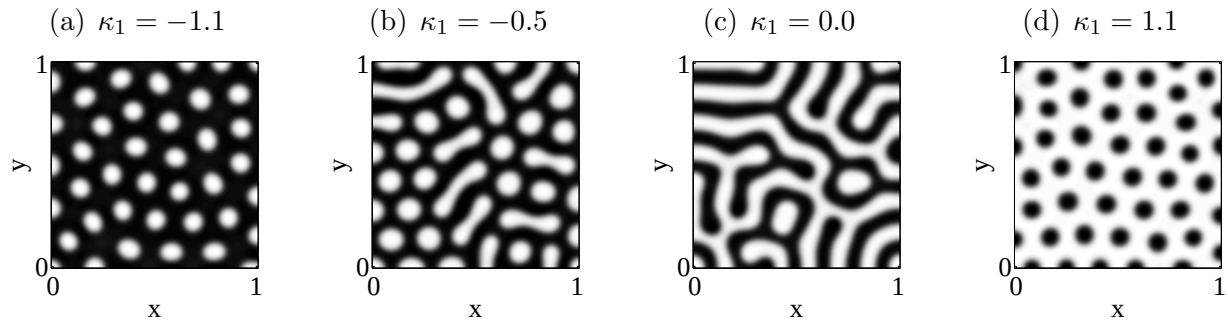


FIG. 6.11: Stationary Turing patterns (the activator distribution is shown) appearing for different values of the control parameter $\kappa_1$ at $\lambda = 0.5$: (a) hexagonal pattern; (b) mixed pattern; (c) labyrinth pattern; (d) dark hexagonal pattern. Parameters: $D_u = 6 \cdot 10^{-5}$, $D_v = 6 \cdot 10^{-3}$, $D_w = 6 \cdot 10^{-2}$, $\lambda = 0.5$, $\kappa_3 = 1$, $\kappa_4 = 3.33$, $\tau = 1$, $\theta = 1$, $\Omega = [0, 1] \times [0, 1]$. Boundary conditions are Neumann.

The third simulation (see Fig. 6.12) is performed for relatively large value of $\lambda$, $\lambda = 1.8$ (further increase of $\lambda$ leads to transition from excitable to bistable region). In this case beyond the threshold initial condition ($t = 0$) evolves to several not periodic in space high-amplitude spots ($t = 100$). The latter are not stable against shape deformations ($t = 200$, $t = 500$) and finally stationary labyrinth-like pattern emerges ($t = 3000$). Notice, that in this case the bifurcation is subcritical, i.e., the amplitude changes not continuously but jump up to some constant value. Moreover, the pattern, obtained for ,e.g., $t = 100$ is not periodic in space, showing the role of the second inhibitor as a local feedback.

Let us remark here that generic Turing structures presented above, like hexagons, stripe and labyrinth-like patterns are "old" structures, observed also in two-component RD systems (with an integral term respectively). Sometimes quite fascinating secondary combination of these objects, specific for multicomponent RD systems can be observed. We only mention so-called "twinkling eyes", i.e., Turing spots, arranged as a hexagonal lattice [Yang et al., 2004]. The structures of this kind are usually a product of interplay of Turing and Hopf bifurcations and is out of scope of the present manuscript.

We are now in a good position to state that a principally new problem posing arises when instead of a regular pattern one tries to investigate one or several localized objects that move and interact with each other, like, e.g., experimentally observed current filaments in a plane discharge [Purwins et al., 2005]. This problem will be investigated more closely in the next chapter.

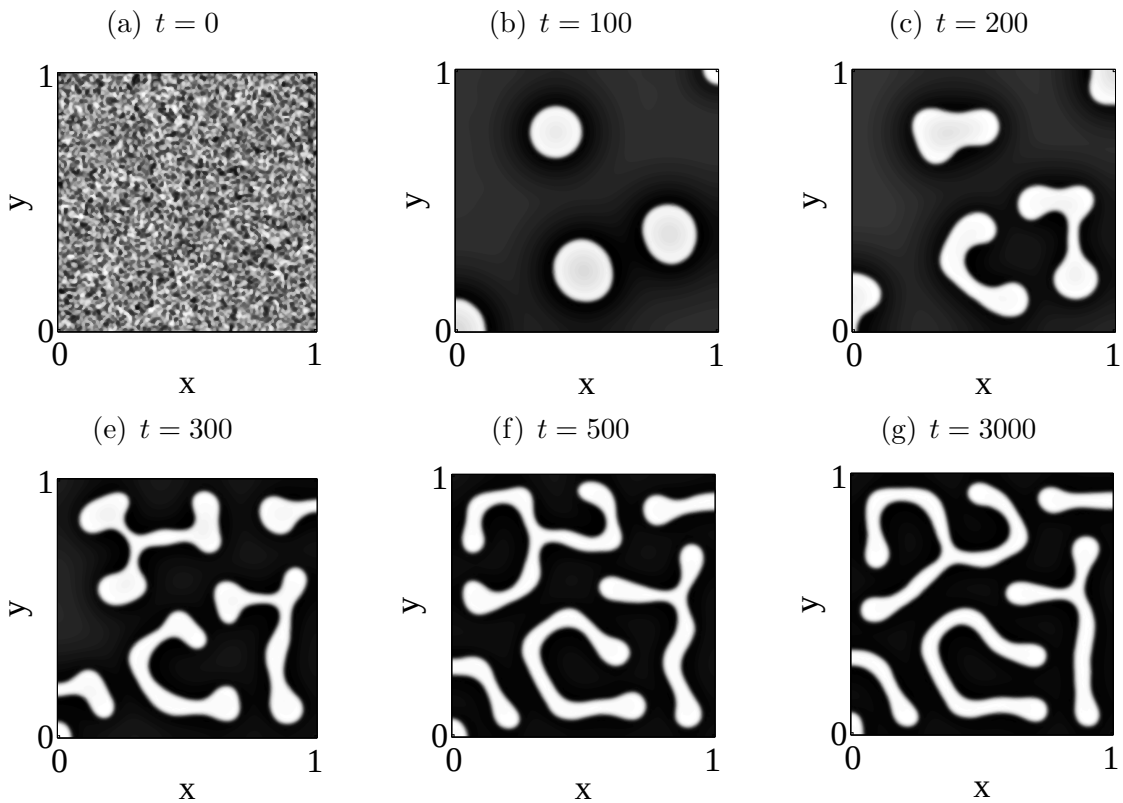


FIG. 6.12: A labyrinth pattern emerged from the noisy homogeneous steady state in a subcritical regime. The plots show the activator distribution at six different times. Parameters: $D_u = 6 \cdot 10^{-5}$, $D_v = 6 \cdot 10^{-3}$, $D_w = 6 \cdot 10^{-2}$, $\lambda = 1.8$, $\kappa_3 = 1$, $\kappa_4 = 3.33$, $\tau = 1$, $\theta = 1.0$, $\kappa_1 = -2.1$, $\Omega = [0,1] \times [0,1]$. Boundary conditions are Neumann.