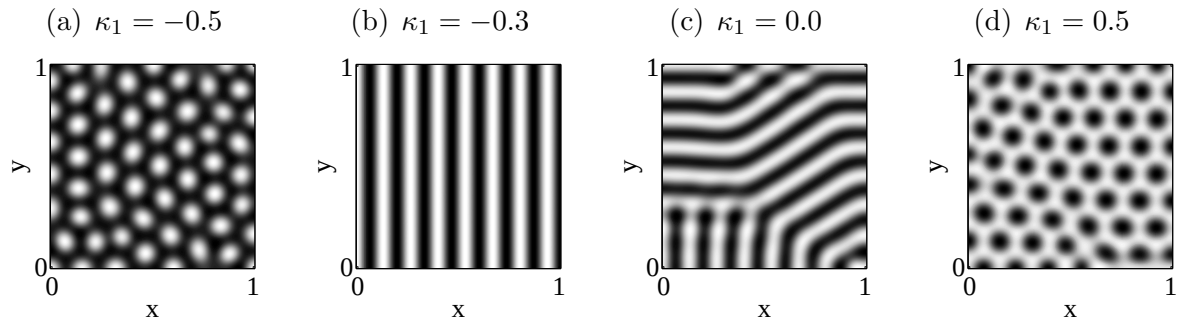

FIG. 6.10: Stationary Turing patterns appearing for different values of the control parameter κ_1 at $\lambda = 0.3$: (a) hexagonal pattern; (b) stripes; (c) stripe-like pattern; (d) dark hexagons. Parameters: $D_u = 6 \cdot 10^{-5}$, $D_v = 6 \cdot 10^{-3}$, $D_w = 6 \cdot 10^{-2}$, $\lambda = 0.3$, $\kappa_3 = 1$, $\kappa_4 = 8.0$, $\tau = 1$, $\theta = 4.0$, $\Omega = [0,1] \times [0,1]$. Boundary conditions are Neumann.

whereas κ_1 is responsible for different patterns inside each simulation. In order to better understand the role of these control parameters we have performed numerical simulations of the system (7.1) for several values of λ, changing in each simulation κ_1 in some interval and keeping other parameters fixed. Notice that Eqs. (7.1) possesses no inversion symmetry. It implies that near the threshold κ_{1c} (or $-\kappa_{1c}$) hexagonal patterns have the maximum growth rate and are preferred towards, e.g., stripes (see Sec. 6.1).

The results of simulations are summarized in Figs. 6.10, 6.11, 6.12. Figure 6.10 shows the stationary activator distribution, calculated for $\lambda = 0.3$. Beyond the threshold $-\kappa_{1c} = -0.51$ the initial conditions in form of random perturbed homogeneous solution results in stationary hexagonal pattern, shown in Fig. 6.10 for $\kappa_1 = -0.5$. By increasing of κ_1 hexagons become unstable and stationary stripe pattern appears ($\kappa_1 = -0.3$). Further increase of κ_1 leads to stripe-like pattern ($\kappa_1 = 0$) and finally to dark hexagonal pattern ($\kappa_1 = 0.5$). Subsequently, after the threshold $\kappa_{1c} = 0.51$ the system converges to a homogeneous stable state.

A similar bifurcation scenario can be observed in the next simulation, performed for $\lambda = 0.5$ (see Fig. 6.11). Here beyond the threshold $-\kappa_{1c} = -1.12$ starting again with random perturbed homogeneous solution one obtains stationary hexagonal pattern ($\kappa_1 = -1.1$). But in contrast to the case $\lambda = 0.3$, further increase of κ_1 leads not to stripes, but to some mixed state, consisting of stable hexagonal structures with stripe-like defects (see Fig. 6.11 for $\kappa_1 = -0.5$). The latter proceeds to labyrinth-like pattern ($\kappa_1 = 0$) and finally dark hexagonal pattern emerges ($\kappa_1 = 1.1$). Again, after the threshold $\kappa_{1c} = 1.12$ the system has a stable uniform state as a solution. Notice that in both cases the corresponding bifurcations have clearly supercritical character. Moreover, it is visible to the naked eye that the amplitude of the structure in the second case ($\lambda = 0.5$) is larger than for $\lambda = 0.3$.