\right]^{-1/2}$
		$u_1 = u_0\frac{v_1}{c}$
		Orthonormality: $u^\mu u_\mu = 1$

Symbol	Name / Description	Definition / Example
V	Potential	Coulomb: $V \sim 1/r$
		Yukawa: $V \sim \frac{e^{-r/l}}{r}$
		Higgs: $V(\xi) = \frac{3}{2}\frac{\mu^4}{\lambda}\xi^2 + \frac{\Lambda_0}{8\pi G_0}$
		$V^*(\phi) = V(\phi) + V_0$
		Constant term: $V_0 = -\frac{3\breve{\alpha}\mu^2}{4\pi\lambda}\Lambda_0$
		Eff. RN-like: $V_{eff} = \frac{\varepsilon \tilde{M} G_N}{r} + \frac{L^2}{2r^2} - \frac{\tilde{M} G_N L^2}{r^3}$
		Formal constant term: $\breve{V} = \frac{3}{2}\frac{\mu^4}{\lambda}$
$\mathcal{V}$	Potential energy density	$V = \int \mathcal{V}\sqrt{-g}d^3x$
v	Ground-state (VE) value	exp: $v \approx 6.07 \cdot 10^4$ (GeV)^2
		$v = \pm\sqrt{-\frac{6\mu^2}{\lambda}}$
v_1, v	Velocity	$u_1 = u_0\frac{v_1}{c}$
v_t	Tangential velocity	$v_t = \sqrt{r\frac{d\Phi}{dr}}$
w	EOS parameter	Barotropic: $p = w\varrho$
		DM equation-of-state: $\hat{w} = \frac{p}{\hat{\epsilon}}$
		SF equation-of-state: $w_\xi = \frac{p}{\epsilon_\xi}$
w	Potential-term mixture	$w = \lambda - \nu$
w_P	Polytropic amplitude	$p = w_P\epsilon^\gamma$
$X(A, B; r^{-n})$	RN 2^{nd}-order correction	$X = \frac{1}{2r^2} + \frac{4K}{3r^3} + \frac{39K^2}{6r^4} + \frac{3\tilde{Q}^2}{4r^4} + \ldots$
$X_1(A, B; r^{-n})$	RN 2^{nd}-order correction	$X_1 = X\left[2\left(1 - \frac{K}{r}\right)^{-1} + \tilde{Q}^2 X\right]$
$X_2(A, B; r^{-n})$	RN 2^{nd}-order correction	$X_2 = X_1 + X_{2a}$
$X_{2a}(A, B; r^{-n})$	RN 2^{nd}-order correction	$X_{2a} = \frac{1}{r^2}\frac{1 - \frac{K}{r} - \frac{\tilde{Q}^2}{r^2}}{\left(1 - \frac{K}{r} + \frac{\tilde{Q}^2}{r^2}\right)^2}\left[1 + \frac{\tilde{Q}^2}{2r^4}\left(\frac{1 - \frac{K}{r} - \frac{\tilde{Q}^2}{r^2}}{1 - \frac{K}{r} + \frac{\tilde{Q}^2}{r^2}}\right)\right]^{-1}$
x	(Cartesian) Coordinate	$x^\mu = x^0 = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z$
$\hat{x}$	Yukawa matrix	$\mathcal{L}_Y = -k_f(\bar{\psi}\phi^\dagger\hat{x}\psi + \bar{\psi}\hat{x}\phi\psi)$
Y	Hypercharge	U(1) strength: $\mathcal{F}_{(1)\mu\nu} = F_{(1)\mu\nu}Y$
		Gell-Mann–Nishijima (without quarkness): $Q = T^3 + \frac{1}{2}Y$
Z	Partition function	$Z_{AHM} = \int \mathcal{D}\tilde{C}_\mu \mathcal{D}\Phi e^{-S_{AHM}[\tilde{C}_\mu, \Phi]}$
Z^μ	Neutral weakon	$Z^\mu = W_3{}^\mu \cos\vartheta_W + A^\mu \sin\vartheta_W$
$\mathbb{Z}^2$	Dual symmetry group	
z	Redshift	$z = \frac{\lambda_0 - \lambda_{emission}}{\lambda_{emission}}$

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Support and publications

This work was supported by the Graduate School of Mathematical Analysis of Evolution, Information and Complexity of the University of Ulm as well as by the Research Group for Cosmology and Quantum Gravitation of the Institute of Theoretical Physics of the same higher-studies institution, under direction of Prof. Dr. Frank Steiner and with special collaboration of Dr. Hemwati Nandan of the Centre of Theoretical Physics in New Delhi and Prof. Dr. Heinz Dehnen of the Department of Physics of the University of Konstanz as well as of Dr. Umananda Dev Goswami of the University of Dibrugarh in India. The following articles were published during the realization of this work:

(i) [21]: N.M. Bezares-Roder, H. Nandan and H. Dehnen; "Horizon-less Spherically Symmetric Solutions in a Higgs Scalar-Tensor Theory of Gravity", *International Journal of Theoretical Physics* ***46(10)***, 2429–2436 (**2007**), DOI 10.1007/s10773-007-9359-5; Ulm Report-TP/07-8.
Pre-Print: arXiv:gr-qc/0609125.

(ii) [22]: N.M. Bezares-Roder and H. Nandan; "Spontaneous Symmetry Breakdown and Critical Perspectives of Higgs Mechanism", *Indian Journal of Physis* ***82(1)***, 69–93 (**2008**); Ulm Report-TP/08-8.
Pre-Print: arXiv:hep-ph/0603168.

(iii) [110]: U.D. Goswami, H. Nandan, C.P. Pandey and N.M. Bezares-Roder; "Maxwell's Equations, Electromagnetic Waves and Magnetic Charges", *Physics Education* ***25(4)*** (**2008**) 251–265.

(iv) [176]: H. Nandan, N.M. Bezares-Roder and H.C. Chandola; "Dual Meissner Effect and QCD Vacuum" in *Proceedings of the LIII DAE-BRNS Symposium on Nuclear Physics* ***53*** (**2008**) 583, Ulm Report-TP/08-9.

(v) [178]: H. Nandan, N.M. Bezares-Roder and H.C. Chandola; "Screening Current and Dielectric Parameters in Dual QCD", *Indian Journal of Pure and Applied Physics* ***47(11)*** (**2009**) 808-814.

(vi) [24]: N.M. Bezares-Roder and F. Steiner; "A Scalar-Tensor Theory of Gravity with a Higgs Potential", in *Mathematical Analysis of Evolution, Information and Complexity* (eds. W. Arendt and W. Schleich), Wiley-VCH **2009**. ISBN-10:3527408304 and ISBN-13:9783527408306.

(vii) [111]: U.D. Goswami, H. Nandan, C.P. Pandey and N.M. Bezares-Roder; "Covariant Formalism of Maxwell's Equations and Related Aspects", *Physics Education* ***26(4)*** (**2009**) 269–278.

Further:

- [179]: "Black Hole Solutions and Pressure Terms in Induced Gravity with Higgs Potential" by H. Nandan, N.M. Bezares-Roder (corresponding author) and H. Dehnen was submitted for publication. Pre-Print: arXiv:0912.4036 [gr-qc].

- [23]: “Scalar-Field Pressure in Induced Gravity with Higgs Potential and Dark Matter” by N.M. Bezares-Roder, H. Nandan and H. Dehnen was submitted for publication.
  Pre-Print: arXiv:0912.4039 [gr-qc].

Another related article fulfilled after Diploma studies, however published during the realization of this work reads

- [20]: N.M. Bezares-Roder and H. Dehnen; ”Higgs Scalar-Tensor Theory for Gravitation and the Flat Rotation Curves of Spiral Galaxies”, *General Relativity and Gravitation (GRG)* ***39****(8)*, 1259–1277 (**2007**). DOI 10.1007/s10714-007-0449-8. Ulm Report-TP/06/4.
  Pre-Print: arXiv:0801.4842 [gr-qc].

Acknowledgements

“The grounding of every disclosure of the Being (*l’être, das Sein*) is freedom [...] Cognizance may exist only in the amount there exists freedom. There exists freedom because every action is defined by the possibility of its opposite [...] Recognition is to bring what is to the light, to act and search on the margin of mistakes, rejecting in that way ignorance and lies related to predisposition. And truth is this progressive disclosure, even though truth itself may be relative to the epoch in which it is achieved.”
– J.-P. Sartre, *Wahrheit und Existenz.*

I am very grateful to Prof. Steiner for the possibility of working within his group and carry out this work under his supervision and kindest confidence and interest in fundamental research and scientific freedom and their value. I am very grateful to the Institute of Theoretical Physics of the University of Ulm as well as to the Graduate School of Mathematical Analysis of Evolution, Information and Complexity for the support throughout this time. I am thankful to Prof. Balser for accepting refereeing this work.
I also declare my deepest gratitude to Prof. Dehnen of the University of Konstanz for his kind support, and especially to Dr. Nandan of the Centre of Theoretical Physics in New Delhi for pulling me up in difficult times and pushing me to new ideas. I am thankful to Dr. Goswami of the University of Dibrugarh in India for some joint work and confidence in my work.
I want to thank the people of the institute for their support, and especially Dr. Lustig for going through part of this long and surely potentially boring work.
And at last, but in no way at least, I thank with all my heart Lizbeth and my family for their tenderness which has been so important for me and for bearing and supporting me all this time. My parents and my sister, together with my brother in-law and niece/goddaughter Sophia Elisabeth Rutschmann Bezares, deserve special reference, for they have encouraged me to all I am today (good and bad) and will always be part of the same. All my gratitude to the Aceves Nafz family which has always been part of mine, but also to all my friends which have stood by my side in some moment or the other or which have let me do the same. I am happy that our ways crossed and hope that they keep on doing so.

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Ulm, den 22. März 2010

(Nils M. Bezares Roder)

Scholar Curriculum

Personal Data

Surname, Given Name	Bezares Roder, Nils Manuel
Date of Birth	January 20^{th}, 1980
Place of Birth	Mexico City, Mexico

Scholar Data

Date	June 1999
Studies	**Deutsche Allgemeine Hochschulreife** (German General Matriculation Standard)
Institution	**Colegio Alemán Alexander von Humboldt Xochimilco** (German School, Mexico City)
Date	August 1999
Studies	**Bachillerato del Colegio de Ciencias y Humanidades** (Mexican Matriculation Standard for Natural and Human Sciences)
Institution	**Colegio Alemán Alexander von Humboldt Xochimilco / Universidad Nacional Autónoma de México**
Date	October 1999 – August 2005
Degree	**Diplom-Physiker (Dipl.-Phys.)** (Diploma in Physics)
Institution	**Universität Konstanz**
Advisor of Diploma Thesis	Prof. em. Dr. H. Dehnen, Fachbereich Physik, Universität Konstanz
Co-Advisor	PD Dr. B. Fauser, Max-Planck-Institut für die Mathematik in den Naturwissenschaften
Date	August 2004 – July 2006
Affiliation	**Research Group for Gravitational Physics**
Institution	Fachbereich Physik, Universität Konstanz
Date	August 2006 – Dato
Studies	**Ph.D. studies (Promotionsstudium Dr. rer. nat.)**
Institution	Fakultät für Naturwissenschaften, Universität Ulm
Advisor of Thesis	Prof. Dr. F. Steiner, Institut für Theoretische Physik, Universität Ulm & Centre de Recherche Astrophysique, Université Lyon 1, CNRS
Co-Advisor	Prof. Dr. W. Balser, Intsitut für Angewandte Mathematik, Universität Ulm
Fellowship	Graduate School for Mathematical Analysis of Evolution, Information and Complexity